Day 19: 3/29
Questions?
Thursday: Thursday: Exam 2 (in class)
Matter Waves and Measurement
Review for Exam 2

PH300 Modern Physics SP11

“IT doesn’t matter how beautiful your theory is; it doesn’t matter how smart you are. IF it doesn’t agree with experiment, it’s wrong.”

– Richard Feynman

Exam 2 is this Thursday, 3/31, in class.
HW09 will be due Wednesday (3/30) by 5pm in my mailbox.
HW09 solutions posted at 5pm on Wednesday

Recently:
1. Electron diffraction and matter waves
2. Wave packets and uncertainty

Today:
1. Finish matter waves/wave function
2. Review for Exam 2

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Superposition

If $\Psi_1(x,t)$ and $\Psi_2(x,t)$ are solutions to a wave equation, then so is $\Psi(x,t) = \Psi_1(x,t) + \Psi_2(x,t)$

Superposition (linear combination) of two waves

We can construct a “wave packet” by combining many plane waves of different energies (different $k$’s).

Superposition

$\Psi(x,t) = \sum A_n \exp[i(k_n x - \omega_n t)]$

Plane Waves vs. Wave Packets

$\Psi(x,t) = A \exp[i(k x - \omega t)]$

$\Psi(x,t) = \sum A_n \exp[i(k_n x - \omega_n t)]$

Which one looks more like a particle?

Superposition

$\Psi(x,t) = \sum A_n \exp[i(k_n x - \omega_n t)]$

Plane Waves vs. Wave Packets

$\Psi(x,t) = A \exp[i(k x - \omega t)]$

$\Psi(x,t) = \sum A_n \exp[i(k_n x - \omega_n t)]$

For which type of wave are the position ($x$) and momentum ($p$) most well-defined?

A) $x$ most well-defined for plane wave, $p$ most well-defined for wave packet.
B) $p$ most well-defined for plane wave, $x$ most well-defined for wave packet.
C) $p$ most well-defined for plane wave, $x$ equally well-defined for both.
D) $x$ most well-defined for wave packet, $p$ equally well-defined for both.
E) $p$ and $x$ are equally well-defined for both.
Uncertainty Principle

In math: $\Delta x \cdot \Delta p \geq \frac{\hbar}{2}$

In words: The position and momentum of a particle cannot both be determined with complete precision. The more precisely one is determined, the less precisely the other is determined.

What do $\Delta x$ (uncertainty in position) and $\Delta p$ (uncertainty in momentum) mean?

A Realist Interpretation:
- Measurements are performed on an ensemble of similarly prepared systems.
- Distributions of position and momentum values are obtained.
- Uncertainties in position and momentum are defined in terms of the standard deviation.

A Statistical Interpretation:
- Photons are scattered by localized particles.
- Due to the lens’ resolving power: $\Delta x = \frac{\lambda}{\theta}$
- Due to the size of the lens: $\Delta p = \left( \frac{\hbar}{4} \right) \left( \frac{\theta}{2} \right) \Rightarrow \Delta x \cdot \Delta p \geq \frac{\hbar}{2}$

A Wave Interpretation:
- Wave packets are constructed from a series of plane waves.
- The more spatially localized the wave packet, the less uncertainty in position.
- With less uncertainty in position comes a greater uncertainty in momentum.

Matter Waves (Summary):
- Electrons and other particles have wave properties (interference).
- When not being observed, electrons are spread out in space (delocalized waves).
- When being observed, electrons are found in one place (localized particles).
- Particles are described by wave functions: $\Psi = \psi(x,t)$ (probabilistic, not deterministic).
- Physically, what we measure is $p(x,t) = |\psi(x,t)|^2$ (probability density for finding a particle in a particular place at a particular time).
- Simultaneous knowledge of $x$ & $p$ are constrained by the Uncertainty Principle: $\Delta x \cdot \Delta p \geq \frac{\hbar}{2}$.
PE summary:

- Current responds instantaneously to light on/off
- Not just a heating effect!
- Color matters: Below a certain frequency: No electrons
- Not just a heating effect!
- Positive voltage does not increase the current (remember the 'pool analogy')
- Negative voltages do decrease current
- Stopping voltage is equal to initial kinetic energy!
- Initial kinetic energy depends on color of the light.
- Current is proportional to the light intensity.

