Student representational competence, self-assessment, and problem solving in physics

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Abstract

Student problem-solving ability is related to the representational format of the problem. In a study of two large-lecture algebra-based introductory university physics classes, we examine student problem solving ability on homework problems given in four different representational formats (mathematical, pictorial, graphical, verbal), with problems as close to isomorphic as possible. In addition to the homeworks, we examine students' assessment of representations by providing follow-up quizzes in which they choose between various problem formats. Subsequently, we look for factors that may influence their ability or choices. As a control, some parts of the classes were assigned a random-format follow-up quiz. We find that there are statistically significant performance differences between different representations of nearly isomorphic quiz and homework problems. We also find that allowing students to choose which representational form they use improves student performance under some circumstances and degrades it in others. We also begin to characterize how and why students choose and make use of the different representations available.

I. INTRODUCTION

Student competence with different representational formats is a popular topic in modern science and mathematics education. By ‘representational formats,’ we refer to the many ways in which a particular concept or problem can be expressed. One can categorize problem representations according to whether they are formal or informal, abstract or concrete, or text-based versus graphics-based, to name just a few. Studies involving representational formats have taken many approaches to this division, including comparisons of mathematical problems couched in words to those stated primarily in equations,[1] comparisons of learning environments that are virtual to those that are physical,[2] and comparisons between verbal, mathematical, graphical, and diagrammatic formats.[3] Scientists can interpret all of these formats effectively and are able to integrate them, translate among them and assess their usefulness in different situations. A possible instructional goal is to develop this cross-representational facility in science students. In physics education research (PER), there have been several studies in which students are explicitly taught to handle multiple representations of the same topic.[4][5][6]

There have been a number of studies (both in[7][8][9] and outside of PER) that examined student performances on particular representations of physics problems (see Meltzer for a short overview[3]). There have also been studies that compare student performance on problems that involve multiple representations to performance on problems that involve isolated representations,[10][11] and studies that investigate student skill at translating between representations.[12][13] There has, however, been less research that compares student performance on different single representations of a problem. Koedinger[1] studied the performance and strategies of middle-school algebra students solving both word- and formula-based representations of problems. He found significant performance differences between the formats, and noted that these differences correlated with the different strategy choices that students made when handling different representations. In Meltzer’s[3] work, he provides students in an introductory level algebra-based physics class with quizzes that have nearly isomorphic problems represented four different ways (verbally, mathematically, graphically, and diagrammatically). His prior lectures and assignments made heavy use of the different representations seen on the quizzes. After averaging over several years of data for the same class, Meltzer found instances where students performed significantly better on one representation of a problem than on another. He also found that students were not necessarily consistent in their performance on the same representations across topics, leaving open the question of why these performance differences exist.

In this study, we directly compare student performance on different representational formats in the style of Meltzer. In addition, we perform the study in two different classroom environments (one mostly traditional, and one reformed). We divide study problems into verbal, mathematical, graphical, and pictoral formats. We do not mean to imply that these representations span the space of all possible representations or that they are mutually exclusive, but we do consider this set to cover most of the representations seen by students in lecture and in standard texts. We also consider these representations to be fundamental in physics generally. We should point out that variations are possible within any particular category of representation; for example, our pictoral format includes both literal pictures of diffraction patterns and schematic pictures of Bohr-model electron orbits. As we will see, the data show that student success with one representation of a problem does not necessarily imply success with another, a result that may have significant implications. Conceptual surveys, for instance, may need to carefully attend to the various possible representations of a topic to avoid false positives where a student is assumed to have mastered a concept based on performance on one or two representational formats. Furthermore, we will see that success with a particular representation of a topic does not necessarily imply success with that same representation of a different topic.

We also broaden the examination by investigating whether students can assess their own representational competence, what motives they have for handling a problem in a particular representational format given a choice of formats, and whether providing this choice affects their performance compared to students given problems in random formats. These questions relate to students' "meta-representational competence," a notion that diSessa has developed in his work.[14] There, diSessa and others investigated students' ability to generate their own representations of a concept or situation, sometimes in cases where they have very little formal training.[15] They have also considered students' ability to assess and critique the representations that they generate.[16] Our study differs from these in that we ask students to assess fairly standard representations that we have provided rather than ones they have generated themselves. We also have them assess their own skills and preferences regarding these standard physics representations, in part by choosing which representational format they would like to work with on a quiz. There are a number of outcomes that we might observe here. It may be that students have well-defined learning styles and are aware of them, enabling them to increase their performance given a choice of representation. It may be that students perform in a relatively consistent way across representational format but are unaware of their strengths, leading to unchanged or even reduced performance when given a choice. Or, it may be that students' performance when given a choice of representations varies and is difficult to predict, with some topics and representations resulting in improved performance and other topics and representations resulting in lower performance. This would suggest a more complicated explanation of how their performance varies as it does, one that attends carefully to the different levels of context present, and this is in fact what we find.

In short, we have four primary goals:

- To further demonstrate that student performance varies, often strongly, across different representations of physics problems with similar content.
- To investigate why students perform differently on these different representations.
- To show that giving students a choice of representational format will change their performance either for better or for worse, depending on the circumstances.
- To begin to explain how providing a choice of representational format results in these performance differences, and to note the possible effect of different instructional techniques.

II. METHODS

We administered our study in recitation to two large (546 and 367 student) algebra-based introductory physics classes at the University of Colorado at Boulder. These courses are composed primarily of students taking the class to satisfy the requirements for life science, social science, and pre-medical programs. The first course in our study was an on-sequence second-semester class (Physics 202) held in the spring of 2004. The format was mostly traditional, albeit with some in-lecture qualitative and quantitative concept tests using a personal response system. Students had three one-hour lectures per week, and met for two hours each week in either a recitation or a lab. The recitation/lab part of the course was directed by a different professor than the lecture portion. The recitations were generally traditional, with students spending most of their time discussing homework and exam questions with a graduate TA. The labs focused mostly on investigation, testing predictions, and completing open ended tasks (that is, tasks where the students were given a general goal but no specific directions for how to accomplish that goal). Students' grades were based on participation in the concept tests, exams, labs, and homework assignments, both online[17] and long answer.

The second course was an on-sequence first semester class (Physics 201) in the fall of 2004. This course precedes 202 in the standard sequence, but this particular 201 section took place the semester following the 202 class mentioned above, and so each group was being exposed to the study for the first time. The 201 course was taught by a different professor, who is familiar with many of the major results of physics education research. The 201 class was largely reformed, with heavy use of interactive concept tests and an emphasis on tightly integrated lecture demos. The students had the same number of lectures, recitations, and labs as the 202 students. The recitation/lab section was taught by the lecture professor and another professor working together. The recitations focused on working through problems rich in context in small groups, with some demonstrations and some time reserved for homework and exam questions. The labs were a mixture of directed work, open-ended questions, and testing predictions. Students' grades were determined in much the same way as in 202.

For the 202 class, we performed the study in two different subject areas: wave optics and atomic physics. In each subject area, the students were assigned four multiple-choice homework questions that covered the same concept in four different representational formats, as well as a one-question multiple-choice quiz given in recitation. The homeworks and quizzes were all based on material covered recently in class and in lab. All four homework problems on a particular assignment were based on the same concept, though the questions and answers differed enough from problem to problem to prevent repetition. These homeworks were assigned online as pre-recitation questions and were turned in at the start of the recitation section. An example of two of four homework problems from one of the two 202 assignments is shown in Figure 1. After turning in the homeworks, the students were given the one-question quiz in one of four representational formats. These problems were isomorphic from format to format, with the answers and distractors mapping from one

