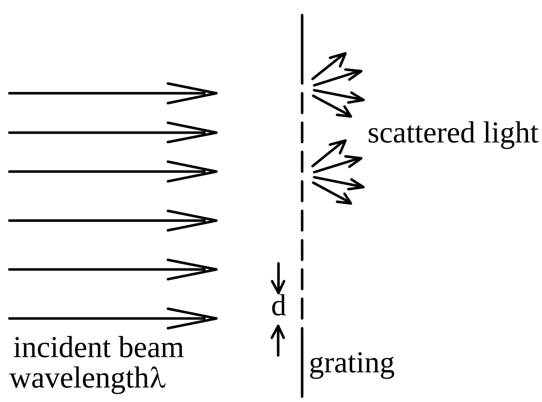
Lab 12: The Hydrogen Spectrum & Rydberg Constant

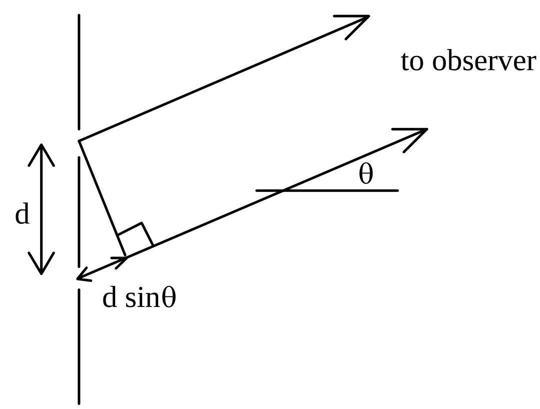
INTRODUCTION & BACKGROUND:

In this experiment you will use a diffraction-grating spectrometer to measure the wavelengths of the emission lines of hydrogen. With these measured wavelengths you will compute the Rydberg constant. To perform this experiment intelligently, you need to understand two things: (1) how a diffraction grating works and (2) the Bohr model of the hydrogen atom.

Diffraction grating review: A diffraction grating is simply a piece of glass or plastic which has a series of very fine scratches or grooves cut in its surface. The grooves are perfectly straight and parallel and are equally spaced so that there are a fixed number of grooves per millimeter, typically around 500 grooves/mm.

A grating behaves essentially like a multi-slit aperture, that is, a mask with many closely spaced slits. If the number of grooves per length is n (grooves per cm), then the separation between adjacent slits is d = 1/n (cm per line or simply, cm). Consider what happens when a beam of monochromatic (single wavelength) light strikes a grating at normal incidence, as shown below. Each groove or slit scatters the light in all forward directions. However, in only certain directions will the light scattered from different grooves interfere constructively, producing a strong beam.





The diagram on the right shows two light rays emerging from adjacent slits in the grating and heading toward an observer (or a point on a screen) at an angle θ from the normal (perpendicular) direction. In traveling to the observer, the ray from the lower slit has to travel an extra path distance; this path difference is Δpath=d·sin(θ). The two rays will interfere constructively only if the path difference is an integer number of wavelengths:

(1) $$d \sin\theta = m\lambda \,,$$

where λ is the wavelength of the light and m is any integer. At only those special angles corresponding to integer m's (m=0, 1, 2,...) will the rays from all the slits interfere constructively, producing a bright beam in that direction. In any other direction, the rays from the various slits interfere destructively and produce no light intensity. The integer m is called the order of the diffraction.

An incident light beam made of a several distinct wavelengths will be split by the grating into its component wavelengths, with each separate wavelength heading in different directions, determined by the condition $d\sin\theta = m\lambda$. In this way, the various wavelengths can be determined by measuring the angles.

Bohr model of the hydrogen atom: In the 19th century, it was known that hydrogen gas, when made to glow in an electrical discharge tube, emitted light at four particular visible wavelengths. In 1885, a Swiss high school teacher named Balmer discovered that the four wavelengths, here labeled λ_i (where i = 1,2,3,4) precisely obeyed a curious mathematical relation:

(2) $$\frac{1}{\lambda_i} = R\left(\frac{1}{2^2} - \frac{1}{n_i^2}\right)$$

where R is a constant, and n_i = 3, 4, 5, 6. The four wavelengths (or "lines") were henceforth called the Balmer lines of hydrogen. **Why** hydrogen emitted only those visible wavelengths and why the wavelengths obeyed the Balmer formula was a complete mystery.

