Expert and Novice student use of computer simulations:

Fourier: Making Waves

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Background

I. Physics Education Research

The field of physics education research has a few primary objectives: to study how physics is learned, to support research-based instructional practices, and to assess how to teach physics in such a way that promotes the development of students learning physics[1]. Discoveries in this field demonstrate that there is a need to revisit the way physics, and science in general, is taught to match modern theories and empirical findings on how students learn. Further, the field has started to attend to student and popular perceptions about the nature of physics. Research in physics education has shown that conventional methods for teaching physics have failed to convey physics concepts in a meaningful way[2]. Traditional "talk and chalk" lectures support only a small percentage of students in learning physics. In general, in traditional teaching environments students find physics concepts inaccessible and/or irrelevant[3]. The challenge is that science isn't just for scientists anymore - the modern move is toward "science for all"[4]. However, in the realm of education, the gap between experts (instructors) and novices (students) presents a challenge. Especially at the university level, professors are often unaware of basic students' confusions and frustrations[5]. Physics education researchers are dedicated to bridging this gap through innovative teaching methods and tools that reflect what has been discovered about how students learn physics.

II. Constructivism and Constructionism

Learning by creating a representation is not a new concept. Analogies to common occurrences or memory tricks, such as mnemonic devices, are everyday tools used for learning a new concept, memorizing the order of the planets, or remembering what you need from the grocery store. Cognitively the notion of making connections to preexisting knowledge and using those connections to productively create a response to presented information is known as constructivism[6]. The constructivism principle is central to long-term memory and recall, and therefore an essential piece of the physics learning puzzle. When physics concepts can be connected and applied to students' preexisting knowledge of the world, the new knowledge is embedded in their long term memory rather than using short term memory to recall representations when an appropriate looking equation pops up[7].

Taking this one step further, constructionism[8], like constructivism, is centered on the notion of making cognitive connections via representations. Constructionism, however, is based on more tangible or "doable" representations. Constructionism supports that learning by doing or creating is particularly effective for learning. When students learn through creating their own projects, they are personally involved with what they are doing. Fun or intrigue makes a concept more accessible and students acquire some of the traditional skills indirectly through their exploration. Constructionism situates learning as a byproduct of something else the students enjoyed doing[8].

III. Computer Based Learning and the PhET Project[9]

Physics education researchers, in their attempts to bring physics education techniques into the 21st century, have found a promising frontier in the realm of computer-based learning. New computer technologies have made it possible to teach physics ideas in novel ways. Computer based simulations (sims) have proven to be powerful tools for not only effectively developing students' conceptual understanding, but also for supplementing and enhancing lecture material and promoting interactive learning environments. Computer sims allow intructors to eliminate some of the distractions students encounter in conventional representations; they can be specifically tailored to integrate qualitative and quantitative features of representations. Likewise, since physics concepts are often very complex, representations for these concepts can be comparably complicated. For example, the concept of an electromagnetic wave is quite complex, but is taught in introductory physics. Consider two representations of a wave- one with a sine function, and another representation by creating a wave on a string. Both representations "show" the same thing. A student sees amplitude, wavelength, and all the physical characteristics of a wave in both representations. However, a sine wave on a piece of paper situated on a coordinate system may represent something purely mathematical, whereas the tangible wave on a string may better represent something physically happening. This may cause the mathematical and physical concepts surrounding waves to remain separate for the student. A computer simulation on waves can show the mathematical elements of waveforms while simultaneously showing a physical model. Computer based sims also give students a means by which to visualize the "invisible", give instant feedback and gratification to the student and provide an overall means by which students can accessibly interact with physics concepts**.** [10]

The Physics Education Technology project (PhET) at the University of Colorado has developed many computer simulations and studied their effectiveness as learning tools. Sims exist for a plethora of topics in introductory physics as well as topics in higher-level physics, chemistry, biology, and other sciences. Each sim is designed independently as different concepts require unique presentations to be effective. PhET sims invoke constructionist and constructivist theory in efforts to make the underlying physical concepts understandable and accessible to students. Sims are designed to be interactive and engaging, to give the students constructive visual representations, to make implicit physical characteristics explicit, and to allow students to explore physical phenomena with constructive guides and constraints but with minimal direct instruction. The effectiveness of PhET sims has been researched in the scope of lectures, labs, recitation/homework, and in general informal settings. Studies show that how a simulation is used affects how much students can learn from it. For example the Circuit Construction Kit simulation is extremely effective in electronics labs[9,10]. Students get the "hands on" feel of constructing a circuit as they build it component by component. However they are not distracted by irrelevant details such as wire color and the physical size of a resistor. They also can see what is going on microscopically as the sim shows electrons moving through the wire, slowing down across resistors, and storing up in capacitors, instead of just seeing the end result of a light going on. Additionally they don't have to deal with the confusions of components not working or shorting out. Other sims are extremely useful in recitation/homework sessions. Students are guided through a sim by way of a worksheet. They may be told what do with a function on a simulation, but then are left to interpret what happens on their own or with a group. In comparison to reading about or looking at diagrams of a particular phenomenon, students interact with the sim and experience the cause and effect relationships directly. When asked to explain not just what they observed but how a certain mechanism caused it, they develop a deeper understanding of the concept. Overall, the sims transcend the "what" part of learning and allow students to internalize the more significant "how" and "why". [9]

The Simulation

I. Overall Simulation

Fourier: Making Waves has three application tabs within the simulation: Discrete, Wave Game, Continuous. The simulation provides a variety of quantitative and qualitative tools to help students learn and understand the concept of Fourier decomposition and concepts relating to it. Within the simulation are built-in instructions for use. The "drag me" or "match the pink wave function" commands (discussed later in this section) give the students minimal guide as they use the sim. There is a help tab that offers more instructions, but in general students only experience these two commands. The guides give the students a jumping off point to begin exploring the sim. The learning goals provided by the PhET team for this simulation are:

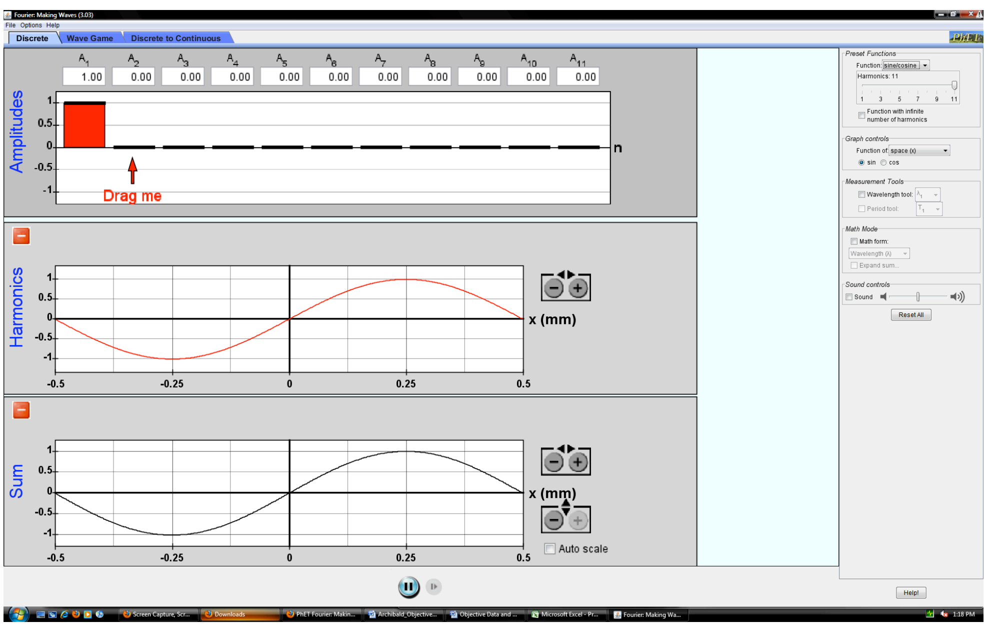
- Explain qualitatively how sines and cosines add up to produce arbitrary periodic functions.
- Recognize that each Fourier component corresponds to a sinusoidal wave with a different wavelength or period.
- Mentally map simple functions between Fourier space and real space.
- Describe sounds in terms of sinusoidal waves.
- Describe the difference between waves in space and waves in time.
- Recognize that wavelength and period do not correspond to specific points on the graph but indicate the length/time between two consecutive troughs, peaks, or any other corresponding points.
- Become comfortable with various mathematical notations for writing Fourier transforms, and relate the mathematics to an intuitive picture of wave forms.
- Determine which aspect of a graph of a wave is described by each of the symbols lambda, T, k, omega, and n.
- Recognize that lambda & T and k & omega are analogous, but not the same.
- Translate an equation from summation notation to extended notation.
- Recognize that the width of a wave packet in position space is inversely related to the width of a wave packet in Fourier space.
- Explain how the Heisenberg Uncertainty principle results from the properties of waves.
- Recognize that the spacing between Fourier components is inversely related to the spacing