Summary PE effect:

1. Current vs. Voltage:
   - High intensity
   - Low intensity

2. Current vs. Intensity:
   - Initial KE vs. f

Summary of what we know so far:
1. If light can kick out electron, then even smallest intensities of that light will continue to kick out electrons. KE of electrons does not depend on intensity.
2. Lower frequencies of light means lower initial KE of electrons & KE changes linearly with frequency.
3. Minimum frequency below which light won't kick out electrons.

(individual atoms vs. atoms bound in solids)

In each case, the blue photon ejects the red electron. Consider the following statements:
I. The work functions are the same in both cases.
II. The KE of the ejected electrons are the same in both cases.

A. I=True, II=True
B. I=True, II=False
C. I=False, II=True
D. I=False, II=False

(Einstein) Need “photon” picture of light to explain observations:
- Light comes in chunks (“particle-like”) of energy (“photon”)
- a photon interacts only with single electron
- Photon energy depends on frequency of light 
- For lower frequencies, photon energy not enough to free an electron

Work function = amount of energy needed to kick out most loosely bound electron out of the metal.
In each case, the blue photon ejects the red electron. Consider the following statements:

I. The work functions are the same in both cases.
II. The KE of the ejected electrons are the same in both cases.

A. I=True, II=True  
B. I=True, II=False  
C. I=False, II=True  
D. I=False, II=False

Conservation of energy
KE of ejected electron = Energy needed to extract electron from metal.

E=hc/λ...

A. ...is true for both photons and electrons.  
B. ...is true for photons but not electrons.  
C. ...is true for electrons but not photons.  
D. ...is not true for photons or electrons.

E = hf is always true but f = c/λ only applies to light, so E = hf ≠ hc/λ for electrons.

define work function Φ = E_b - E_e, where E_b is the binding energy and E_e is the electron energy.

Balmer had a mathematical formula to describe hydrogen spectrum, but no mechanism for why it worked.

\[ \lambda = \frac{91.19 \text{nm}}{m^2/n^2} \]

where m=1,2,3 and where n = m+1, m+2

Hydrogen atom-Balmer series

Balmer (1885) noticed wavelengths followed a progression

\[ \lambda = \frac{91.19 \text{nm}}{m^2/n^2} \]

As n gets larger, what happens to wavelengths of emitted light?

a. gets larger and larger without limit  
 b. gets larger and larger, but approaches a limit  
 c. gets smaller and smaller without limit  
 d. get smaller and smaller, but approaches a limit

\[ \text{Bohr Model:} \]

Success! What it can do: predict energy levels fairly precisely (although not fine splittings discovered short time later) for any one electron atom.

Problems: It cannot:

explain anything about jumps between levels, atomic lifetimes, why some transitions stronger than others, atoms with multiple electrons.

If calculate angular momentum of atom L = mvr, for orbit, get L = nh/2\pi.

Angular momentum only comes in units of h/2\pi = h ("quantized" in units of "h-bar")

Basic connections between r, v, and energy:

F = ma = F_{coul} = \frac{kq^2}{r^2}, \text{mv}^2/r = \frac{ke^2}{r^2}, \text{mv}^2 = \frac{ke^2}{r}

\text{Ans)} b) F_{coul} = \frac{ke^2}{r^2}

Equate to Coulomb force, \text{mv}^2/r = \frac{ke^2}{r^2}, \text{mv}^2 = \frac{ke^2}{r}
Compare the energy of the photon given off when the electron goes from the n=2 level of H to the ground level (n=1) with the energy difference between the n=4 level and the n=2 level.

\[ E_{2-1} / E_{4-2} = ??? \]

a. 2, b. 4, c. \( \frac{1}{2} \), d. \( \frac{1}{4} \), e. \( \frac{3}{16} \)

Electron Energy levels:

At 0 eV, electron has escaped atom. (So no transitions from 0 eV back down)

In discharge lamps, one electron bashes into an atom.