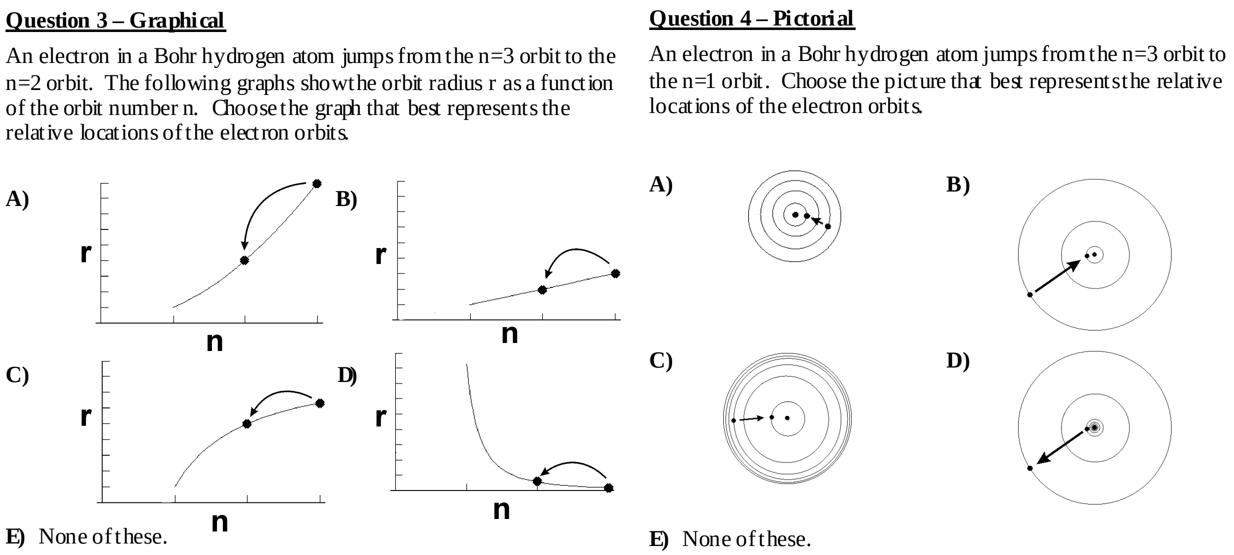

FIG. 1: Isomorphic homework problems (in graphical and pictoral/diagrammatic formats) regarding Bohr-model electron orbit radii.

format to the next. It is worth noting that we use the word 'isomorphic' to mean isomorphic from the point of view of a physicist. A student may have a different view of the similarity (or lack thereof) between these problems.[18]

Nine of the thirteen 202 recitation sections were allowed to choose from the four representational formats on the quiz without getting to see the problems before they selected. The idea was that the students would make their choice based on their previous experience with representations in classes and on the homework assignment. The other four sections had quiz formats randomly distributed to the students; these students served as a control group. The choice and control sections did not change from one subject area to the next, and the students in the two groups performed similarly on the study homeworks, the course exams, and in the course overall. Both the quizzes and homeworks included a Likert-scale on which the students could rate the perceived difficulty of the question, and the quizzes included a section where the students were asked to write about why they chose the format they did (if they had a choice) or which format they think they would have performed best at given the choice (if they had a random assignment). Both the quizzes and the homeworks counted towards the students' recitation scores for participation but were not otherwise graded.

	Verbal format	Mathematical	Graphical	Pictoral
202 Diffraction/Interference HW (N = 241)	0.52	0.61	0.46	0.54
202 Bohr model HW (N = 218)	0.84	0.83	0.76	0.62
201 Mechanics/energy HW (N = 333)	0.54	0.70	0.50	0.49

TABLE I: Fraction of students answering a homework problem correctly, broken down by representational format and topic. Standard errors vary but are on the order of 0.02.

The study was conducted in much the same way in the 201 class. We covered two subject areas: energy (in particular, kinetic and potential energies and their connection to motion) and pendulums. For the energy and motion topic, the students received a four-question pre-recitation homework and an in-recitation quiz, with the students in nine of the eighteen recitation sections having a choice of representation and the rest of the students receiving one at random. For the pendulum topic, we gave the students a recitation quiz only (no homework) in order to satisfy schedule constraints. Again, the choice and control groups were the same from one topic to the next, and the two groups performed similarly on homeworks, exams, and the class overall.

In this paper, we restrict our attention to students who completed a homework (when there was a homework) and the corresponding quiz for a topic, which amounts to roughly 240 and 220 students in the first and second 202 studies, and 330 students in each of the two 201 studies. All of the homework and quiz questions can be found in Appendix A.

III. DATA

In this section, we focus on comparisons of student performances on isomorphic (or nearly isomorphic) problems in different formats and comparisons of student performance in choice and random-assignment (control) recitation sections. We also examine why students made use of the representations they did and how they used multiple representations when they did.

Table I shows the fraction of students (in both choice and control sections) that answered each of the twelve homework problems (four formats in three different topics) correctly. In all cases, the problems on a homework set were as similar as possible except for the answer and the representational format. Table II shows the performances of the students on each

	Control (random format) group				Choice group			
	Verbal	Math	Graphical	Pictoral	Verbal	Math	Graphical	Pictoral
202 Diff.	0.24 (17)	0.56 (18)	0.25 (16)	0.58 (19)	0.35 (17)	0.37 (57)	0.04 (26)	0.82 (59)
202 Spec.	0.32 (19)	0.13 (15)	0.53 (17)	0.83 (18)	0.81 (21)	0.90 (42)	0.96 (27)	0.39 (58)
201 Springs	0.56 (43)	0.41 (39)	0.69 (42)	0.58 (40)	0.55 (11)	0.57 (102)	0.88 (17)	0.77 (39)
201 Pend.	0.55 (42)	0.30 (40)	0.64 (39)	0.67 (43)	0.62 (21)	0.39 (28)	0.65 (40)	0.78 (80)

TABLE II: Quiz performance of students from the random-format recitation sections (left) and from the recitations sections that had a choice of formats (right). The number of students taking a quiz is in parentheses. The quiz topics are diffraction, spectroscopy, springs, and pendulums. Standard errors vary and are not shown.

format of each in-recitation quiz, grouped by whether they were in a choice or control section. The number of students in each subgroup appears in parentheses.

A. Analysis: Performance across representational format

All statistical significance tests involving student success rates are two-tailed binomial proportion tests. We shall use to the following terminology: A difference with $p > 0.10$ is referred to as not significant, p between 0.10 and 0.05 is marginally significant, p between 0.05 and 0.01 is significant, and $p < 0.01$ is highly significant.

In examining the homework data, we note that in several cases there were differences in performance from format to format on a particular assignment. When there was a difference in performance between two formats, the mathematical format was often one of the formats involved. This was the only format to require an explicit calculation. The other formats involved conceptual reasoning supported by descriptive language, graphs, or pictures. We see that on average students were most successful with the mathematical homework format, which is consistent with the notion that first-year university physics students are more comfortable with 'plug 'n chug' types of problems than with conceptual problems.[19][20] We should point out that a mathematical format need not always involve numerical calculation; indeed, one of the math-format quiz questions (to be described later) was best solved through conceptual reasoning supported by the qualitative use of equations. Nevertheless, in this study the mathematical format usually involved direct calculation.

We also see that there are some noticeable performance differences among the more conceptual formats. For instance, consider the graphical and pictoral problems on the Bohr model assignment, shown in Figure 1. Both require knowledge of how the electron orbit radius varies with the principal quantum number in the Bohr model. The questions differ only in which specific transition is being presented and in whether the problem and solutions are expressed in graphs or pictures/diagrams. Of the 218 students who answered both problems, 77% answered the graphical problem correctly and 62% answered the pictorial problem correctly. This difference is highly significant statistically ($p = 0.006$) and is particularly interesting in that the graphical representation is a rather non-standard one. Students had not seen any graphs of orbital radius versus quantum number, but the pictorial representation of electron orbits should have been somewhat familiar since it is featured in both the textbook and the lectures that preceded this quiz. Further examination of the individual student answers on these two questions indicates that this performance difference can be attributed almost entirely to the 36 students who answered the graphical problem correctly and missed the pictorial problem by choosing the distractor C. This distractor bears a strong resemblance to the energy-level diagrams seen in the Bohr model section of the text and lectures. Since the problems are so similar and the same distractors are present in each problem, it appears that in this case representational variations may be traceable to a very topic-dependent cueing on visual features of one of the problems.

Another potentially intriguing homework pair exists on the 202 interference homework. There is an 8% difference between the graphical and pictoral formats, problems that were again extremely similar. This difference, however, is only marginally significant statistically ($p = 0.08$). It is notable, though, that in this case the *direction* of the effect was different. Students were more successful with the pictoral format, and less successful with the graphical format.