between wave packets, and that a continuous distribution of Fourier components leads to a single wave packet.

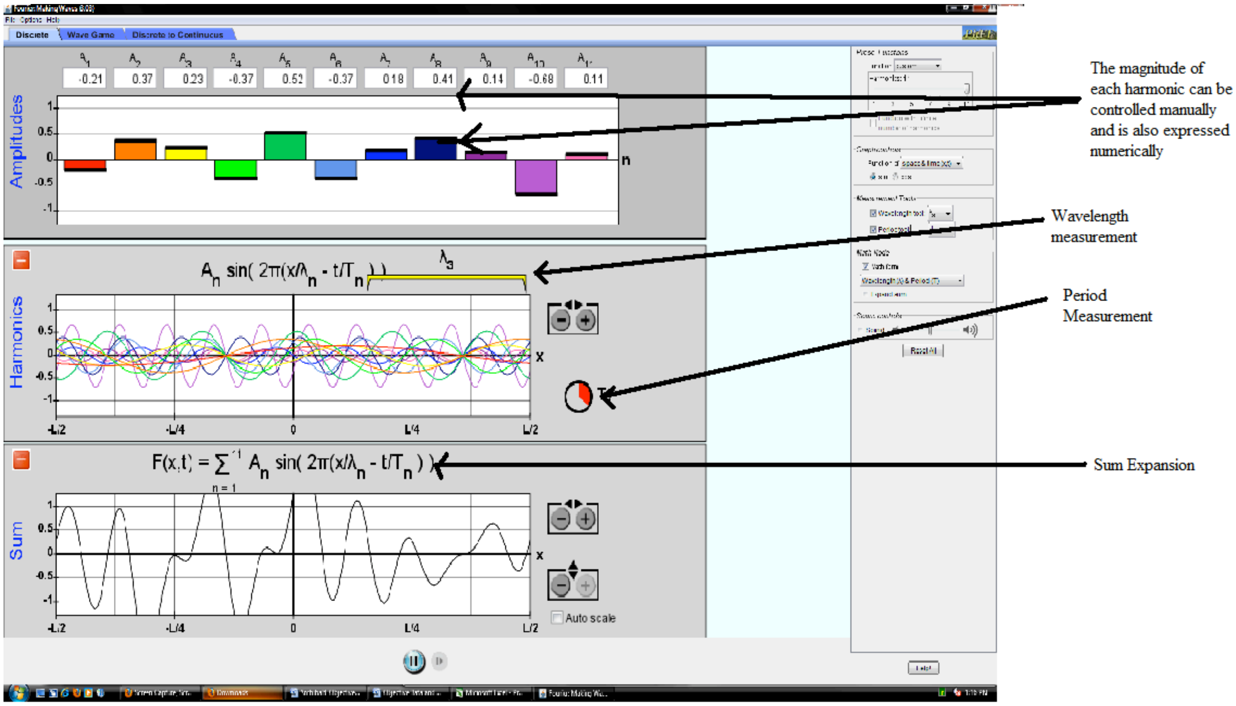
In this experiment, only the Discrete and Wave Game applications are examined.

II. The Discrete Tab

In the "Discrete" application on the simulation there is a wide variety of quantitative functions that allow students to explore the phenomena of adding waves more thoroughly. (Img. 1) Functions include amplitude adjusters for each of the harmonic waveforms, wavelength measurement, period measurement, scale adjustment, changing from sine to cosine, space/time/space and time coordinate options, expression for the summation expansion, and more. When the application is opened, initially it directs the student to "drag here" on the amplitude adjustment function. The student drags a bar up/down vertically and the corresponding harmonic's amplitude adjusts to this magnitude. There are 11 harmonics to choose from, and users can create a variety of waveforms (Img. 2)



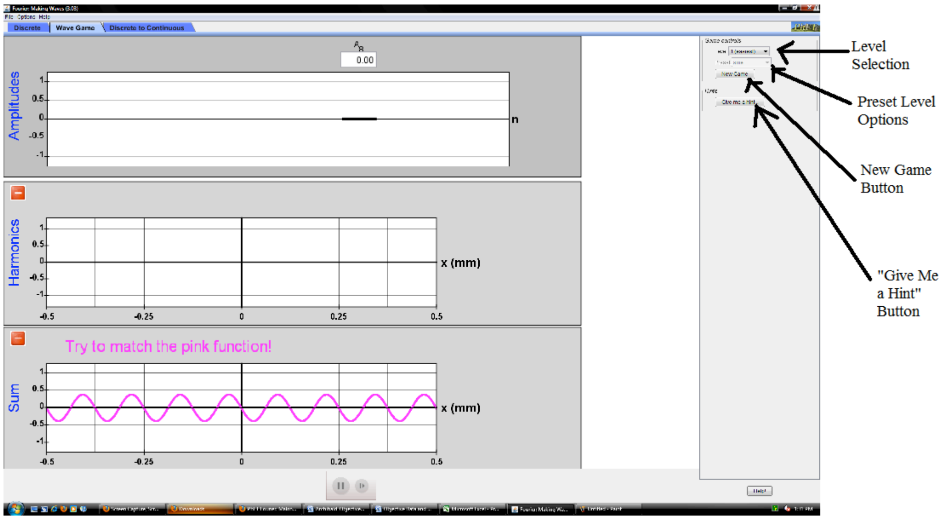
Img1. This screen is what appears upon opening the simulation. Users can vary the magnitude of each harmonic (1-11, left to right) in the top panel. Each harmonic in waveform is shown in the middle, and the sum of the harmonics at the bottom. The function controls are in the control panel on the right. .



