Cognitive Issues in Upper-Division Electricity & Magnetism

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We report on common student difficulties in junior Electricity & Magnetism I (Electro- and Magneto-Statics). As part of our efforts to systematically improve this course we developed a conceptual test, the CUE (Colorado Upper-division Electrostatics), in order to assess impacts of the transformed curriculum and systematically document student difficulties. We find persistent errors in students' ability to conceptually understand and visualize E&M, to accurately communicate that understanding, and to appropriately apply problem-solving strategies. Our work underlines the need for further research on the nature of student learning – and appropriate instructional interventions –at the upper division.

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INTRODUCTION

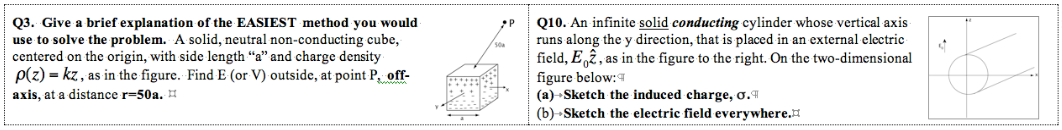
The PER community has established a large and growing research base which includes developing and evaluating curricula to improve our teaching of introductory physics topics. In the field of electromagnetism (E&M), much effort has focused on the introductory level[1], including assessment[2], but less attention has been focused on the upper division[3]. At CU Boulder, we are engaged in a long-term effort to extend PER methods to an investigation of junior-level E&M[4]. This work involves three strands: articulation of learning goals in collaboration with a broad cross-section of faculty, development of materials and teaching practices to address those goals, and construction of appropriate assessment tools.

In this paper, we present some initial results arising from our development of the Colorado upper-division Electrostatics assessment (CUE.) The CUE serves multiple purposes. First, it provides direct measurement of a limited (but not narrow) range of course goals valued by working faculty. Much as the FCI provides a "litmus-test" of the effectiveness of lower-division intro courses, the CUE may serve as a comparative instrument to assess *courses* in upper-division E&M. Second, and again following on a role played by the FCI, the questions are designed to seem straightforward to an expert. Thus, poor performance by students may drive faculty to pay more attention to their own goals and teaching methods. Third, the open-ended nature of the instrument provides potential to examine a variety of aspects of student thinking and to elicit student ideas. This provides opportunities for education researchers to learn more about specific areas where students are struggling at this level. Such deepened understanding of student learning difficulties is a prerequisite for meaningful course reforms. In this paper, after a brief introduction to the CUE, we focus on a few select topical and procedural areas of student difficulties uncovered by the CUE. This work is preliminary, and intended more as a demonstration of the potential of this instrument than as a summary of outline of outcomes. We also hope to encourage other faculty to extend this work, as the community expands its focus on upper-division transformations.

METHODS

Learning goals for the course were developed in a series of meetings with a group of PER and non-PER faculty. These learning goals externalize the often implicit expectations that faculty believe that a junior level physics student should gain in this and other upper division courses, such as "choose and apply the appropriate problem-solving technique" and "sketch the physical parameters of a problem."[5] These goals guided development of the CUE and course approach.

The CUE is a 17-question test (15 questions on electrostatics, and 2 questions on magnetostatics) testing students' performance on the learning goals of the course: ie., to choose a problem-solving method and defend that choice, sketch electric field patterns, graph electric field strength and potentials, and explain the physics and mathematics underlying steps in



Q3. Give a brief explanation of the EASIEST method you would use to solve the problem. A solid, neutral non-conducting cube, centered on the origin, with side length “a” and charge density $\rho(z) = kz$, as in the figure. Find E (or V) outside, at point P, **off-axis**, at a distance **r=50a.**

Q10. An infinite solid ***conducting*** cylinder whose vertical axis runs along the y direction, that is placed in an external electric field, $E_0\hat{z}$, as in the figure to the right. On the two-dimensional figure below:

(a) Sketch the induced charge, σ.

(b) Sketch the electric field everywhere.

FIGURE 1. Two problems from the CUE, Q2 (5 points) and Q10 (10 points). Total exam is worth 118 points.

common problems. A pre-test was developed from a subset of the questions on the CUE. With the exception of one multiple-choice question the exam is open-ended; 3 additional questions give students a multiple-choice alternative and require students to explain their answer to receive credit. The CUE is given in a single 50-minute lecture. Fig 1 shows two sample questions from the CUE[6]. The CUE was validated and the independent graders agree on the overall CUE score within 95% for all students, and on the question score within 80% on all but 3 questions[7].

The CUE was given in-class to 203 students in 4 semesters at the University of Colorado (CU) and 5 non-CU courses[7]. Because the CUE was modified from semester to semester, the “Comparison CUE” score (88 points total out of 118). Comparison CUE Scores for the 5 courses with N>20 are shown in Figure 2. Students in courses using transformed curricula perform better than those in traditional courses, suggesting the CUE measures differences that we care about.