We can find another example of performance variation across isomorphic problems in the second 202 quiz, which deals with the emission spectrum of a Bohr-model hydrogen atom. The students were prompted to recall the spectrum of hydrogen, and were asked how that spectrum would change if the binding of the electron to the nucleus were weaker. The questions, answers, and distractors were the same on each quiz except for their representation. Figure 3 shows the problem setups and one distractor for the verbal and pictorial formats (performance data are in Table II). Note that one week previous to the quiz, students com-

Spectroscopy Problem – Pictorial Format

The Balmer series of spectral lines is shown below, as seen through a spectrometer:

Now suppose we are in a world where electric charges are weaker, so the electron is not held as tightly by the nucleus and the ionization energy is 13 eV instead of 13.6 eV. Choose the picture that best represents what the new spectrum would look like.

B)

Spectroscopy Problem -- Verbal Format

Consider the Balmer series of spectral lines from hydrogen gas. Now suppose we are in a world where electric charges are weaker, so the electron is not held as tightly by the nucleus. This means that the ionization energy for the electron will be smaller. What will happen to the Balmer lines that we see?

B) The spectral lines will all shift to shorter wavelengths (toward the bluer colors).

FIG. 2: Setup and second answer choice for the verbal and pictoral format quizzes given in the second trial. The other distractors match up between the formats as well.

pleted a lab covering emission spectroscopy, and the quiz images match what students saw through simple spectrometers. Nineteen students in the control group were randomly assigned a verbal format quiz, and 18 were assigned a pictorial format quiz. 32% of the verbal group answered the question correctly, while 83% of the pictorial group answered correctly. This difference is highly significant (p = 0.0014). Answer breakdowns indicate that eight of the ten students in the verbal group that missed the question chose the distractor corresponding to the spectral lines moving in the wrong direction (pictured in Figure 2). Only one student from the pictorial group made this error. It is not clear why there would be such a split, especially since the pictoral format shows numerically larger wavelengths as being on the left, opposite the standard number line convention. A possible hypothesis is that students connect the pictoral format more closely to the lab, giving them additional resources with which to handle the problem. However, as we will see, the students that were given a choice of format had significantly worse performance on this pictoral format, and so easy identification with the lab cannot be a complete explanation.

Next, consider the performance of control group students on the mathematical formats of the 201 and 202 quizzes. In three of the four quizzes, the average success rate on the math quiz was significantly lower than the average success rate on the other three formats combined. For the spectroscopy quiz, the average verbal/graph/pictoral score was 0.56 versus 0.13 on the math format, a difference significant at the $p = 0.004$ level. For the 201 spring quiz, the difference was 0.61 vs. 0.41 ($p = 0.03$), and for the 201 pendulum quiz, the difference was 0.62 vs. 0.30 ($p = 0.0004$). The difference between the average verbal/graph/pictoral score and the average math score on the 202 diffraction test was not significant. It is somewhat surprising that students were less successful with the randomly assigned math format given their generally higher performance on the equation-based homework problems, though we should note that the students took the quiz in recitation with a time limit (about fifteen minutes) and without access to a textbook, making the environment much different than that in which they would do a homework problem. We should also note that the math problem on the 201 spring quiz was difficult to solve through explicit calculation, and was more easily handled by using the equations qualitatively. This gives it a different character than the other math-format problems, which is a point we shall return to later.

B. Analysis: Effect of student choice of representation

In Table II, we see a format-by-format comparison of the students who received a quiz at random and the students who were allowed to choose a quiz format. There were a total of sixteen choice/control comparisons available (four trials with four formats each). Of the eight from the 202 class, six showed a statistically significant performance difference. These data, along with the significances of the choice/control differences (or lack thereof) in the 201 class, are summarized in Table III.

These results are notable in that the effects are in some cases quite strong. For instance, 90% of the choice group answered the math format question correctly for the spectroscopy topic, while 13% of the control group answered the same problem correctly. In addition, the direction of the effect can vary. In four of the six cases, giving students a choice of formats significantly increased performance, while in two of the six cases it resulted in a significant decrease. Furthermore, when comparing across content areas we see reversals in the direction of the effect. On the diffraction quiz, students in the choice group do better than the control group on the pictorial representational format and worse on the graphical representational format, while on the spectroscopy quiz the students in the choice group do worse on the pictorial representation and better on the graphical representation. As

Quiz subject	Verbal	Math	Graphical	Pictoral
202 Diffraction	**X**	X	0.04	**0.03**
202 Spectroscopy	**0.002**	**0.0001**	**0.0004**	0.001
201 Springs	X	**0.09**	**X**	**0.07**
201 Pendulums	**X**	**X**	**X**	**X**

TABLE III: Statistical significance of the quiz performance differences between the format choice and control groups in the 2020 and 2010 sections. Numbers are p-values using a two-tailed binomial proportion test. X denotes a p-value of greater than 0.10. Bold indicates that the choice group had higher performance than the control group.

we can see, giving students a choice of format does not result in consistently increased or consistently decreased performance relative to the control groups. Rather, the direction of the effect appears to vary strongly across both topic and representation, which suggests two things. First, these students do not have the meta-representational skills necessary to consistently make productive representational choices under these circumstances. Second, a complete explanation of these performance differences will likely be non-trivial and will not be able to rely much on broad generalities.

We can further characterize student performance in these cases by considering which distractors they chose. As was mentioned above, the control groups for the pictoral and verbal formats of the 202 spectroscopy quiz (see Figure 2) showed a significant performance difference, with the errors made by the verbal format control group being concentrated almost entirely on the distractor B in which the spectral lines move in the wrong direction (other distractors include the lines compressing, the lines staying the same, and none of the above). The corresponding choice groups did the reverse. The verbal format group had 17 out of 21 people answer correctly, with three choosing the distractor B. The pictoral group had 23 of 58 students answer correctly, with 27 students selecting the distractor B. Thus, we see that the students who chose a verbal-format quiz performed in very nearly the same way as the students who received the pictoral format at random, both in terms of success rate and choice of distractors. Similarly, the students who chose a pictoral quiz performed in the same way as the students who were randomly assigned a verbal quiz.

In general, of the six statistically significant 202 choice/control comparisons, the performance difference in two of them (spectroscopy verbal and pictoral) was mainly attributable to students focusing on a particular distractor. In the other four (spectroscopy mathematical and graphical, diffraction mathematical and verbal) the incorrect answers were split among two or more distractors. Note that the quiz distractors map from one format to the others, so this is not simply a case of some of the problems not having any attractive distractors, though apparently (and perhaps not surprisingly) different representations of a problem can make different distractors more or less attractive.

Next, consider the eight choice/control comparisons from the 201 section (Tables II and III). None of the pairs showed different performance at a $p = 0.05$ significance level. Two were marginally significant (the math format spring quiz at $p = 0.09$ and the pictoral format spring quiz at 0.07). There was very little difference in performance between the choice and control groups on the pendulum quiz, which was given four weeks after the spring quiz. The difference between these data and the corresponding 202 data is pronounced. Students in 201 did roughly as well regardless of whether they received their preferred format or a format at random, suggesting that their representational skills are more balanced. That is, they are less likely to have much more trouble with a random representation than with their representation of choice. Since one of the major differences between the 201 and 202 groups was the method of instruction, it may be that the instruction contributed to the effect. We shall return to this point in the discussion. We also should note that the 201 and 202 studies involved different topics, which may have contributed to the different performances. We are currently running a comparison of two 202 sections with different professors teaching the same topics. The results will be part of a follow-up paper.

C. Student self-assessment and assessment of the representations

In this section we consider data intended to address two related questions. First, how do students assess and value the different representations available here? Second, how (and how successfully) do they assess their own representational competence?

2020 Diffraction Quiz		2020 Spectroscopy Quiz	
Verbal 10	3: Preference for qualitative analysis. 1: Connected it to the pre-recitation homework. 1: Prefers concepts to math.	Verbal 12	3: Preference for concepts/words over math/pictures. 3: Don't like pictures. 2: The format supports the concepts.
Math 24	9: Preference for "plug 'n chug" problems. 4: Find equations/numbers easy to work with. 3: Preference for mathematics over pictures.	Math 18	5: Preference for "plug 'n chug" problems. 4: Preference for mathematics over concepts. 3: Preference for mathematics over pictures.
Graphical 12	3: Visual learners/people. 2: Like having a visualization provided. 2: Connected it to the pre-recitation homework.	Graphical 10	2: Visual learners/people. 2: Preference for visuals over math. 2: Connected it to the pre-recitation homework.
Pictoral 51	17: Visual learners/people. 12: Connected it to lab. 9: Find other formats (esp. math/words) difficult.	Pictoral 35	12: Liked the colors/found it attractive 8: Like having a visualization provided. 6: Connected it to lab.