Img2. This image shows the many quantitative functions offered on the Discrete tab being utilized

III. Wave Game

The Wave Game is designed to allow students to explore increasingly complex wave forms as they progress through 10 levels of a game. (Img. 3) The game instructs students to "match the pink function". On level one (Img. 4), students are given one harmonic amplitude to adjust to match the wave. On level two students are given 2 harmonics and must choose the correct one to match the wave. On level 3 all eleven harmonics are given, students must choose the correct one. Level 4 is the first one that gives a wave that is made of more than one harmonic. Students are given 2 harmonics to adjust to match the function. Level five gives all eleven harmonics, students must choose the correct 2 harmonics to match the wave. This pattern continues through the levels: level 6 gives 3 harmonics to adjust and all three are used to match, level seven is given all 11 harmonics, choose the correct 3, level 8 gives 4 harmonics and all four must be used, level 9 is given all 11 harmonics, choose the correct 4, and level 10 gives all 11 harmonics and the wave is a sum of all 11. There is also a level called "preset" which gives a rough square, saw tooth, or triangle wave the student must match. Also in the game are hints. When the student clicks "give me a hint" it informs the user how many harmonics are needed to match the function. “Give me another hint” will inform which specific harmonics are needed. “Cheat”, the third hint, will give the student the exact value of the amplitude of each harmonic.



Img3. The opening screen of the wave game is shown here. The vertical 'Amplitude', 'Harmonic', and 'Sum' screens are displayed identically to the Discrete tab. Similarly, the controls are all on the right panel, though there are far fewer.

Study

I. Motivation

The intent of this project is to investigate patterns of student usage surrounding one particular PhET sim. *Fourier: Making Waves* consists of a mostly qualitative approach to the subjects of wave superposition and Fourier decomposition. The theory underlying Fourier decomposition involves rigorous and complicated mathematics. Populations of varying mathematical and scientific backgrounds were observed to determine if their knowledge of the theory effected the students' reactions to and/or use of the simulation.

II. Research Questions

The questions motivating this research include:

- What are the *differences* in patterns of interaction with the sim between the expert and inexperienced populations?

- What are, or are there, *similarities* in usage and interaction with the sim across both populations?
- Is there a correlation between mathematical/scientific background and *enjoyment* of the simulation?

III. Methodology

a.) Populations

Subjects were categorized as expert or novice based on the following criteria: the novice population (N=3) includes freshman or sophomore level science majors. Subjects in this population had mathematical backgrounds up to and including third year calculus and no higher than introductory physics and chemistry. Novice subjects had no prior knowledge of Fourier decomposition theory and had never seen the “Making Waves” simulation, though all subjects expressed moderate familiarity and comfort with basic waveforms. The expert population (N=3) included upper division physics undergraduate students. Each subject had taken courses that introduced Fourier decomposition and its applications. One subject had taken a course specifically in Fourier analysis. All the subjects had seen the sim used as an example in class, though none had used it themselves. All expert subjects expressed that they had merely seen the sim used once in the past and felt that they were unfamiliar with the workings of the sim.

b.) Research Protocol

Data for this project were accumulated via one-on-one interviews and videotaped interaction with the simulation. Each interview lasted about 40 minutes. The interviews began with preliminary questions about each subject’s background in math and science. Subjects were informed that the nature of the study was to investigate the effectiveness of the sim as a learning and teaching tool. Novice subjects were given the initial question to consider: “can any continuous function be represented as a combination of sines and cosines only?” The expert subjects were also asked to offer any knowledge they had regarding Fourier decomposition; responses varied, but all included the idea that any continuous function could be described by a sum over sines and cosines, negating the need for an initial question. Each subject was then video taped as he or she used the sim and was encouraged to express their thinking out loud as they explored the sim (T~20minutes). Subjects were informed to use only the “discrete” and "wave game" tabs of the simulation. Though students were not given any external instructions for using the sim, the instructions within the framework of the simulation discussed in the previous section do provide guidance; therefore, students were minimally guided as they used the sim. While taping students were asked questions regarding particular choices they made; those being more case specific will be discussed in the data section.

After each subject used the simulation he or she was asked a series of follow up questions. Both populations were asked a non-pointed question about their opinions of the sim in general, then were asked to hone in on specific likes and dislikes and any difficulties or frustrations they encountered. Finally each group was asked content specific questions. Expert subjects were asked to assess how the sim affected their confidence in the concept in general and the underlying mathematics and if they felt it would be a useful tool in the future. Novice subjects were asked more basically to describe what they noticed about wave interactions after using the sim. In addition to what the answer to the question was, I also noted *how* each subject answered the questions asked. How they answer offers insight into what they learned in the sim or to what extent they understood what they saw. All subjects were also asked if there was anything they would change about the sim and if they felt they could use it as a tool to explain the concepts to others. A sample protocol is included in Appendix A.

c.) Data Analysis:

Video-recordings were examined to look specifically for exclamatory and emotional commentary; over-all trends of simulation use including: starting place, switching between functions, total time spent of each function, use of qualitative and quantitative features of the simulation; and trends of use in the wave game including: highest level reached, progression through levels, time spent on each level, and emotional response to the game overall. Follow up questions during the interview assessed the subjects impression of the simulation overall, particular likes and dislikes, and frustrations. Responses to these questions were used to assess connections between user opinions of the sim by population. Responses to these questions also supplemented video observations regarding user enjoyment.

d.) Data summary

The data for this study include quantitative statistics taken from the videos of each subject using the sim, my interpretation of emotional responses from reviewing the videos coupled with commentary of the subjects regarding certain responses, answers to the survey questions, and general observations I made during each interview. Collected data was divided into two categories: quantitative and qualitative. The quantitative data primarily examines the wave game and its levels. It includes time counts, levels attempted and succeeded, etc. The qualitative section focuses on population dependent and independent usage trends, special cases of unique instances, and user feedback. Analyses of data are coupled with the presentation of the raw data.

Quantitative Data and Analysis

I. Time Spent on Each Application Tab

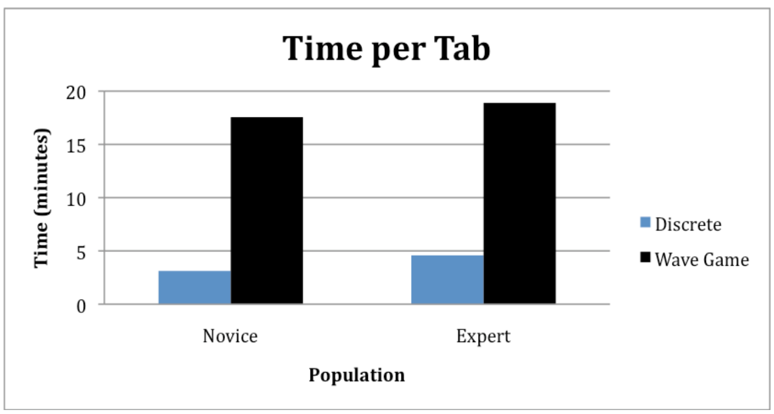


Fig.1: Time per Tab. Novice Population: Discrete Tab average time=3.1min; Wave Game average time=17.6min. Expert Population: Discrete Tab average time=4.6min; Wave Game average time=18.9min

In general, all students spent about 80% of their time on the wave game regardless of where they started, how often they switched back and forth between tabs, or how they used the simulation individually. This shows that though the Discrete application simulation offers more to do in terms of different functions, buttons to push, etc, the Wave Game application held the attention of students significantly longer.

II. Functions Used:

When using the sim, students either explored all the functions, or only the amplitude adjustment function. 2 experts and 1 novice used all of the functions while 1 expert and 2 novices used only the amplitude adjustment. Observations show that the use of these functions varied more by individual use rather than by trends in each population, these observations will be discussed further in the Qualitative Data section.