What energy levels for electrons are consistent with this spectrum for "Finkelnolium"?

What energy levels for electrons are consistent with this spectrum for "Finkelnolium"?

Electron Energy levels:

Electron energy levels = PE + KE

Since PE = 0 at infinity (e.g. electron escaping from atom)
A positive total energy would mean that KE-PE and electron would leave atom!

In discharge lamps, one electron bashes into an atom.

If atoms fixed at these points in tube, what will you see from each atom?
A. All atoms will emit the same colors.
B. Atom 1 will emit more colors than 2 which will emit more colors than 3
C. Atom 3 will emit more colors than 2 which will emit more colors than 1
D. Atom 3 will emit more colors than 2. Atom 1 will emit no colors.
E. Impossible to tell.

Magnetic Moment in a Non-Uniform Magnetic Field

Classically, we expect:

1. Magnetic moments are deflected in a non-uniform magnetic field
2. Magnetic moments precess in the presence of a magnetic field, and the projection of the magnetic moment along the field axis remains constant.
3. If the magnetic moment vectors are randomly oriented, then we should see a broadening of the particle beam
Stern-Gerlach Experiment with Silver Atoms (1922)*

Analyzer 2 is oriented at an angle $\Theta$:

Ignore the atoms exiting from the minus-channel of Analyzer 1, and feed the atoms exiting from the plus-channel into Analyzer 2.

What is the probability for an atom to exit from the plus-channel of Analyzer 2?

For $\Theta = 0$, 100% of the atoms exit from the plus-channel.

For $\Theta = 90^\circ$, 50% of the atoms exit from the plus-channel.

For $\Theta = 180^\circ$, 0% of the atoms exit from the plus-channel.

Probability

When tossing a die, what is the probability of rolling either a 1 or a 3?

In general*, the word "OR" is a signal to add probabilities:

$P_1 + P_2$

Simultaneously toss a die and flip a coin. What is the probability of getting 2 and Tails?

In general*, the word "AND" is a signal to multiply:

$P_1 \cdot P_2$

*Only if $P_1$ & $P_2$ are independent

Probability

If the setting is in the +z-direction (A), then the probability to leave through the plus-channel is 1.

For either of the two other settings (B & C), the probability of leaving through the plus-channel is 1/4. [cos($120^\circ/2$)]

If the settings are random, then there is 1/3 probability that the analyzer is set to a specific orientation.

The probability of leaving through the plus-channel is:

$\frac{1}{3} x \frac{1}{2} + \frac{1}{3} x \frac{1}{4} + \frac{1}{3} x \frac{1}{4}
\begin{align*}
&= \frac{4}{12} + \frac{1}{12} + \frac{1}{12} = 6/12 = 1/2
\end{align*}$

*Nobel Prize for Stern in 1943
Continuous Probability Distributions

- $p(x)$ is a probability density (not a probability). We approximate the probability to obtain $x$ within a range $\Delta x$ with:
  \[ P_x = \frac{\Delta x}{\Delta x} \begin{cases} x \leq x \leq x + \Delta x \end{cases} = p(x) \cdot \Delta x \]

- The probability of obtaining a range of values is equal to the area under the probability distribution curve in that range:
  \[ P(a \leq x \leq b) = \int_a^b p(x) \, dx \]

- For $x_1, x_2, x_3, \ldots x_n$ (discrete values): $\langle x \rangle = \sum x_i P(x_i)$
  $\langle x \rangle$ = average value of $x$

- For continuous $x$: $\langle x \rangle = \int x \, p(x) \, dx$

Hidden Variables

If you believe the magnetic moment vector always points in some direction, the value of $m_x$ for an atom in the state $\left| \frac{1}{2} \right>$ would be called a hidden variable.