FIG. 3: Reasons 2020 students gave for choosing a particular representation of a quiz. Only the largest categories are presented here. The number in each format box is the number of usable responses.

1. *Student comments*

The students in the format choice groups were asked to "Write a few sentences about why you chose the problem format you did." We then coded these responses, separating them into categories that developed as the coding proceeded. In figures 3 and 4 we present the three dominant categories for each quiz. Appendix B shows the complete comment data. Some remarks regarding our categorization methods: Students in the "visual learners" category have explicitly identified themselves as visual learners or visual people. People that expressed a preference for "plug and chug" problems used language that clearly indicated the insertion of numbers into formulas in a simple fashion, and always used the words plug and/or chug. Students that remarked that they simply found equations or mathematics easier to handle or more straightforward were placed in other categories. In some cases, there is a category for those that chose a format because they were attracted to it and a separate category for those that chose a format because they were avoiding a different one. Many of the responses were too vague to be useful; "Pictures are pretty," for example. These were discarded.

There are a few notable trends. First, 72% of all the choice group students (including those who did not make comments) selected either a math or pictoral format quiz. We also see that the vast majority of students who cited their lab in explaining their choice chose the pictoral format. This is despite the fact the recent lab included representations that corresponded to each quiz format.

There are a fair number of students that chose the mathematical format expecting a plug

2010 Spring Quiz		2010 Pendulum Quiz	
Verbal 9	6: Preference for concepts/intuition 3: Other formats are difficult.	Verbal 17	4: Preference for concepts/words over math/pictures. 4: Clear/ordered presentation. 3: Other formats are difficult.
Math 74	16: Preference for mathematics over concepts. 12: Like the definite/straightforward nature. 11: Comfortable handling equations. 7: Preference for "plug 'n chug" problems.	Math 22	7: Comfortable handling equations. 6: Like the definite/straightforward nature. 2: Preference for mathematics over concepts.
Graphical 11	3: Visual learners/people. 2: Like having a visualization provided. 2: Connected it to the pre-recitation homework.	Graphical 27	6: Visual learners/people. 6: Like having a visualization provided. 3: Didn't like the math format before.
Pictoral 26	12: Visual learners/people. 7: Like having a visualization provided. 5: Find other formats difficult (esp. math).	Pictoral 62	15: Visual learners/people. 13: Didn't like the math format before. 12: Like having a visualization provided.

FIG. 4: Reasons 2010 students gave for choosing a particular representation of a quiz. Only the largest categories are presented here. The number in each format box is the number of usable responses.

and chug style of problem, except in the case of the 201 pendulum quiz, which followed the 201 quiz on springs. The 201 quiz on springs was unique in that the mathematical format quiz was difficult to handle through explicit calculation alone, and favored qualitative reasoning supported by equations. Eighteen students taking the 201 pendulum quiz mentioned that they didn't like the math format for the earlier spring quiz, with 13 of these choosing the pictoral format the second time. It would appear that in this case there was a mismatch between the students' conception of what constituted a math problem (plugging and chugging) and our conception (either calculation or using equations as a conceptual tool), and the students responded accordingly.

In the 202 class, 9% of the people who initially chose a verbal format quiz stayed with that format for the second quiz. Twenty-nine percent of the graphical, 42% of the pictoral, and 46% of the mathematical groups stayed with their format. For the 201 section, 73% of the verbal, 25% of the math, 71% of the graphical, and 79% of the pictoral groups stayed with their choice of format from the first quiz to the second. For all formats but the math (which, for the 201 spring quiz, was different in character from the other math problems in this study), the 201 section was substantially more likely to stay with their choice of format. Of the 76 students in 201 that changed from math on the first quiz to a different format on the second, 11 chose verbal, 22 chose graphical, and 43 chose pictoral. The strong preference for the pictoral format during this switch, the fractions of the class selecting either a math or pictoral quiz, and the the student comments overall are all consistent with the notion that students perceive the mathematical and pictoral formats to be dominant and antithetical. That is, when considering the different possible representations of a physics problem, students appear to think primarily of pictoral and mathematical formats (and not so much of others) and to think of these formats as opposites in a sense.

In both the 201 and 202 sections, many of the students who selected a graphical or pictoral format identified themselves as visual learners (15 and 7 of the students on the first and second 202 quizzes, and 15 and 21 students on the first and second 201 quizzes). No students identified themselves as any other type of learner, save one that identified himself as a kinesthetic learner and chose a mathematical format. In the cases of the pictoral formats of the 202 diffraction quiz, the 201 spring quiz, and the 201 pendulum quiz, there were enough self-identified visual learners to compare their performance to the other people choosing the same format. There were 18 self-identified visual learners in the 202 diffraction quiz, who had a success rate of 0.89 as compared to the success rate of 0.79 for the other 41 students. This difference is not statistically significant. For the 201 spring quiz, there were 12 self-identified visual learners, who had a success rate of 1.00 as compared to 0.67 for the other 27 students. This difference is significant at the $p = 0.02$ level. For the 201 pendulum quiz, there are 15 self-identified visual learners. These had a success rate of 0.87, as compared to 0.75 for the other 65 students. This difference is not statistically significant. Averaging the above, the self-identified visual learners had a success rate of 0.91 versus 0.75 for the other students, which is significant at the $p = 0.02$ level. There are a number of confounding factors that leave us hesitant to draw conclusions based on these data alone. Both the students' abilities to assess their own competencies in this fashion and the overall usefulness of categorizing people as different types of learners are somewhat unclear here, and we also note that the students made these identifications (or didn't) *after* having succeeded or failed at visual and/or non-visual tasks.

D. Students' use of multiple representations

The students in this study were provided with single representations of quiz problems, but in many cases the students' papers showed that they had made explicit use of supplementary representations in solving the quiz. This was most often a picture that they drew in support of a mathematical or verbal format problem. Some students wrote equations in

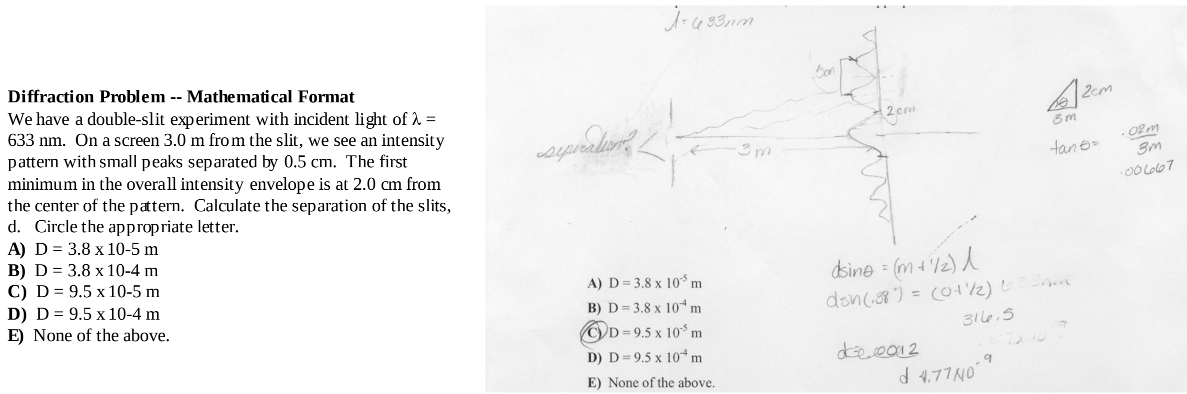

FIG. 5: A student's use of a supplementary representation (hybrid graph/picture) to solve the math format 2020 diffraction quiz.

support of non-mathematical formats and a handful wrote out physical principals longhand or drew a graph. It is a common goal in physics education to teach the use of multiple representations,[6][4] so it is interesting here to compare the performance of students that produce supplementary representations to those that do not. We should emphasise that neither of the courses studied here made an explicit attempt to teach the use of multiple representations in the style of the just-mentioned references. We should also note that the students who had no explicit supplementary representation may well have used multiple representations in their solution to some extent (it is hard to conceive of a student that can think strictly in terms of one representation and no other), but we shall focus on the students that made these explicit.