III. Time Spent on Each Level:

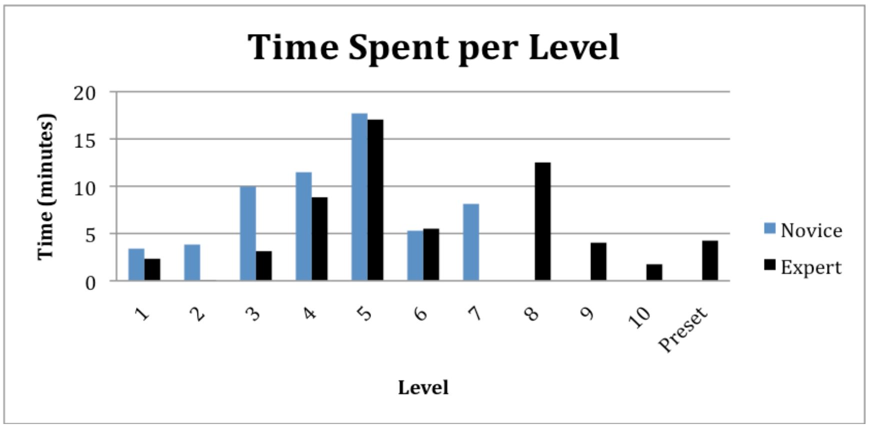


Fig 2. Time Spent per Level. Times represent the total time spent on each level per population. Total time, not average, is shown as not every subject tried every level.

Novice students tended to spend more time on each level compared to the experts. Novices tended to progress linearly through the levels (i.e. level 1 then 2, 3, 4, etc) where as experts would start with level 1 and then explore the higher levels in no particular order. With both populations, as the levels increased, so did the amount of time students spent on them. Level 5 proved to be where all students started to have a more difficult time solving the puzzle. The drop off from time on level 5 to level 6 is most likely because of the structure of the level. Level 5 gives a wave that is a combination of two harmonics but all eleven harmonics are given, where level 6 gives a wave that is a combination of 3 harmonics and gives only the 3 harmonics that are needed to match the function. Novices and experts alike did not notice the pattern in which levels progressed and when all eleven harmonics were offered they looked for solutions using all of them. This seemed to make level 5 more challenging than 6. From level 6 onward, time spent on levels drops over all. On the higher levels students tended to resort to hints and/or give up more quickly than on the lower levels. Students appeared to put forth less concerted effort into these levels because of the difficulty of these puzzles. Though level 5 was clearly a challenge, students completed more levels than on the higher levels. Students spent time on the highest level for less time because the puzzles were less solvable, a result that can be seen in the next section. The break between levels 5/6 and the higher levels maybe the threshold at which the levels move from being hard to too hard.

IV. Games "Won" by level

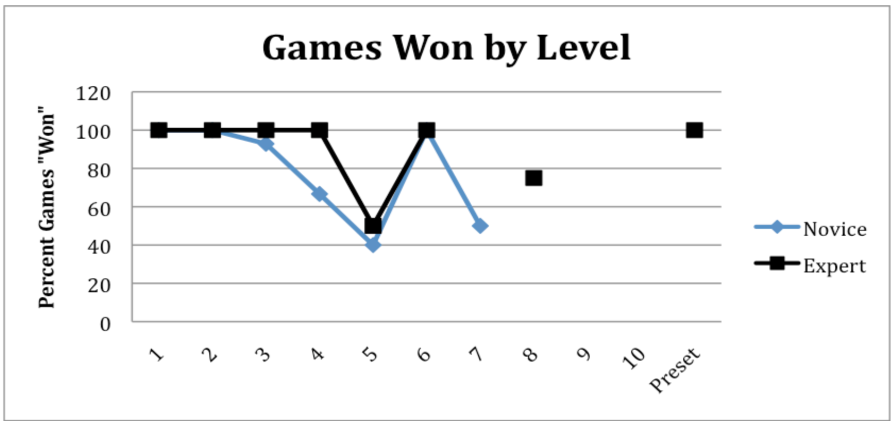
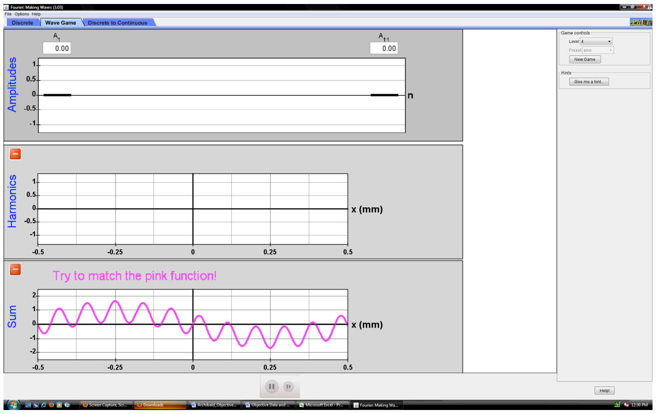
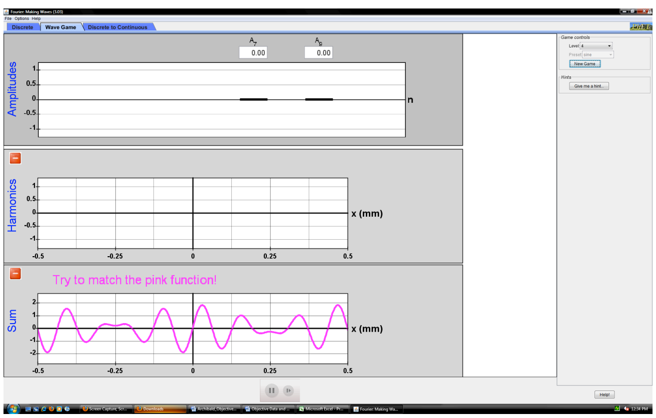


Fig.3: Games Won by Level. This graph shows the percent of successfully completed puzzles per number of attempted puzzles by level for each population.

The game also offers a "new game" that allows students to start on a new puzzle without having to complete the one they are working on. Only puzzles that were actually tried, not passively clicked through, are counted as attempts. A completion counts both successfully completed puzzles and puzzles that were close enough (Often, students had found solutions that appeared correct but not exactly enough for the computer to recognize it). Looking at the success rate for each level, the previously mentioned difficulties with level 5 are more apparent. Experts had no difficulty with levels 1-4, and achieved a 100% completion rate. Novices started to exhibit difficulties with level 3 and 4. Again, level 3 gives all eleven harmonics though the function to match exhibits a single wave and level 4 gives a superpostion of 2 harmonics and gives only the 2 needed to solve it. On level 4 there was a distinct pattern to which "types" of puzzles were not completed. Puzzles where the two harmonics were very different were much easier for novices to solve.



Img4. This image shows what novices chararcherized to be an easy level. Harmonics 1 and 11 are given. Because the harmonics are so different, the characteristics of both waveforms are distinct.



Img5. This image is an example of a hard puzzle on level 4. Harmonics 7 and 9 are given and then characteristics of each harmonic individually are not apparent.

Though the appropriate two harmonics are initially given on level 4, novices were still unable or unwilling to solve some puzzles because the wave looked too complicated. Often novices would opt for a new game until an easier puzzle came up.

Level 5 shows a significant decline in success for both populations. Because the waves in this level are combinations of two harmonics and all eleven harmonics are offered as choices, level five appears to be much harder than four. Also, since all eleven harmonics were given, students attempted to find solutions that utilized all of the harmonics. This quickly gave studnets very complicated waves with only small parts that resembled the function they were given.