If $m_x$ has some real value at any given moment in time that is unknown to us, then that variable is hidden:

- The value of $m_x$ exists, but we can’t predict ahead of time what we’ll measure ("up" or "down").

Locality

EPR makes one other assumption, but is it really an assumption?

Suppose we have two physical systems, 1 & 2.

If 1 & 2 are physically separated from one another, locality assumes that a measurement performed on System 1 can’t affect the outcome of a measurement performed on System 2, and vice-versa.

Entanglement

Suppose we have a source that produces pairs of atoms traveling in opposite directions, and having opposite spins:

- Total spin = 0
  \[ \uparrow \downarrow \rightarrow \downarrow \uparrow \]

OR

If $\downarrow \downarrow$ and $\downarrow \downarrow$ sum to zero, then $\uparrow \downarrow$ and $\uparrow \downarrow$ must also sum to zero.
• We can’t predict what the result for each individual atom pair will be.

- \( |\Psi_{12} \rangle = |\uparrow_1, \downarrow_2 \rangle \) and \( |\Psi_{12} \rangle = |\downarrow_1, \uparrow_2 \rangle \) are both equally likely.

- **Quantum mechanics** says to describe the quantum state of each atom pair as a **superposition** of the two possible states:

  \( |\Psi_{12} \rangle = |\uparrow_1, \downarrow_2 \rangle + |\downarrow_1, \uparrow_2 \rangle \)

- When we perform the measurement, we only get one of the two possible outcomes, each with a probability of 1/2.

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**Experiment One**

- If the second photon (\( \nu_2 \)) is detected by PMA, then the photon must have traveled along Path A.
- If the second photon (\( \nu_2 \)) is detected by PMB, then the photon must have traveled along Path B.
- The photon must decide at BS1 whether to take Path A or Path B.

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**Experiment Two**

- Whether the photon is detected in PMA or PMB, both Path A or Path B are possible.
- When we compare data from PMA & PMB, we observe interference – the photon must decide at BS1 to take both paths.

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**Delayed-Choice**

How can the photon “know” when it encounters BS1 whether one or two paths are open? (whether we’re conducting Experiment One or Experiment Two?)

What if the photon encounters BS1 while we are conducting Experiment One?
- There is only one path to the second beam splitter.
- It must “choose” to take that one path at BS1.

Suppose we open up a second path to BS2 (switch to Experiment Two) while the photon is still in the apparatus, but before it reaches a detector.

When we look at the data, we observe interference, as though two paths had been available all along.

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**The EPR Argument**

- Both Albert & Niels agree that we must describe the atom-pair with the superposition state \( |\Psi_{12} \rangle = |\uparrow_1, \downarrow_2 \rangle + |\downarrow_1, \uparrow_2 \rangle \)

- Albert says the real state of the atom-pair was \( |\Psi_{12} \rangle = |\uparrow_1, \downarrow_2 \rangle \) and the measurements revealed this unknown reality to us.

- Niels says that \( |\Psi_{12} \rangle = |\uparrow_1, \downarrow_2 \rangle + |\downarrow_1, \uparrow_2 \rangle \) was the actual, indefinite state of the atom-pair and the first measurement instantly collapsed the state into \( |\Psi_{12} \rangle = |\uparrow_1, \downarrow_2 \rangle \)

---

**Three Experiments with Photons**

- **Experiment One** says photons behave like particles.
- **Experiment Two** says photons behave like waves.
- **Delayed Choice** says photons do not behave like particles and waves at the same time.
**Particle or Wave?**

Bohr: Complementarity – We can only observe one type of behavior or another. The photon interacts with the entire measurement apparatus (including any business about switching between experiments).

Dirac: The photon always behaves like a wave at the first beam splitter. A (random) interaction with one of the PMT’s causes the state to collapse instantaneously into one point in space.