On the 202 diffraction quiz, 51% of the 172 choice-group students made explicit use of some supplementary representation. These students had an average success rate of 0.47, compared to 0.54 for the students that did not explicitly use a supplementary representation. This difference is not statistically significant (p = 0.12). Breaking it down by format, 35-40% of each of the verbal, graphical, and pictoral groups used a supplementary representation, and in each case the group using such a representation did not do significantly better or worse than the group that did not explicitly use an additional representation. Of the students that chose a math format, 75% (43 out of 57) used a supplementary representation, which was always a picture. These students had a success rate of 0.28, compared to 0.64 for the 14 that did not. This performance difference is significant at the p = 0.014 level.

That the performance difference should favor students that didn't draw a picture is surprising, and so we examined the problem in more detail. The math format quiz question is displayed along with a student drawing in Figure 5. This is a question regarding the diffraction pattern coming from two finite-width slits illuminated by monochromatic light, a topic that was featured in a lab but was covered minimally in lecture. The pattern will have a narrow peaks separated by a distance X governed by the slit separation D, and a longer-period envelope with peaks separated by a distance x governed by the slit width d. Given the distance from the slits to the screen L, the wavelength of the incident light λ, and either X or x, one can calculate either D or d using $(d, D)\sin(\theta) = n\lambda$. Most student errors involved mixing up D and d. We examined each student picture (which was often a hybrid of a picture and a graph) and categorized it. Almost no one drew a correct two-slit diffraction pattern, suggesting that this topic was not well understood at this point. Of the 35 students that drew a picture and answered the question wrong, thirteen students drew a picture that represented a single-slit diffraction pattern with peaks separated by x, which led to a mix-up of D and d in the equation. There were also eight students that drew such a picture with peaks separated by X and then used the equation appropriately, answering the question correctly. Fourteen of the students that drew a picture drew a single-slit diffraction pattern with both x and X labeled as follows: X was marked off between two peaks far from the center, and x was marked off as the distance from the centerline to the first minimum. These students did appear to notice that the distance labeled 0.5cm on their paper was roughly twice as wide as the distance labeled 2cm. This drawing was an apparent misinterpretation of the phrase "The first minimum in the overall intensity envelope is at 2.0 cm from the center of the pattern" present in the problem statement. This language is similar to that of the text and of the lab that covered this topic, though this is no guarantee that it would be understood. Of these fourteen students, two answered correctly and twelve answered incorrectly, calculating d instead of D. These twelve students account for much of the performance difference between the picture and no-picture groups.

These data recall Chi's[18] finding that in some cases novice problem solvers draw more pictures than experts while making more errors. This suggests that one motivation for using multiple representations is to work through something perceived to be difficult. However, the students that drew a picture rated the problem to be just as difficult as the students that did not draw a picture. On a Likert scale from 1 (easiest) to 5 (hardest), the students that drew a picture gave this problem an average rating of 3.76 while those that did not draw a picture gave a rating of 3.79. It is thus not clear whether the students that struggled with this problem were more likely to draw a picture. There have been other studies in which including multiple representations of a problem resulted in lower performance than using single representations. This was interpreted broadly either as an increase in cognitive load when the representations are separated[11][21] or as an increase in load stemming from an inability to map from one representation to the next.[10] The case here is somewhat different in that it appears to be tied to the specific contextual features of the problem and the problem-solvers. It seems that here the higher error rate among students using multiple representations is traceable to a particular misunderstanding of the problem statement that was much more likely to be detrimental if it was expressed in a pictoral manner. If the problem or the general background of the students on this topic had been slightly different, this likely would not have occurred.

On the 202 spectroscopy quiz, 10 out of 148 students in the choice group used a supplementary representation. This is too small a sample for analysis. We do find it notable that there would be such a large difference from topic to topic, with 51% of students using an explicit supplementary representation for the diffraction quiz and only 7% using one for the spectroscopy quiz. The average success rate across all students on this quiz (choice and control) was 0.62, which is significantly greater than the 0.42 for the students on the diffraction quiz. In contrast, the choice and control students gave the spectroscopy quiz a difficulty rating of 3.60 averaged across all formats, compared to the rating of 3.47 for the diffraction quiz. Thus, it appears that the spectroscopy quiz was easier for the students, though they did not rate it as such. It is not clear whether this influenced their decision to use an explicit supplementary representation.

On the 201 spring quiz, 74 of the 169 students in the choice group used a supplementary representation. Sixty-nine of these were students that had chosen a math-format problem. These students had a success rate of 0.55 as compared to 0.61 for the 33 students that chose a math format and did not use a supplementary representation. This difference is statistically insignificant. The use of supplementary representations was less common but somewhat more spread out for the control group on the same quiz: 9, 23, 9, and 4 students used a supplementary representation on the verbal, math, graphical, and pictoral formats respectively. This is 45 students out of 164, or 27%. The 23 students that used a supplementary representation on the math format had a success rate of 0.43, which did not differ significantly from the success rate of 0.38 achieved by the 22 students that did not use a supplementary representation. These data were very similar for the 201 pendulum quiz.

In summary, students that use explicit supplementary representations on these quizzes are roughly as successful as those that do not, with one case in which they are less successful. This is consistent with the cognitive science results mentioned previously in which researchers found that multiple representations do not necessarily increase performance. Rather, multiple representations are tools that students can either use productively or not.

IV. DISCUSSION AND CONCLUSION

This study attempts to answer a number of questions. First, does student problem solving performance vary with representational format? We see evidence that it does, often strongly. In the case of the Bohr-model homework problem, the performance difference between the nearly-isomorphic graphical and pictoral problems is due to students selecting a particular distractor. This distractor is one that superficially resembles energy-level diagrams that they have seen associated with this material, but only when it is represented pictorally. We also see students in the random-format groups doing much better on a pictoral format spectroscopy quiz than on a verbal format of that same quiz. It is less clear what might have triggered this. While the choice groups make it clear that students connect the pictoral format more closely to their laboratory experiences, the lab did not ask them to consider this particular concept (though it did make use of similar representations). This issue is further confounded by the fact that the students that were allowed to choose a pictoral format, in the process perhaps identifying themselves as students connecting more strongly with the laboratory, did significantly worse than the students that chose a verbal format.

We note that students that were randomly assigned a mathematical quiz did significantly worse in three of the four cases than students assigned any other format. This was true in two cases when the math problem involved simple calculation. This is surprising since the selections and comments of the choice groups, in particular the reasons cited for the move away from the math format in response to the first 201 quiz (a problem that was not "plug 'n chug"), suggested that many of the students preferred plug 'n chug problems to other sorts. The poorer performance on the mathematical format was also present on the aforementioned 201 quiz where the mathematical format was more easily handled with conceptual reasoning. In that case, the equations appear to have provoked the students to spend time on unhelpful calculations instead of thinking about the problem. Students calculating without thinking has been observed many times before, and has been attributed in part to a lack of meta-level skills.[22]

Given that students do perform differently on different assigned representations of problems, we should like to explain why. The data suggest that performance on different representations depends on a number of things, including student expectations, prior knowledge, metacognitive skills, and the specific contextual features of the problems and the representations. This dependence on specific context also seems to be responsible for the reduced performance of some 202 students that made use of multiple representations, as compared to students that did not. It may also be that different problem representations are prompting different problem-solving strategies, as Koedinger[1] has observed in young algebra students. The strategies of our students cannot be consistently inferred from the data presented here, and so we have interviewed students in-depth as they solve these sorts of problems. The results of these interviews will be part of a follow-up paper.

We also set out to determine whether allowing students to choose which representation they worked in had an effect on their performance. The data show that giving them this choice for a quiz did indeed result in performance differences as compared to the random-format students; however the direction of that effect turned out to be inconsistent. In some cases, students given a choice of representation did much better than the students that were assigned a format at random; in other cases, they did much worse. Furthermore, whether the choice group did better or worse than the control group for a particular format sometimes varied from one quiz topic to the next, as was the case for both the graphical and pictoral groups in the 202 section. This could possibly be explained by the movement of a group of students that is good at choosing from one format to another, but analysis of the students that switch formats shows that this is not the case. Students that stayed with these formats did approximately as well on the second quiz as students that switched to these formats.