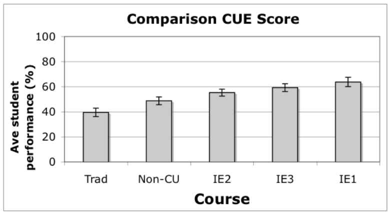


FIGURE 2. Comparison CUE score (Total N=203). “Trad” (N=26) is a traditional lecture-taught course, and IE1-3 (N=21, 48, and 27) used research-based course transformations, all at CU, whereas “Non-CU” (N=91) is the average of two large lecture courses at another public university, *not* taught using our transformed curriculum.**Error! Bookmark not defined.**

STUDENT PERFORMANCE

The CUE taps into four main learning goals for the course, which can be subcategorized as in Table 1.

TABLE 1. Learning Goals addressed by the CUE. *Due to slightly different versions of the test, the number of questions varies slightly by course (generally “Trad” consists of 1-2 questions fewer than the IE courses).*

Learning Goal	Description	# of Questions
1. Math/Physics connection	Physical meaning of equations	3
2. Visualizing the problem	Sketching, graphing	3
3. Communication	Explanations and justifications	9 (Trad) 11 (others)
4. Problem-Solving		
(a) Appropriate method choice	Correct method choice for the problem	6 (Trad) 7 (others)
(b) Techniques and skills	Superposition, Approximations, Symmetry, Sep. of Var.	12 (Trad) 14 (Others)

Figure 3A shows student scores on these different learning goals. Performance in all areas is low (never over 60%), though students in the transformed courses do better overall. This is a challenging test. The CUE was also given at a small private liberal arts college (N=12); those students were better prepared (as measured by the CUE pre-test) and performed 10-20% better than Ave IE on all areas except Math/Physics Connection. Thus, it is possible for different student populations to perform well on these measures.

In the remainder of this paper we briefly point to a few preliminary observations about common student errors on the skills tested by the CUE.

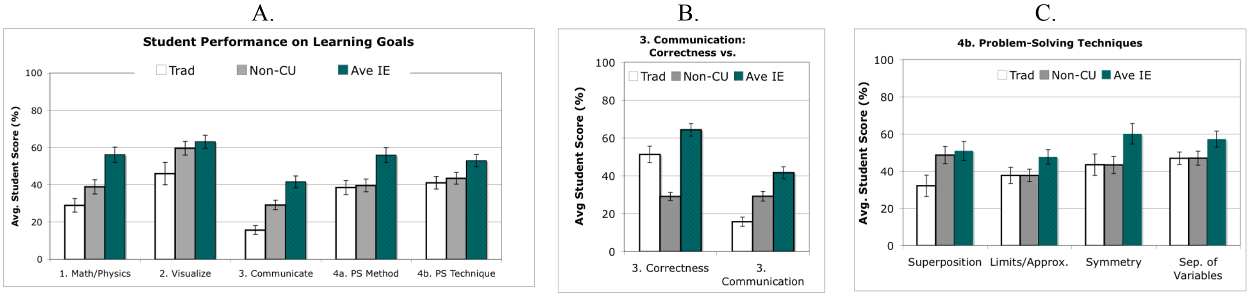


FIGURE 3. (A) Student performance on Learning Goals. “Ave IE” is the average of transformed CU courses IE1-3. All questions are given equal weights for purposes of this analysis, regardless of the points they contribute towards the final CUE score. (B) Communication score compared to correctness score. (C) Problem-solving techniques broken out by category..

Communication

One learning goal that students clearly struggle with in all courses is that of Communication. This category consists of questions where students are asked to justify and explain an answer, as in Q3 (Figure 1). In all such questions, the "correctness" of the answer (did they choose the right problem solving method, did they choose the correct multiple choice response) was coded and graded separately from their explanation. This allows us to separate the quality of the explanation from the factual content knowledge (Figure 3B). In all courses, there is a marked difference between students' ability to correctly answer the question (e.g., "Dipole Approximation" for Q3) and their ability to satisfactorily explain or justify their choice (e.g., "You are far away from the cube, so the dipole is a good approximation; lack of symmetry for using Coulomb's Law.") This is not surprising, as students' answer-making skills are typically cultivated more strongly than their ability to explain those answers.

There are a variety of types of erroneous explanations that students can give. Using Q3 as an example, some explanations were insufficient ("because it looks like the monopole term would be zero," "looks like dipole"), used inappropriate equations ("$\nabla^2 V = 0$ *and then* $E = -\nabla V$") or simply gave the incorrect or partially correct answer ("direct integration, no symmetry"). A more complete explanation, for example, would be "distance is great enough to just use dipole"

Additionally, many students in the Trad course simply wrote the name of the method (e.g. "Multipole Expansion") without an explanation for their choice of strategy, which certainly lowered their scores. The fact that the transformed curriculum has an effect on students abilities to explain their answer likely reflects both (a) enhanced conceptual understanding due to the focus of the transformed course approach, and (b) the value that those instructors placed on reasoning throughout the course.