Students did not seem to notice only two harmonics were nessecary. 4 out of the 6 students attempted level 5 and all of them ended up relying on the “hint” option. The one expert who used a hint only used the first level hint which informed him that only two harmonics were needed. He noticed quickly that of the 6 harmonics he had adjusted 2 of them were more prominent than the other 4 and easily solved the puzzle. All the novices had to use the second level hint which informed them which two harmonics specifically needed. This is similar to the challenge of level 4. After using the second hint, novices could easily solve the puzzles. Novices who could not complete a level 5 puzzle tended to revert back to level four, this trend will be discussed more in Qualitative data.

From the rate of success shown in Figure 3, it is clear that level 6 was much easier than level 5. Both populations had 100% completion of the attempted puzzles. Because levels 4 and 6 are so similar (and only add one more hamonic), it may be possible to attribute the increase in success by novices from level 4 to level 6 to an increased amount of comfort with the game. Level 6 was the last level attempted by the novice students who tried it, they also both proceeded through the levels in order. When the subjects were on level 4 it was their first experience dealing with the interfering wave forms. At first this may have been challanging and confusing which lead to novices giving up on hard puzzles in search of easier ones, on level 6 they seemed to have a better sense of what they were working with, and were able to complete the puzzles. Even though the novices were more successful on level 6 than level 4, it is not suggestive that level 6 was easier for them than level 4. Novices had 18 attempts at level 4 and only 3 attempts at level 6. Also, as seen in the next section, the time spent on each game on level 6 was about 4 times greater. However, being more familiar with the idea of interfering waves towards the end of their sessions, lead novices to stick with each puzzle in level 6 until they completed it. The decline in success from level 6 to 7 mirrors that of the shift from level 4 to level 5. This further suggests that levels that provided all eleven harmonics were significantly more difficult for students to solve than those that only included the appropriate number of harmonics. Also, though experts were the only ones to explore the higher levels they were rather unsuccessful. All completed attempts in level 8 were by way of hints, and no attempts were completed in level 9 or 10. Only one student attempted the pretest level, but it is not included in the analysis because its structure is much different from the other levels .

V. Time Spent on Each Game Attempt

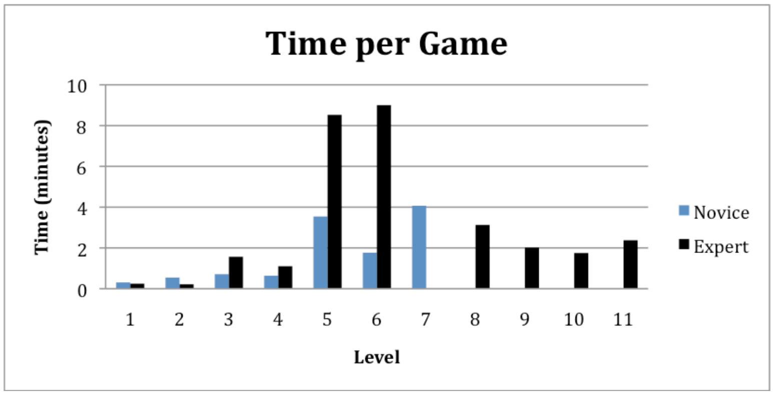


Fig.4 Time per Game. This table shows the average time spent on each game attempt by level for each population. (Total time spent on level divided by attempts).

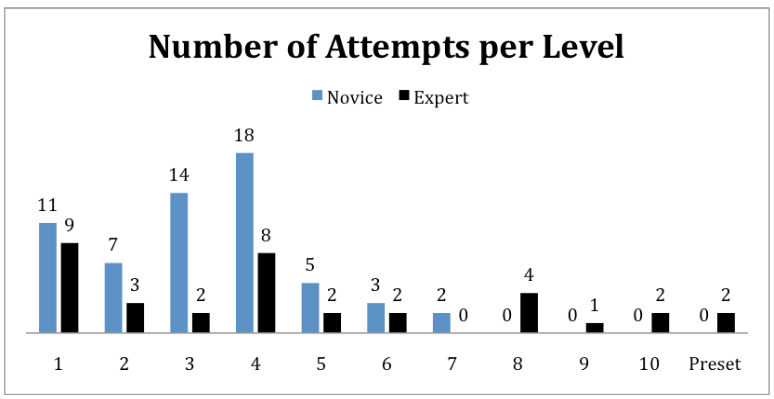


Fig.5. Number of Attempts per Level. Counts of attempts on each level are shown. This data is supplimentary to the discussion of the Time per Game data but will not be discussed exclusively in this section.

Games that are counted in Figure 4 include all attempted levels, but does not include levels that were passively skipped when students clicked "new game" looking for easier puzzles. The ease with wich students progressed through each level can be intrepreted comparing the time they spent on each level to the rate at which they completed puzzles in each level and how many times the challenge level is attempted. As dicussed previously levels 1-4 have shown to be quite easy for both populations. Times to solve the puzzles on these levels are about 30 seconds each for both populations. On level 3 and 4 it seems as if the experts had a harder time than the novices. However, on level 3, the experts only attempted the level twice before moving on, and had a 100% success rate. Novices attempted level 3 fourteen times. After a few times attempting level 3 they were able to notice that only one harmonic was needed to solve the puzzle and could move through the puzzles more quickly, which brought the average time down considerably. Similar patterns can be seen for level 4. Experts averaged about a minute per game but were 8/8 with completion, where as novices spent less than a minute but only completed 12/18 attempts. The decreased average time can be attributed to the levels novices quickly gave up on. Notably the times for both novices and experts is relatively short through level 4.

What is interesting is the divergence between the experts and novices on levels five and six. Experts spent two to three times the amount of time on these levels as novices. Also, at these levels, number of attempts and completion percentages were rather similar for these two populations. Again, the novices made more attempts, and both populations had a 50% succcess rate. As noted before, both the experts and novices used hints on level 5 puzzles. However, the way they two groups used the hints was different. Not only did the experts spend more time trying the puzzle before resorting to hints, they also only used the first hint. Novices used the second hint that told which harmonics to use, which makes the challenge more similar to that of Level 4. The time to complete a level 4 puzzle is much less than that of a level 5 which is why the novices have such a smaller average time. Similarly, on level 6 the experts solved the puzzles without the aid of hints. Novices used hints on this level because, as one student stated, they were "kind of close to the right shape" but wanted to see the answer. This decreased the average novice time compared to the experts who worked through the level unaided. The perseverence of the novices did not transcend into higher levels. Experts spent 2-3 minutes per game on levels 8-10. Of the 7 attempts on these levels, only three attempts in level 8 were successful, one of these successful attempts was without the use of hints. Experts gave up on the higher levels expressing that the waveforms were too complicated and they didn't expect to be able to solve them.

Qualitative Data and Analysis

Students were initially directed to use the simulation applications in whichever way they chose. The intent of students' minimally guided use was to see if any trends, or inclinations, existed among the two populations. After the initial observations of each subject it seemed as though each subject's interaction with the simulation was quite unique. However, upon further comparison, many trends did indeed exist within each population and over the group as a whole.

In addition to 'novice' and 'expert' discussions, N1, N2, N3, E1, E2, or E3 will be used to designate discussion of case specific incidences.