In trying to explain the effect of student choice, we examined the students' comments. It appears that, correct or otherwise, students generally view mathematical and pictoral representations as dominant and opposite, at least out of the set of representations presented here. Most students selected one of these two formats. Students that switched from a mathematical format typically switched to a pictoral format. Student comments regarding their choice of quizzes frequently pitted mathematics against pictures, with one being favored versus the other. These same comments suggest that students connect pictoral representations quite strongly with 'concepts,' which students appear to view as unconnected to the mathematics.

One could suppose that student choices and performances are guided by intrinsic learning styles. While it is the case that self-identified visual learners performed better on one of the pictoral format quizzes than other students (which is cause and which is effect is not clear here), the arrangement of the performance variations seen here suggests that the bulk of the data cannot be explained by a simple alignment of student choices with some individual learning style. For example, the students that chose pictoral format quizzes for both the diffraction and spectroscopy topics had success rates of 0.86 and 0.33 on these quizzes. This is a dramatic difference since it appears that the diffraction quiz was easier overall, as the choice group had respective success rates of 0.48 and 0.70 averaged across all formats. A learning styles explanation would expect the same students to perform reasonably consistently on the same formats relative to the rest of the class. Considering the complexity of the performance data, it appears likely that an explanation of the effect of giving students a choice of representation will need to carefully attend to the context of the problem, much as we argued above regarding a complete explanation of student performance on different representations.

The fact that the 201 students showed a much smaller performance difference between their choice and control groups may be a function of the broader context, including the methods of instruction. As we described before, the 201 class included more reforms and the features of the course may have provided students with a more varied set of representational skills. This could have leveled out students' performances on their preferred representations as compared to other representational formats. This might also explain the 201 students' much greater tendencies to remain with a format from quiz to quiz, as students with broader representational skills may be less likely to be dissatisfied with a particular representation.

From the above, it appears that a complete understanding of student representational competence will need to attend to the specific and general features of the problems, the courses, and the learners. Put another way, we believe that we will need an in-depth analysis of the different layers of the context present.[23]. To this end, our next paper will synthesize the results shown here, the results of student problem-solving interviews, and an analysis of the lectures and exams of the 201 and 202 sections. Our goal will be to begin to develop a coherent theoretical view of how physics students handle multiple representations in solving problems, with a particular focus on the context of such learning. We will also further investigate the role that different instructional techniques play in developing students' representational and meta-representational competencies.

Acknowledgments

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APPENDIX A: STUDY HOMEWORKS AND QUIZZES

Note that some of the problem elements have been rearranged or omitted (for instance, the Likert scales on problems after the first) in order to save space.

Physics 2010 HW – Energy and motion

Question 1 - Verbal Format

A professor drops a ball from the top of an eight-story physics building. At what point has the ball reached half of the speed it has just before it hits the ground? Neglect air resistance.

A) The ball has reached half of its final speed when it has fallen two stories.
B) The ball has reached half of its final speed when it has fallen four stories.
C) The ball has reached half of its final speed when it has fallen six stories.
D) The ball has reached half of its final speed at some other point.

How difficult did you consider this question? (Circle the best number)

Easy **1** **2** **3** **4** **5** Hard

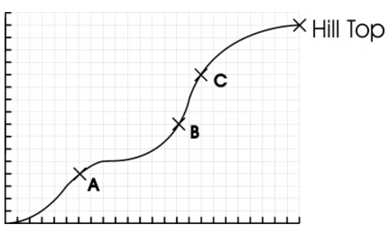
Question 2 - Mathematical Format

A dumbbell (m = 15 kg) is dropped from a height of 12 meters. Calculate the speed of the dumbbell when it has fallen to a height of 3 meters. Neglect air resistance, and round to the nearest 0.1 m/s. Note that $g = 9.8\ m/s^2$.

A) 9.4 m/s
B) 13.3 m/s
C) 7.7 m/s
D) None of the above.

Question 3 - Graphical Format

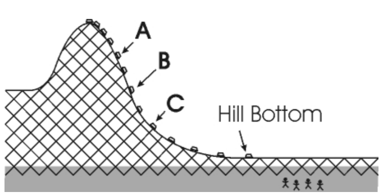
A roller coaster car approaches a hill going just fast enough to reach the top and stop. A graph of the hill's height versus its horizontal coordinate is shown.
At what point has the car slowed down to half of its original speed? Neglect friction.



A) Point A B) Point B C) Point C
D) Somewhere else.

Question 4 - Pictorial Format

A roller coaster car comes to rest at the top of a hill before starting down the other side. At what point on the track is the car moving at one half of the speed it has at the bottom of the hill? Ignore friction.



A) Point A B) Point B C) Point C
D) Somewhere else.

Physics 2020 Diffraction/interference HW

Question 1 - Verbal Format

We have two very flat pieces of glass on top of each other. The left side of the top piece of glass is propped up very slightly, so that there is a thin layer of air between the pieces. When we shine light on the glass from above, we observe light and dark interference fringes evenly spaced across the glass.

Now we prop up the right side of the top piece of glass by the same amount, so that both sides of the top piece of glass are held up by the same amount. What do we observe?

A) Now that both sides are propped, we will see twice the effect, and so there will be twice as many fringes.
B) Now that both sides are propped, we will see half the effect, and so there will be half as many fringes.
C) The change we made was symmetric, and so we will see no change in the fringe pattern.
D) The spacing between the pieces is constant, and so we will see no fringes at all.
E) The new setup is exactly out of phase with the old one, so we will see light where before there was dark, and dark where before there was light.

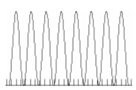
Question 2 - Mathematical Format

We have light of wavelength λ = 600 nm shining on two very flat pieces of glass on top of each other. The top piece is propped up on one side, and we see bright and dark interference fringes across the glass. By how much does the thickness of the air gap change as you go from one bright fringe to the next bright fringe?

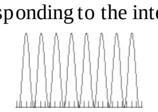
A) t = 150 nm
B) t = 300 nm
C) t = 450 nm
D) t = 600 nm
E) t = 1200 nm

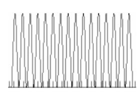
Question 3 - Graphical Format

We have two very flat pieces of glass on top of each other. One side of the top piece of glass is propped up very slightly, and we shine light on the glass from above. We observe fringes whose intensity vs. position can be graphed as:

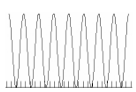


Now we change things by halving the distance by which the side of the top piece is propped up. Choose the graph corresponding to the intensity of the fringes we now see:

A) 

B) 

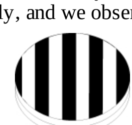
C)

D) 

E) 

Question 4 - Pictorial Format

We have two very flat pieces of glass on top of each other. One side of the top piece of glass is propped up very slightly, and we observe the following interference fringes:



Next, we prop up the side of the top piece twice as far as before. Choose the picture representing the resulting fringe pattern.

A) 

B) 

C) 

D) 

E) 

Physics 2020 Bohr-model HW

Question 1 - Verbal Format

An electron in a Bohr-model hydrogen atom is in the ‘orbit’ with the lowest possible energy. How does the radius of the electron orbit change if it moves up to the third energy level?

A) The radius of the new orbit will be three times the original radius.
B) The radius of the new orbit will be nine times the original radius.
C) The radius of the new orbit will be one-third the original radius.
D) The radius of the new orbit will be one-ninth the original radius.
E) None of these.

Question 2 - Mathematical Format

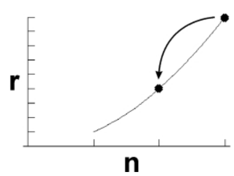
The Bohr radius for an electron is $r_1=0.529\text{x}10^{-10}$ m. Calculate the radius of the n=4 energy level.

A) $r_4=0.033\text{x}10^{-10}$ m
B) $r_4=0.132\text{x}10^{-10}$ m
C) $r_4=2.116\text{x}10^{-10}$ m
D) $r_4=8.464\text{x}10^{-10}$ m
E) None of the above.

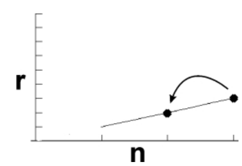
Question 3 – Graphical

An electron in a Bohr hydrogen atom jumps from the n=3 orbit to the n=2 orbit. The following graphs show the orbit radius r as a function of the orbit number n. Choose the graph that best represents the relative locations of the electron orbits.