**Double-Slit Experiment**

\[ P(x) = |\Psi(x)|^2 = A_x^2 + A_2^2 + 2A_xA_2 \cos \frac{2\pi}{\lambda} (x_2 - x_1) \]

\[ P_{2x}[x] = |\Psi_1 + \Psi_2|^2 \]

- In quantum mechanics, we add the individual amplitudes and square the sum to get the total probability.
- In classical physics, we added the individual probabilities to get the total probability.

**Distance to first maximum for visible light:**

Let \( m = 1, \quad \lambda = 500 \text{ nm}, \quad D = 5 \times 10^{-4} \text{ m} \)

\[ \theta = \frac{\lambda}{D} = \frac{5 \times 10^{-7}}{5 \times 10^{-4}} = 0.001 \text{ rad} \]

If \( L = 3 \text{ m}, \) then \( H = L \cdot \theta = 3 \text{ mm} \)

**Hydrogen atom-Balmer series**

Balmer (1885) noticed wavelengths followed a progression

\[ \lambda = \frac{91.19 \text{ nm}}{n^2} \]

where \( n = 3, 4, 5, \ldots \)

As \( n \) gets larger, what happens to wavelengths of emitted light?

- a. gets larger and larger without limit
- b. gets larger and larger, but approaches a limit
- c. gets smaller and smaller without limit
- d. get smaller and smaller, but approaches a limit
Compare the energy of the photon given off when the electron goes from the \( n=2 \) level of H to the ground level (\( n=1 \)) with the energy difference between the \( n=4 \) level and the \( n=2 \) level.

\[
E_2/E_4 = \text{???}
\]

a. 2, b. 4, c. \( \frac{1}{2} \), d. \( \frac{1}{4} \), e. \( \frac{3}{16} \)

If electron going around in little orbits, important implications from classical physics

Basic connections between \( r, v, \) and energy!

\[
F = \frac{ma}{r} = ?
\]

\begin{itemize}
    \item a) \( -\frac{mv^2}{r} \)
    \item b) \( \frac{mv^2}{r} \)
    \item c) \( -\frac{v^2}{r^2} \)
    \item d) \( -\frac{mv}{r} \)
    \item e) don’t remember learning anything related to this
\end{itemize}

Equate to Coulomb force, \( = kq \cdot q/r^2 \),

\[
\frac{mv^2}{r} = \frac{kq^2}{r^2} \]

\[
mv^2 = \frac{kq^2}{r}
\]

\section*{Bohr “Electron in orbit with only certain particular energies”}

This implies that an electron in Bohr model of hydrogen atom:

a. is always at one particular distance from nucleus
b. can be at any distance from nucleus

c. can be at certain distances from nucleus corresponding to energy levels it can be in.

d. must always go into center where potential energy lowest

e. All of the above.

Note that both A & B are used in derivation of Bohr model.

\section*{deBroglie Waves}

What is \( n \) for electron wave in this picture?

\begin{itemize}
    \item A. 1
    \item B. 5
    \item C. 10
    \item D. 20
    \item E. Cannot determine from picture
\end{itemize}

Answer: C. 10

\section*{deBroglie Waves}

Given the deBroglie wavelength \( (\lambda = h/p) \) and the condition for standing waves on a ring \( (2\pi r = n\lambda) \), what can you say about the angular momentum \( L \) of an electron if it is a deBroglie wave?

\begin{itemize}
    \item A. \( L = \frac{nh}{r} \)
    \item B. \( L = \frac{nh}{\lambda} \)
    \item C. \( L = \frac{nh}{2} \)
    \item D. \( L = 2nh/r \)
    \item E. \( L = nh/2r \)
\end{itemize}

(Recall: \( h = \hbar/2\pi \))
Analyzer 2 is now oriented downward:

```
1 +
  
2 +
  
    all
  
  +
  
  none
```

Ignore the atoms exiting from the minus-channel of Analyzer 1, and feed the atoms exiting from the plus-channel into Analyzer 2.