I. Population Independent Trends

- 5/6 students started on the Discrete tab and then moved to the Wave game

- All students used the Harmonic Amplitude adjustment first when on the Discrete tab

- All students started with level 1 on the wave game

The first three points are similar in nature. The Discrete tab is the application that the simulation opens up to by default, the amplitude adjuster is the first accessible function within the Discrete tab, and a Level 1 puzzle is what the Wave Game initially opens up to. Though unguided by the confines of the experiment, every student reacted to the guides within the simulation. After each student responded to the guides within the sim, they then digressed into their own exploration of it. *Fourier: Making Waves* as a whole gives users very little explicit direction, but all subjects responded to the few guides it has. The tendency of the students to follow the guides of the simulation before exploring on their own reflects their inclination to follow directions and demonstrates the instructional potential of these kinds of simulations.

- Students either used all of the functions within the Discrete application or none

Half the students explored the functions on the Discrete tab, while the other half used the initial amplitude adjuster and proceeded on to the game. The students who did explore the discrete functions tried most all of them. They went through all of the functions rather quickly, noting what each one did along the way, but never taking too much time to consider the quantitative aspects of each of the functions. Their interactions with these functions were very passive and there didn't seem to be any correlation between the use of these functions and performance on the game. The functions on the discrete page are characteristically quantitative. Without anything to directly apply the functions on the Discrete tab to, students appeared to be less engaged with the information.

- Students rarely went back to the Discrete tab after beginning the wave game.

Most students made only one or two transitions between the Discrete tab and the Wave Game, and as seen in the previous section, the majority of time spent overall was on the Wave Game. The Wave Game proved to be more engaging for a few reasons. Primarily, the interactive nature of the game held students attention better than exploring quantitative features that they had no direct use for. Students expressed a desire to beat challenging levels, stayed involved long enough to move on to higher levels, and demonstrated excitement when they won. Also, most of the students expressed an affinity for puzzle type games in general which motivated them to stay involved with the Wave Game.

- No subject made the connection that the number of the harmonic equaled the number of wavelengths.

Interestingly, no student noticed that the number subscripted for each harmonic represented the number of wavelengths of the wave given, yet all the students realized that wavelengths on the left side were longest and got progressively shorter to the right side of the list. The students' active engagement with the qualitative aspects of the simulation and disengagement with the quantitative aspects seemed to affect their ability to make basic quantitative connections.

- Every student executed a "trial and error" strategy for the Wave Game

For both experts and novices, students struggled in their attempts to develop a strategy when using the wave game. All students knew that long wavelengths were on the left, progressively getting shorter towards the right. However, all students admitted that guessing and checking was their primary approach to the wave game. Even in levels like the one depicted in Fig. 4, where the one wave length of the first harmonic and the eleven small wavelengths of the eleventh harmonic are very clear, students didn't actually count the wavelengths. They approached it more qualitatively, noticing a big wave or a lot of little ones. Students not counting wavelengths may be connected with the fact that none of them notices that the number associated with each harmonic was the number of wavelengths. Since they did not make that connection, counting the wavelengths they saw wouldn't have helped if they didn't know how to use that information. The pattern of guessing and checking by both populations shows that knowledge of Fourier decomposition theory isn't necessarily a tool for doing well in the game.

II. Population Dependent Trends

- Novices progressed through wave game levels in order, Experts skipped quickly to higher levels
- Novices spent more time "mastering" a level before progressing to higher levels
- Only experts attempted the high levels

The way each population moved through the levels demonstrates the level of confidence with which they approached the game and mirrored how well they understood superposition of waves. In the preliminary interview questions, novices all expressed that they had some knowledge of wave forms. They all demonstrated basic knowledge of amplitude, period, wavelength and for the most part were able to articulate what they noticed about these properties in the simulation. However, when novice subjects were challenged to think about any function being represented by combining only sines and cosines as they used the sim, they appeared shocked and confused by the proposition. The expert subjects all offered the information up front that a continuous function can be expressed as a sum of sines and cosines. Also the experts demonstrated a high level of understanding and confidence with the ideas of wave forms in general and the qualitative aspects of superposition. Experts were able to talk about wave properties in a way that exhibited a high level of mastery of the concepts. The initial hesitation of the novice group can be seen in their interaction with the wave game. Novices progressed through the levels in order and did each level more than once before proceeding. The confidence with the experts is similarly demonstrated as spent little or no time on the introductory levels and immediately attempted higher levels. Novices demonstrated "mastery" of each level before moving on to the next. A few times the novices would get stuck on a level, move back to the preceding level, and then try the harder level again. This mirrors the need for them to feel comfortable with the basic properties of the concept before exploring more complex elements of the concept. Since the experts knew how complex Fourier decomposition can be, they moved immediately to the higher levels assuming the lower levels were overly basic. However, confidence in the concept did not translate into success for the experts. They had very little success on the higher levels in general. Even level 5, which only added two harmonics, was just as difficult for the experts, based on rate of successful attempts, as it was for the novices. I think the divergence between the confidence and success on levels of the experts can be attributed to a disconnect between what they know about Fourier decomposition in a theoretical sense and how to tangibly use that knowledge. The experts know the general concept of Fourier decomposition, the formulas to expand the sum and find amplitude coefficients, and other technical workings of the theory. However, none of the information seemed to be directly useful to solving the qualitative puzzles in the Wave Game.

III. Specific cases

a) N3 flipped between the discrete tab and the wave game many times.

This student used flipping back and forth between the discrete tab and the wave game as a strategy. They failed to notice that what was shown on the screen in terms of amplitudes, harmonic graphs, and the sums, were all represented in the same way on both screens. When this student was faced with a difficult looking puzzle they would go back to the discrete tab and adjust different harmonics until they found one that "looked like the one [they] needed" and then go back to the game. This seems like a more difficult approach since amplitudes of harmonics can be adjusted and seen on the wave game in the same fashion, though the student did not notice that. The quantitative characteristics surrounding the graphs may have influenced this student on the discrete tab. Though they were doing essentially the same action of adjusting amplitudes, the quantitative environment may have seem more like it was giving an answer than the qualitative environment of the wave game. The student might also have felt that the discrete tab offered more of a "practice" environment, and that trying incorrect harmonics on the game its self may have caused them to lose.

b) E3 expressed a lack of desire to complete levels

E3 was more advanced than the other expert subjects. This student had not only had Fourier theory introduced in advanced physics courses, but had completed a Fourier Analysis course as well. Overall, this student seemed the least engaged with the simulation. With the wave game specifically this student expressed a lack of interest in completing levels. They said that solutions they found were "close enough" to moved on to a new puzzle or didn't finish because it wasn't required. Additionally, they said that they didn't particularly enjoy puzzles anyway. The students' disengagement with the simulation overall was due to boredom with the overly basic nature of the content in relation to how advanced their studies were. This shows that this simulation is probably best suited for beginning and intermediate students who are attempting to learn or refresh their knowledge of the content. The student's disinterest in puzzles shows that this type of game is not for everyone. On its own (i.e. without outside instruction or requirements) the puzzle aspect of the game proved to be engaging to most students, but this quality may be a deterrent for students not afflicted by puzzle-type games.

c) E1 and E2 demonstrated underlying knowledge of Fourier decomposition by exploring complex waveforms.

E1 was the only student to attempt the "preset" level on the wave game. (The preset level has puzzles with square, saw tooth, etc shaped waves. This student was very successful on these levels and was able to articulate clearly what was characteristics of the harmonics were necessary to achieve each waveform. E2 began the simulation by trying to make complex waves. After initially trying the amplitude adjustment function, they then proceeded to attempt to make a saw tooth wave and were rather successful after just a few minutes. These two students' ability to take what they knew about the theory going into the simulation and clearly demonstrate it shows that, in addition to the instructional traits of the simulation, there may be uses for the simulation to test student understanding.