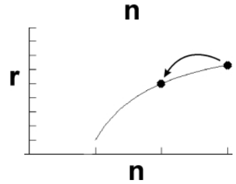
A)



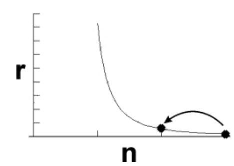
B)



C)



D)

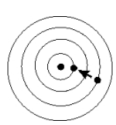


E) None of these.

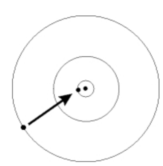
Question 4 – Pictorial

An electron in a Bohr hydrogen atom jumps from the n=3 orbit to the n=1 orbit. Choose the picture that best represents the relative locations of the electron orbits.

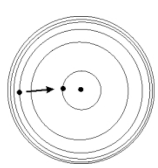
A)



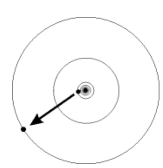
B)



C)



D)



E) None of these.

Diffraction Problem -- Verbal Format

We have a double-slit experiment set up. A helium-neon laser is shining on a pair of finite-width slits, and we see a corresponding intensity pattern on a screen. The pattern consists of narrow, closely spaced spots that get brighter and dimmer as you look across the screen, periodically dropping to nothing. I take the slits away and replace them with a pair of slits that are the same width, but are twice as far apart. What happens to the intensity pattern? Circle the appropriate letter.

A) The entire pattern squishes together so that it is half as wide.

B) The narrow peaks are half as far apart, and the rest of the pattern is unchanged.

C) The narrow peaks are the same distance apart, but the places where the peaks drop away to nothing are twice as far apart.

D) The narrow peaks are the same distance apart, but the places where the peaks drop away to nothing are half as far apart.

E) The narrow peaks are twice as wide, and the rest of the pattern is unchanged.

How difficult did you consider this question? (Circle the appropriate number)

Easy **1** **2** **3** **4** **5** Hard

Diffraction Problem -- Mathematical Format

We have a double-slit experiment with incident light of λ = 633 nm. On a screen 3.0 m from the slit, we see an intensity pattern with small peaks separated by 0.5 cm. The first minimum in the overall intensity envelope is at 2.0 cm from the center of the pattern. Calculate the separation of the slits, d. Circle the appropriate letter.

A) D = 3.8×10^{-5} m

B) D = 3.8×10^{-4} m

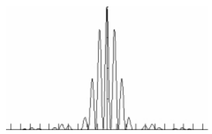
C) D = 9.5×10^{-5} m

D) D = 9.5×10^{-4} m

E) None of the above.

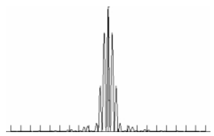
Diffraction Problem -- Graphical Format

We have a double-slit experiment setup. A helium-neon laser is shining on a pair of finite-width slits, and we see a corresponding intensity pattern on a screen. The graph of the intensity of this pattern versus position is:

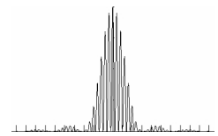


I take the slits away and replace them with a pair that has slits that are of the same width, but twice as far apart. What happens to the pattern on the screen? Circle the appropriate letter.

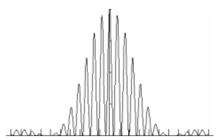
A)



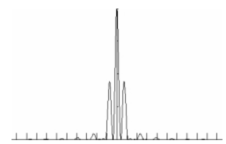
B)



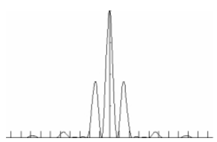
C)



D)

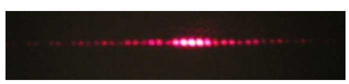


E)



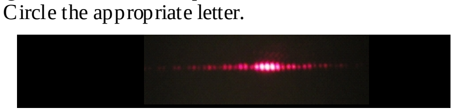
Diffraction Problem -- Pictorial Format

Suppose we have a double-slit experiment set up. A helium-neon laser is shining on two slits of finite width. On the screen behind the slits we see an intensity pattern that looks like:

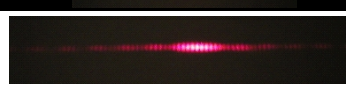


Now we change the double slit setup so that the slits are twice as far apart. Which of the following intensity patterns will we see? Circle the appropriate letter.

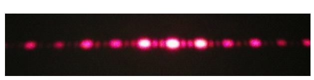
A)



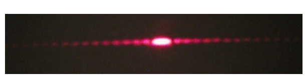
B)



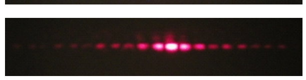
C)



D)



E)



Spectroscopy Problem -- Verbal Format

Consider the Balmer series of spectral lines from hydrogen gas. Now suppose we are in a world where electric charges are weaker, so the electron is not held as tightly by the nucleus. This means that the ionization energy for the electron will be smaller. What will happen to the Balmer lines that we see?

A) The spectral lines will remain the same.

B) The spectral lines will all shift to shorter wavelengths (toward the bluer colors).

C) The spectral lines will all shift to longer wavelengths (toward the redder colors).

D) The spectral lines will all shift toward the center of the visible spectrum.

E) Something else.

Spectroscopy Problem -- Mathematical Format

Suppose that we change the hydrogen atom so that the ionization energy for the electron is 11 eV instead of 13.6 eV. Calculate the energy of the photon emitted when the electron moves from the $n = 4$ to the $n = 2$ orbit.

A) 3.15 eV

B) 2.75 eV

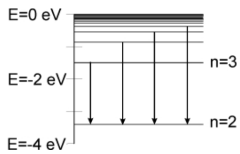
C) 2.06 eV

D) 2.55 eV

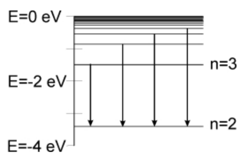
E) None of the above.

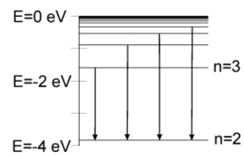
Spectroscopy Problem -- Graphical Format

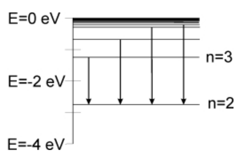
The energy level diagram below shows the electron transitions that lead to the Balmer series:

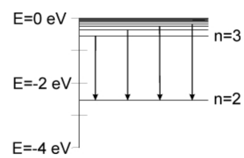


Now suppose we are in a world where electric charges are weaker, so the electron is not held as tightly by the nucleus and the ionization energy is 11 eV instead of 13.6 eV. Choose the graph that best represents what the new energy levels would look like.

A) 

B) 

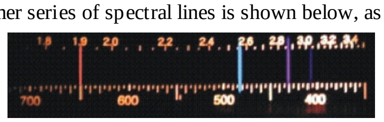
C) 

D) 

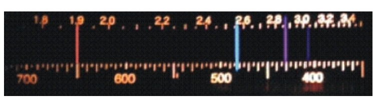
E) Something else.

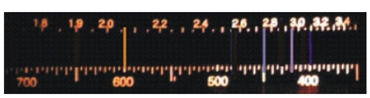
Spectroscopy Problem -- Pictorial Format

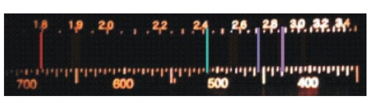
The Balmer series of spectral lines is shown below, as seen through a spectrometer:

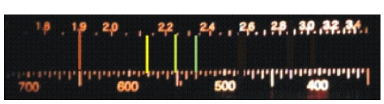


Now suppose we are in a world where electric charges are weaker, so the electron is not held as tightly by the nucleus and the ionization energy is 13 eV instead of 13.6 eV. Choose the picture that best represents what the new spectrum would look like.

A) 

B) 

C) 

D) 

E) Something else.

Spring Problem -- Verbal Format

A ball is hanging on a spring, and is oscillating up and down. At which point is the ball moving fastest?

A) The ball is moving fastest when it is at its highest point.

B) The ball is moving fastest when it is at the midpoint of its motion and is moving down.

C) The ball is moving fastest when it is at its lowest point.

D) None of the above are true.