The mystery was solved in 1913 by the Danish physicist Niels Bohr. According to the Bohr model, the electron orbiting the proton in a hydrogen atom can only exist in certain orbital states labeled with a quantum number n (n=1, 2, 3, 4...). When the electron is in orbit n, the total energy of the hydrogen atom is given by the formula:

(3) $$E_n = -R\,hc \cdot \frac{1}{n^2} = -\frac{13.6\text{eV}}{n^2},$$

where c is the speed of light, h is a constant (Plank's constant), and R is a number predicted by the Bohr model to be $R = 1.09737 \times 10^7\ \text{m}^{-1}$. The different energies E_n correspond to different orbital states of the electron. Smaller-radius orbits correspond to lower values of n and lower, more negative, energies. The n=1 state is the lowest possible energy state and is called the ground state.

When an electron makes a transition from an initial state of higher energy E_i to a final state of lower energy E_f , the atom emits a photon of energy

(4) $$E_\gamma = hf = h\frac{c}{\lambda} = E_i - E_f\,.$$

Here we have used the expression for the energy of a single photon: E = hf, where h is Planck's constant and f is the frequency of the light. From equations (3) and (4), the wavelength of the emitted photon is related to the initial and final quantum numbers like so:

(5) $$\frac{hc}{\lambda} = E_i - E_f = -R\,hc\left(\frac{1}{n_i^2} - \frac{1}{n_f^2}\right) \quad , \quad \frac{1}{\lambda} = R\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right).$$

This is none other than Balmer's formula! Transitions between any pair of states such that $n_i > n_f$ produces a photon; however, only those transitions with $n_f = 2$ and n_i = 3, 4, 5, or 6, happen to produce photons in the visible range of wavelengths. Using the measured wavelengths of the Balmer series and equation (5), one can compute the Rydberg constant R.

PART I: QUALITATIVE OBSERVATIONS

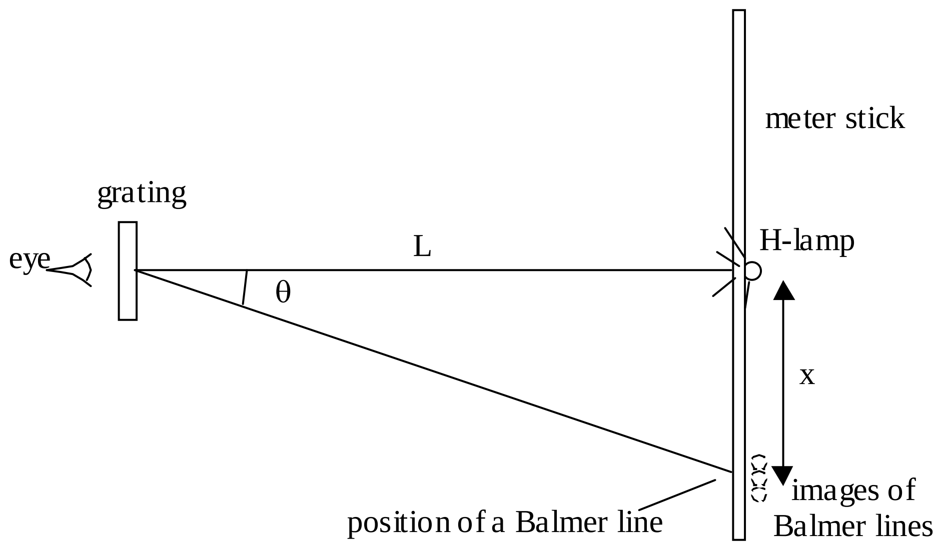
Begin by using the "Project Star" cardboard spectrometers provided to examine **qualitatively** the spectrum of the mercury lamp, the hydrogen lamp, the white light filament, and the overhead fluorescent lights. Record what you see, as descriptively as possible. Note the range of visible wavelengths.

Do these observations make sense with what you saw in class with the diffraction grating? Are there fundamental differences between the different light sources? Which sources appear to be emitting photons from specific transition lines, and which appear to be emitting a blackbody spectrum?

PART II: MEASURING THE λ'S OF THE BALMER LINES IN HYDROGEN

Most people can only see three of the four Balmer lines, because the 4^{th} line is faint and very close to the violet edge of the visible spectrum. Look at the hydrogen source through the grating provided. In the first order spectrum (m=1), you should clearly see three lines: red, blue-green, and violet (some people can see a second violet line, if the room is dark.) Record what you can see.

Arrange the hydrogen lamp and the diffraction grating on your lab bench, as shown below. Set them on supports to place them at a convenient height for your eye. Position the grating exactly L=1.00 m from the lamp and orient the grating so that it is perpendicular to the line from the lamp (that is, have the grating squarely face the lamp). Place your eye very close to the grating and look through it toward the hydrogen lamp. On **both** sides of the lamp, you should clearly see the images of the first-order Balmer lines. (Be careful to keep the grating facing the lamp. Move your eye position, not the grating, to see the lines to the side of the lamp.)