IV. Student Frustrations/Comments

a) "Pickiness" of the wave game

During their interviews nearly every student neglected to finish a puzzle because they felt there answer was "close enough". N1 and N2 specifically noted the selectivity of the wave game solutions as both the thing they disliked least about the simulation and if they could change the simulation in any way. However all students commented on this feature in some way and had they not been confined by the interview to continue playing the game, the frustration from that resulted from this feature certainly would have been a reason to stop playing in N1 and N2 if not all cases. The need for a correct solution to these puzzles is evident in terms of making sure correct information is being produced by the simulation. However, with a game that is very qualitative in nature, an apparently correct solution may be indeed be good enough to provide users with understanding of the concept.

b) User Error with Unguided Exploration

When asked in the follow up questions "would you change anything about the sim" most of the responses involved adding or changing something that was actually part of the simulation but the user had not discovered it themselves. For example E1 complained that the hints were useless. The first time "hint" is clicked the number of harmonics needed to solve the puzzle is given. The second hint tells specifically which harmonics are needed and the cheat option gives the answer. E1 only attempted level 1, 6, 8, and preset. The structure of each of these levels gives only the harmonics needed to solve the puzzle, so in this student's case, the hints were useless, and appeared to be so circumstantially. N3 suggested that the simulation offer more quantitative explanation of goings on with the harmonics and sums. This student, the same one that moved back and forth between the discrete and wave game tabs often, never explored any of the quantitative functions on the discrete tab beyond the amplitude adjustment. What students disliked about the simulation it seems to be a direct result of the unguided manner in which they used the simulation. Though they were able to figure out some things on their own, had they been given more instruction or information on what was available to them in the simulation they may have gotten more out of it.

Final Discussion and Concluding Remarks

I. What are the differences in simulation usage between novice and expert populations?

The differences between expert and novice populations were characterized by the confidence with which each population approached how the simulation was used. All experts expressed a sound understanding of the underlying concepts of Fourier decomposition and wave properties in general. Also, though they had never used the simulation first hand, they had seen it used. Together these gave the experts confidence and comfort as they approached the simulation. The ones who used the quantitative functions on the Discrete tab were able to describe what they saw articulately. On the Wave Game experts quickly moved to higher levels, though with mixed success. The novices had basic knowledge of wave properties, no prior knowledge of Fourier decomposition, and had never seen the simulation before. Also, when challenged to think about the sum of sines and cosines being used to represent any continuous function, the novices seemed overwhelmed and perplexed by the proposition. This led to a feeling of discomfort and less confidence about the simulation among the novices. On the wave game, novices slowly and cautiously worked their way up in levels and would backtrack if they got stuck. When discussing what the noticed quantitatively about the waveforms they were far less articulate than the experts. Many of them interchanged the words "wavelength" and "frequency". One novice pointed to the sum of two waves and commented, "it's almost like this one and this one (each harmonic) are added together". Despite their elementary explanations, the novices were still able to make significant connections between the amplitudes of the harmonics and the superimposed waveforms.

II. What are the similarities in usage across all populations?

In general, the students did not connect the quantitative and qualitative aspects of the simulation. The structure of the simulation physically divided the quantitative and qualitative features into the Discrete tab and Wave Game respectively. However, when students explored the quantitative functions on the Discrete tab they did so mechanically. Each student went down the sidebar of functions, saw what each one did, and moved on without actively engaging with the information they were given. When on the Wave Game, though a primarily qualitative application, the students failed to make basic quantitative connections. For instance, no students recognized that the number subscripting each A on the label of each harmonic represented the integer number of wavelengths of the harmonic. Also, though the super positioned waveform was labeled "SUM", students did not notice that if the sum of the wavelengths was zero at the origin, for example, that the harmonics should all pass through the origin or sum to zero.

III. Is there a connection between the mathematical background of the students and their emotional response to the simulation?

Besides the notion of confidence and comfort with which the students approached the simulation and the slight boredom of one expert previously mentioned, there didn't appear to be a distinct correlation between experience and enjoyment/dislike of the simulation. Additionally, had the novices not been prompted with the information that the simulation involved a theory beyond their level of training the feeling of apprehension they demonstrated may not have been present. For the most part students were captivated by the game, save the one student who did not like puzzles. They all expressed excitement when they beat a level and frustration when they could not. Experts expressed that the sim was a great way to visualize such an abstract concept. Novices said that it was very interesting to see these properties of waves even if they weren't entirely sure what it meant.

IV. Suggestions for future use of *Fourier: Making Waves*

a) Minimally Guided Exploration vs. Guided Use for Students

Based on the results of this study, it seems that the minimal environment was not conducive to students fully exploring the simulation. Once each student started the wave game, they remained on the game almost exclusively for the remainder of their session. Also students who explored the functions on the discrete tab did so very passively. They would click each function to see what it did, but failed to actively consider what bit of information it offered them. It seemed that not having an external motivator or goal, caused both of these behaviors. The wave game has the “built in” goal of beating levels. It’s this characteristic that makes the wave game engaging and gives users something to do. On the other hand, functions on the Discrete tab offer valuable information for understanding waves and Fourier decomposition, but it has no applicability, as it stands on its own. Since the students had nothing to do with the information found in the discrete tab, they dismissed it as unimportant for the situation they were in. It’s fair to assume that in a different starting environment, perhaps if the students were given a quantitative goal, they would have spent more time on the discrete tab interacting with those features and less time on the wave game. The simulation itself has much to offer in terms of illustrating abstract mathematical and physics concepts. The functions on the discrete tab allowed students to adjust many quantitative functions simultaneously and have the effects of each represented graphically. This is an invaluable tool for making connections between the quantitative and qualitative aspects of such a complex theory. However, without anything to apply these functions to (i.e. a worksheet, homework assignment, etc) users failed to actively engage with these functions if they even used them at all. Furthermore, the majority of time spent during unguided exploration was on the Wave Game. The ability of the game to engage users intensely while they interacted with physics concepts is a powerful tool. It’s likely this engagement comes in part from the acknowledgement of a clear goal. However, spending time on the qualitative aspects of the theory is not as beneficial if the quantitative aspects go unnoticed. Finally, students made comments about parts of the sim they wished to change, add, or disliked in general. All of the features they wanted were in fact part of the simulation; they just had not explored far enough to find them. This further shows that unguided use of this simulation may not be the most effective means of use. Because of the many important functions available, the separation of qualitative and quantitative ideas between tabs, and the complexity of the underlying theory overall, use if this simulation by means of a worksheet, assignment, or other guiding tool may prove to be a more effective means for learning.

b) Wave Game Levels and Features

i). Level Progression

The general lack of success on level 5 across both populations indicates that the structure of the levels in the wave game may need to be changed. Though there is a clear pattern of progression from level to level, this pattern may not have been clear to the students. Furthermore, the presence of all 11 harmonics as choices was a distracter to students. Even when the previous level only required 2 harmonics to be used, when given 11 harmonics, students attempted to find solutions that utilized all of them. This caused novice students to rarely advance beyond this level, and often return to level four.

ii) Correctness of Responses

An overwhelming complaint by students was that they wave game was too "picky" with solutions. Upon further investigation, it turns out that solutions are counted as correct when the magnitudes of harmonics are off by +/-.03. To make the game less sensitive would not be beneficial, as it is important for students to be accurate with their solutions. However, when presented solutions are off by magnitudes of .03 or less, it can become difficult to determine what is different between the solution wave and the correct waveform. This is because the lines of the waves are relatively thick and the screen where the solution is shown is quite small. Perhaps methods of making the "wave to match" and the sum wave the users adjust have a higher resolution or larger viewing screen. If the waves on the screen were larger, subtle differentiations between the two waveforms would be easily seen. Likewise, higher resolution lines for the waves would reduce the appearance of the two waves overlapping when they should not.