Spring Problem -- Mathematical Format

A ball is hanging from a spring at rest at y = 0 cm. The spring is then compressed until the ball is at y = 5 cm, and is then released so that the ball oscillates. Up is in the positive-y direction. At which point y is the ball moving fastest? Note that

$$K = \frac{1}{2}mv^2 \qquad U_{spring} = \frac{1}{2}k\left(y - y_0\right)^2 \qquad U_{gravity} = mgy$$

where y_0 is the unstretched length of the spring.

A) y = -5 cm

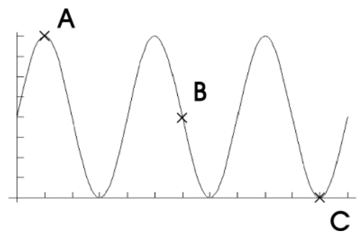
B) y = 0 cm

C) y = +5 cm

D) None of the above.

Spring Problem -- Graphical Format

A ball is hanging on a spring and oscillating up and down. The height of the ball as a function of time is graphed below. At which point is the ball moving fastest?



A) The ball is moving fastest at point A.

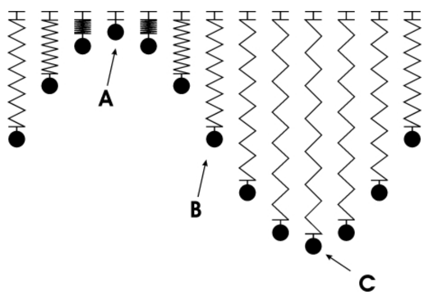
B) The ball is moving fastest at point B.

C) The ball is moving fastest at point C.

D) None of the above are true.

Spring Problem -- Pictorial Format

A ball on a hanging spring is oscillating up and down as shown in the following snapshots.



At which point is the ball moving the fastest?

A) The ball is moving fastest at point A.

B) The ball is moving fastest at point B.

C) The ball is moving fastest at point C.

D) None of the above are true.

Pendulum Problem -- Verbal Format

I set up a pendulum in front of you and pull it back (to your right), and then let it go. The pendulum takes one second to reach the point opposite from where it started.

Now I lengthen the pendulum's string until it is four times as long as it was, with the mass unchanged. I pull the pendulum back to the right again (far enough that the string is at the same angle as before), and let it go. Where is it after one second? Circle the correct answer.

A) Straight up and down, and moving left.

B) Opposite from its starting position.

C) Straight up and down, and moving right.

D) Back in its starting position.

E) Somewhere else.

Pendulum Problem -- Mathematical Format

A pendulum of length L = 1 m starts at x = +5 cm and is released at t = 0. At t = 1 s, it is at almost exactly x = -5 cm. Now suppose we change the length of the pendulum to L = 4 m without changing the mass. We pull it back and release it from x = +20 cm at t = 0. Find x and the sign of the pendulum's velocity at t = 1 s.

Possibly useful equations:

$$x = A\cos\left(\frac{2\pi t}{T}\right) \qquad v = -A\frac{2\pi}{T}\sin\left(\frac{2\pi t}{T}\right) \qquad T = 2\pi\sqrt{\frac{L}{g}}$$

A) x = 0 cm, v is negative.

B) x = -20 cm, v is zero.

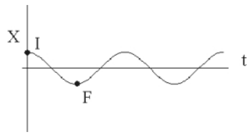
C) x = 0, v is positive.

D) x = +20 cm, v is zero.

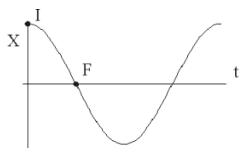
E) None of the above.

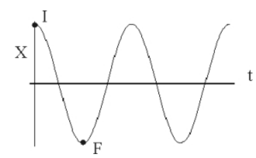
Pendulum Problem -- Graphical Format

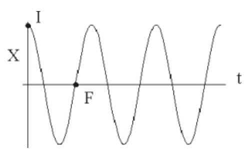
The following graph shows the motion of a pendulum after we let it go at t = 0. The horizontal axis shows the time t and the vertical axis shows the position of the pendulum X. It starts at point I and is at point F at t = 1s.

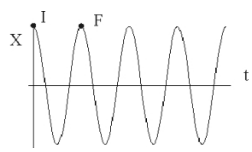


Now I change the pendulum so that it is four times as long as before, but the same mass. I pull it back to the same angle it was at before, and let it go. The graphs below show several possible motions of the pendulum. Let point F again represent t = 1s. Circle the graph that correctly describes the motion.

A)


B)


C)


D)


E) None of the above.

Pendulum Problem -- Pictorial Format

I pull a pendulum back to the position shown below on the left and let it go. It takes one second to swing into the position shown below and on the right.

Start:

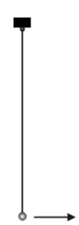

After one second:


Now I change the pendulum so that it is four times as long as before, with the same mass. I pull the pendulum back to the same side to the same angle as before and then let it go.

Select the picture that corresponds to the position of the new pendulum after one second. If the pendulum is straight up and down, select the picture that indicates the correct direction of the motion.

A)


B)


C)


D)


E) None of the above.

APPENDIX B: STUDENT REASONS FOR SELECTING A REPRESENTATIONAL FORMAT

2020 Diffraction quiz	
Verbal 10 usable	3: Preference for qualitative analysis. 1: Connected it to the pre-recitation homework. 1: Prefers concepts to math. 1: Not good at math. 1: Thought it would be good practice.
Math 24	9: Preference for "plug 'n chug" problems. 4: Find equations/numbers easy to work with. 3: Preference for mathematics over pictures. 2: Connected it to the lab. 1: Connected it to the pre-recitation homework.
Graphical 12	3: Visual learners/people. 2: Like having a visualization provided. 2: Connected it to the pre-recitation homework. 1: Connected it to the lab. 1: Like having a qualitative/quantitative hybrid
Pictoral 51	17: Visual learners/people. 12: Connected it to lab. 9: Find other formats (esp. math/words) difficult. 7: Like having a visualization provided. 5: Preference for concepts/concepts over math.

2020 Spectroscopy quiz	
Verbal 12	3: Preference for concepts/words over math/pictures. 3: Don't like pictures. 2: The format supports the concepts.
Math 18	5: Preference for "plug 'n chug" problems. 4: Preference for mathematics over concepts. 3: Preference for mathematics over pictures. 2: Like the definite/straightforward nature. 1: The format supports the concepts.
Graphical 10	2: Visual learners/people. 2: Preference for visuals over math. 2: Connected it to the pre-recitation homework. 1: The format supports the concepts. 1: Like having a qualitative/quantitative hybrid
Pictoral 35	12: Liked the colors/found it attractive 8: Like having a visualization provided. 6: Connected it to lab. 5: Visual learners/people. 2: Preference for concepts/concepts over math. 2: Thought it would be good practice.

2010 Spring quiz	
Verbal 9	6: Preference for concepts/intuition 3: Other formats are difficult.
Math 74	16: Preference for mathematics over concepts. 12: Like the definite/straightforward nature. 11: Comfortable handling equations. 7: Preference for "plug 'n chug" problems. 7: Find other formats difficult. 7: Connected it to the pre-recitation homework
Graphical 11	3: Visual learners/people. 2: Like having a visualization provided. 2: Find other formats difficult. 2: Connected it to the pre-recitation homework. 1: The format supports the concepts. 1: Like having a qualitative/quantitative hybrid
Pictoral 26	12: Visual learners/people. 7: Like having a visualization provided. 5: Find other formats difficult (esp. math). 3: Preference for pictures over equations/numbers.

2010 Pendulum quiz	
Verbal 17	4: Preference for concepts/words over math/pictures. 4: Clear/ordered presentation. 3: Other formats are difficult. 2: Eases visualization of the problem. 2: Didn't like the math format before.
Math 22	7: Comfortable handling equations. 6: Like the definite/straightforward nature. 2: Preference for mathematics over concepts. 2: The format supports the concepts. 1: Preference for "plug 'n chug" problems.
Graphical 27	6: Visual learners/people. 6: Like having a visualization provided. 3: Like having a qualitative/quantitative hybrid 3: Didn't like the math format before. 2: Suits the topic well. 1: The format supports the concepts.
Pictoral 62	15: Visual learners/people. 13: Didn't like the math format before. 12: Like having a visualization provided. 5: Don't like math generally. 5: Preference for pictures over equations/numbers. 3: Connection to real life. 3: Connection to lab and lecture demos. 2: The format supports the concepts.