With a meter stick as close as possible to the lamp, as shown, measure the x-positions (on **both** sides of the central position) of each of the three lines as accurately as possible. With your measured L and x's, compute the angle θ of each of the first-order Balmer lines. Make a table of your results on the next page.

Color	x_{hi}	x_{lo}	x_{avg}	θ_{hi}	θ_{lo}	θ_{avg}
Red (left)						
Red (right)						
Blue/Green (left)						
Blue/Green (right)						
Violet (left)						
Violet (right)						

From the spread in your x measurements, estimate an error-bar for θ. (This can be done simply by calculating θ based the largest and smallest possible x value for each line – including the uncertainty in each measurement and the dual measurements for each line.) Fill these in in the θ_{hi}, θ_{lo}, and θ_{avg} columns in the table

The number of lines per mm is marked on the grating. From this, you can compute the spacing **d** of the grating. Using your measured θ's and computed d, compute the wavelengths of each of the three Balmer lines.

Using the same technique as you did to "propagate" the error in θ, estimate the error in each of the computed wavelengths. State your three measured wavelengths here, including the error bar (i.e. $\lambda = 555 \pm 4$ nm). Don't include significant figures that are irrelevant based on your error-bar.

Using your experimentally determined wavelengths, use equation (5) to determine the Rydberg constant R. Each of your λ's produces an independent value of R. Compute the average R, and from the spread in your values of R, estimate your experimental uncertainty in R.

Describe the comparison of your results with the known value $R = 1.09737 \times 10^7\ m^{-1}$. Does the known value agree with your value, within your experimental uncertainty? If not, can you think of any systematic errors in your measurements that might account for the discrepancy?

PART III: SOLAR SPECTRUM

If you have time, take the Project Star Spectrometer outside (after signing the sign-out sheet) and observe the solar spectrum by pointing the spectrometer toward a white cloud or at the blue sky near the Sun.

DO NOT POINT THE SPECTROMETER AT THE SUN! THE SUN IS VERY BRIGHT! DIRECT SUNLIGHT CAN CAUSE EYE DAMAGE!

You will observe that the solar spectrum is continuous (all the colors of the rainbow, like an incandescent light bulb). If you look closely, you will see some ***dark*** lines in the spectrum where certain colors are missing. This is an example of an *absorption spectrum*. The spectrum of the hydrogen lamp and the other lamps you observed were examples of *emission spectra*. Your TA will explain the difference.

Before leaving, return the Spectrometer to your TA so she can cross your name off the sign-out sheet. No calculations are required for this part.

PRELAB QUESTIONS: (to be turned in upon arriving at lab)

1. For a given order (say m=3), is the angle of diffraction for red light larger or smaller than the angle for blue light? Explain.

2. In part 2 of this experiment, how will you determine the spacing d of your diffraction grating?

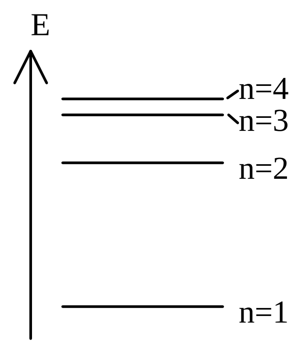
3. What is the relation between the energy E and the frequency f of a photon?

4. The colors of the four lines of the hydrogen spectrum are: red, blue-green, and two shades of violet. Which initial states n_i = 3, 4, 5, or 6 correspond to these colors? (Hint: The colors of the visible spectrum, from longest to shortest wavelengths, are: red, orange, yellow, green, blue, violet.)

5. Transitions to the n=1 (ground state) level from higher levels in the hydrogen atom never produce visible wavelength photons. Are the wavelengths produced by transitions to the n=1 level longer or shorter than visible λ's? Explain.

6. In part 2 of this experiment, which quantities are given, which are measured directly, and which are calculated?

POTENTIAL EXAM QUESTIONS:



1. A partial hydrogen atom energy level diagram is shown. How many different lines in the spectrum of hydrogen correspond to transitions among the bottom four energy levels (n=1,2,3, and 4)?

a) 3

b) 4

c) 6

d) 12

e) None of the above

2. Referring to the diagram above, consider the following four transitions in the hydrogen atom: n=2→n=1 , 3→2 , 4→3 , 4→1. How do the wavelengths of the photons emitted by each of these transitions compare? Order the emitted photons from shortest wavelength to longest wavelength.

a) (shortest λ)	2→1	3→2	4→3	4→1	(longest λ)
b)	4→1	4→3	3→2	2→1	
c)	4→3	3→2	2→1	4→1	
d)	4→1	2→1	3→2	4→3	
e)	3→2	4→3	4→1	2→1	