V. Comments

Both populations struggled to make key connections between the qualitative and quantitative aspects of the simulation. These struggles were independent of the students' background with Fourier decomposition theory as both novices and experts were lacking in this respect. PhET simulations and built on a foundation of constructivist ideals. The simulations' purposes are to be tools that facilitate learning as students build and explore. It seems that the minimally guiding framework of the sim's infrastructure impedes upon the students abilities to make these essential connections between quantitative and qualitative themes. As students built and explored by way of 'minimal guidance' at times the absence of a well-defined learning goal was counterproductive to the constructivist intention. Students focused on the game because they knew of the internally constructed goal of winning. However as they built and manipulated the sums of the harmonics, they did so disconnected from underlying theory of Fourier decomposition. They developed skills to beat the levels but did not translate those skills into a broader understanding of Fourier decomposition and its applications. In order to allow students to build and explore both the quantitative and qualitative features of this simulation more guidance must exist during use of this simulation. These guides could exist with in the sim, namely with in the discrete tab, to connect qualitative concepts, quantitative features, and goal orientation. The guides could also exist externally from the simulation. On a small scale, for instance on the lower levels of the wave game, when students could identify wave patterns they were able to assess what they needed to do to match it and execute it successfully. This characteristic needs to transcend into the whole of the simulation and encompass all of the information about Fourier decomposition the sim has to offer.

Appendix A: Research Methodology Worksheet

Research Question 1:

What are the differences in patterns of interactions with the Fourier sim between expert/experienced and the novice populations?

Research Question 2:

- What are the similarities in usage and interaction with the sim across all populations?
- For both of these taking data will be purely observational. Things I will specifically be monitoring and looking for when reviewing the videos:
    - where each population starts (which tab)
    - how often each population switches back and forth between tabs
    - total time spent on each tab
    - highest levels reached on the wave game
    - avg. time spent on each level on the wave game/patterns of progression through levels

Research Question 3: Do students from each of the three populations exhibit different performance abilities between using the sim and talking about Fourier transforms?

- This question will involve more one on one questioning
- some kind of preliminary question (possibly just the motivating question of the interview)
- open ended questions during interview, while they are using the sim
- what are you doing there? why did you do that? type questions
  (some of this can also be gathered from reviewing the video, as subjects
  will be encouraged to verbalize what they are doing as they use the sim)
- follow up questions (again, really open ended since I'm not looking for a specific kind of answer but rather, can they answer)
- what can you tell me about adding waves together?
- did you notice anything interesting about waves?

Research Question 4: Is there a correlation between math/science background and affect/emotional response to the sim?

- Prelim questions before using sim
- describe your background in math science (though we will already "know" their background, but how they answer the question might give insight to their level of confidence also)
- Observations during sim use:
    - any kind of emotional commentary (saying something is cool, neat, etc.)
    - emotional responses to beating levels on the wavegame
- Response questions to observations
    - ask pointed questions relating to any outward remark (cool!, for example) made
        - what was cool? why?
- Follow up questions at end of interview (start general, move to more pointed questions)
    - what did you think? (really broad, could say any number of things)
    - from there, kind of have to move w/ their response for like/dislike
        - why?
            - anything in particular?
        - then ask opposite side (if they liked it ask them if there was anything they disliked?

        - what, why?
    - did you find the sim fun?
        - specifics?
    - where you ever frustrated or confused?
    - were you able to figure it out/remedy the situation?
      (this last question might overlap w/ question 3-if students were confused by Fourier concepts, were they able to articulate what confused them)

Research Question 5: do students exhibit differential preference / affect (emotional response) for working with the sims versus mathematical formula (specifically the college experienced population)

- prelim question, almost in a word association format
    - I say Fourier Decomposition, you say....
- observations:
    - do they spend more time physically moving the waves or attempt to plug in numbers?

    - Do they use the other analytical functions (phase shift, changing to cosines)
- follow up questions:
    - How do you feel about Fourier decomp. after working with the sim
    - Do you feel more/less confident in your understanding of the concept?
    - Do you feel more/less confident in your understanding of the mathematical basis?

Appendix B: Interview Protocol Outline

Intro (~10 min):

- Welcome participant and thank them for their time
- Introduce the project and let them know how the interview will be conducted
    - "the purpose of my research is to find out if this particular sim is useful as an educational tool. I am going to have you play with the simulation for about 20-30 minutes. I will be videotaping your interaction simply as a resource that I can refer back to at a later time. After that time, I will ask you a series of follow up questions. There is nothing specific you "have" to do. Please only use the first two tabs on the simulation and explain out loud what you are thinking as you go through the activities the best you can. I will be taking notes on your interaction and on any questions I ask, please don't let this startle you. I also might ask you questions about what you are doing as we go along. Do you have questions before we begin (answer any questions)
- Preliminary questions:
    - Please briefly describe your background in math/science
    - (for college/experienced group) Tell me anything you can about Fourier decomposition

Student-Sim Interaction (~20-30 min)

- have sim open and waiting and video camera set up
- Pose leading question (based on skill level)
    - young/novice: "Sine and cosine waves are shaped like this (show sine wave). Do you think any shaped line can be made by combining different sized sine and cosine lines?"
    - college/novice & experienced: "Can any continuous function be represented by a combination of sine and cosine waves?"
- Things to look for during interaction period (to notice, but mostly analyze when reviewing video):
    - exclamatory/emotional commentary
        - note: feeling evoked, what they were doing, context (e.g. time into use)
    - over all trends of use
        - where did they start?
        - how fast did they move from feature to feature?
        - flipping back and forth between tabs?
        - total time spent on each tab
        - use of qualitative vs quantitative features
    - wave game trends
        - highest level reached
        - time spent on each level
        - emotional response to game

Questions to ask while student uses sim

- if the student exhibits a significant emotional/affect kind of response ask what specifically makes them think that/why?
- open ended questions such as "what are you doing now?" "why did you do that?" just to keep them talking about their thought process out loud

Follow up questions (~10 minutes)

- general questions
  - what did you think?
  - from there, kind of have to move w/ their response for like/dislike
    - why?
    - anything in particular?
  - then ask opposite side (if they liked it ask them if there was anything they disliked?
    - what, why?
  - did you find the sim fun?
    - Specifics?
  - where you ever frustrated or confused?
    - about what?
    - were you able to figure it out/remedy the situation?
- content questions
  (experienced)
  - How do you feel about Fourier decomp. after working with the sim
  - Do you feel more/less confident in your understanding of the concept?
  - Do you feel more/less confident in your understanding of the mathematical basis?
  - Did you think this could be useful for you in the future

  (novice)
  - what can you tell me about adding waves together?
  - did you notice anything interesting about waves?
  - is there anything you would change about the sim?
  - Could you use the sim to explain the idea of Fourier decomposition to others? How?

    - note ability to convey ideas for content questions

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