Visualization

Two questions ask students to graph or sketch E fields or potential. Students perform relatively well on this learning goal (at least compared to others), but even here there are difficulties. One of these questions (Q10) asks students to sketch the E-field outside an uncharged metal cylinder placed in an electric field. Students score an average of 62% on this question. The most common student responses are given in Figure 4, below. Students often fail to correctly superpose E-fields to obtain the full E-field outside, or correctly show that E=0 inside the cylinder. Their mistakes on this question also reveal difficulties in contemplating the considering the interaction between fields and charged particles. Students are, for the most part, able to correctly draw the distribution of charge on the cylinder *resulting* from the external E-field (81% average score), but deciding how those charges *create* an E field outside the cylinder, which superposes on E_0 (average score 50%), proves a greater difficulty.

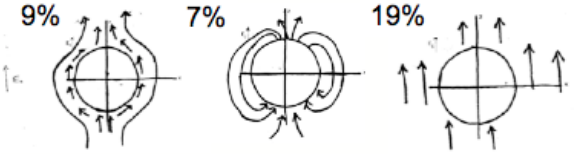


FIGURE 4. Sample student responses (with % of students drawing this figure) for Q10: E-field of metal cylinder in E_0.

Problem-solving skills

The learning goal "Students should be able to choose and apply the problem-solving technique that is appropriate to a particular problem" is a central tenet of all upper division courses. Most questions on the CUE can be characterized as falling under the rubric of "problem solving" to some degree. We broke these questions into two parts: (a) "choosing the right problem solving method", and (b) "demonstrating understanding of a problem-solving technique/skill".

Choosing the right problem solving method

Seven questions on the CUE (such as Q3) require students to extract the central features of a problem and recognize which solution method will best fit the problem at hand. For example, Q2 asks students how to solve for E of a charged cube (same physical situation as the one shown in Figure 1, Q3) when point P is *on*-axis and *close* to the cube. In order to arrive at the correct answer ("Coulomb's Law") the student has to recognize that, while the problem is symmetric, that symmetry is not of the appropriate type to use Gauss' Law. They also must recognize that because the point of interest is close by, the dipole term of the multipole expansion will not yield an accurate approximation. However, among the 203 students whose results are reported in Figure 3, these are the most common incorrect responses: 19% answer "Multipole Expansion" and 13% of respondents choose "Gauss' Law." Only 42% of students receive full credit for the correct answer. When point P is moved off-axis, and far away, only 32% of students get full credit for their choice of method (though 57% mention dipole or multipole in some way), with 22% answering "direct integration." While direct integration is not *wrong* (and received partial credit) it's certainly not the preferred method to solve this problem. Thus, students struggle with extracting the general features of problems when they are taken out of the context of the presentation of that material.

Problem-solving techniques

We can break problem-solving techniques down into several subcategories, i.e., superposition (3 questions), limits/approximations (3-5 questions), symmetry (2 questions), and separation of variables (4 questions). As an example, "Limits and Approximations" consists of the following questions:

- Q3 (Figure 1): Solve for E (or V) above a dipole-like cube when P is very far away and off-axis
- Q6: Solving for B (or A) above a current loop
- Q9 (Figure 1): Determine the sign of V in the limit that r $\rightarrow\infty$ from a positively charged sphere
- Q12: The value of E of a charged disk for z<<R and the functional dependence of E on z as z>>R.
- Q14: Stating how the properties of a capacitor would change in the limit that the inserted dielectric becomes a conductor

In both Q3 and Q6, students fail to recognize the implications of the choice of an observation point that is (a) far away and (b) off-axis. The fact that the observation point is off-axis implies that direct integration will be particularly challenging, yet many students fall back on this "failsafe" method of problem solving: 22% for Q3 and a full 40% for Q6 (probably due to the decreased familiarity with a current loop as a dipole). However, a physicist will recognize that the choice of an observation point that is far away suggests a much easier method – that of the dipole approximation. Yet only 57% of students mention multipole or dipole in their answer for Q3, and just 25% in Q6. Indeed, many students who answered "Multiple Expansion" or "Direct Integration" for when the point was close to the cube answered "same thing" when the point was far away.

In Q12, a surprising number of students fail to correctly evaluate the E-field of a disk when z<<R (infinite plane; average score 43%) and when z>>R (point charge; average score 59%), with large variation by instructor. For z>>R, several students fail to give the functional $1/z^2$ dependence required for full credit, stating just correctly but incompletely that it looks like a "point charge" or goes to zero. This error points to student misunderstanding of what it means to give a "limiting form."

For z<<R, some students get lost in mathematical manipulations as they attempt to solve Gauss' Law for a plane of charge. Several students state that E goes to infinity as z<<R – an error repeated when they sketch E for the disk – 27% draw an E-field which is infinitely valued at z=0. Thus, many students have not completely grasped the difference between point charges and an idealized surface charge (which has no singularity at the origin).

CONCLUSIONS

Through administration of the CUE, we identify several serious student difficulties with the skills expected of junior physics students. While interactive, conceptually-based curricula improve student performance on these learning goals, improved instructional techniques are still needed in order to improve students' ability in these areas. We particularly identified problems in communicating understanding to others, visualizing fields, choosing and applying sophisticated problem solving techniques, and appropriately using skills such as limiting forms. This paper shows the potential of the CUE to assist us in investigating student thinking, through analysis of student responses. This work will be extended in more detail in future publications.

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