What happens when these atoms enter Analyzer 2?

A) They all exit from the plus-channel.
B) They all exit from the minus-channel.

Analyzer 2 is now oriented horizontally (+x):

```
1 +
  
2 +
  
    ???
```

Ignore the atoms exiting from the minus-channel of Analyzer 1, and feed the atoms exiting from the plus-channel into Analyzer 2.

What happens when these atoms enter Analyzer 2?

A) They all exit from the plus-channel.
B) They all exit from the minus-channel.
C) Half leave from the plus-channel, half from the minus-channel.
D) Nothing, the atoms’ magnetic moments have zero projection along the +x-direction.

Analyzer 2 is oriented at an angle Θ:

```
1 +
  
2 +
  
    θ
  
  Θ
  
  ?
```

Ignore the atoms exiting from the minus-channel of Analyzer 1, and feed the atoms exiting from the plus-channel into Analyzer 2.

What is the probability for an atom to exit from the plus-channel of Analyzer 2?

\[ P[\uparrow_\sigma] = \cos^2 \left( \frac{\Theta}{2} \right) \]

Repeated spin measurements:

```
1 +
  
2 +
  
    ???
```

Ignore the atoms exiting from the minus-channel of Analyzer 1, and feed the atoms exiting from the plus-channel into Analyzer 2.

Ignore the atoms exiting from the minus-channel of Analyzer 2, and feed the atoms exiting from the plus-channel into Analyzer 3.

What happens when these atoms enter Analyzer 3?

A) They all exit from the plus-channel.
B) They all exit from the minus-channel.
C) Half leave from the plus-channel, half from the minus-channel.
D) Nothing, the atoms’ magnetic moments have zero projection along the +z-direction.

Instead of vertical, suppose Analyzer 1 makes an angle of 30° from the vertical. Analyzers 2 & 3 are left unchanged.

What is the probability for an atom leaving the plus-channel of Analyzer 2 to exit from the plus-channel of Analyzer 3?

A) 0%  B) 25%  C) 50%  D) 75%  E) 100%

Hint: Remember that \[ P[\uparrow_\sigma] = \cos^2 \left( \frac{\Theta}{2} \right) \]

Instead of horizontal, suppose Analyzer 2 makes an angle of 60° from the vertical. Analyzers 1 & 3 are unchanged.

What is the probability for an atom leaving the plus-channel of Analyzer 2 to exit from the plus-channel of Analyzer 3?

A) 0%  B) 25%  C) 50%  D) 75%  E) 100%

Hint: Remember that \[ P[\uparrow_\sigma] = \cos^2 \left( \frac{\Theta}{2} \right) \]
Probability
If the setting is in the z-direction (A), then the probability to leave through the plus-channel is 1.
For either of the two other settings (B & C), the probability of leaving through the plus-channel is \( \frac{1}{4} \). \( \cos^2\left(\frac{120^0}{2}\right) \)
If the settings are random, then there is \( \frac{1}{3} \) probability that the analyzer is set to a specific orientation.
The probability of leaving through the plus-channel is:
\[
\left[ \frac{1}{3} \times 1 \right] + \left[ \frac{1}{3} \times \frac{1}{4} \right] + \left[ \frac{1}{3} \times \frac{1}{4} \right] = \frac{4}{12} + \frac{1}{12} + \frac{1}{12} = \frac{6}{12} = \frac{1}{2}
\]

What if the probability curve is not normal?

- A) 0
- B) \( \frac{1}{2} \)
- C) 1
- D) Not defined, since there are two places where \( x \) is most likely.

What kind of system might this probability distribution describe?
- Harmonic oscillator
- Ball rolling in a valley
- Block on a spring

Experiment One
If the second photon (\( \nu_2 \)) is detected by PMA, then the photon must have traveled along which path?
- A) Path A (via Mirror A)
- B) Path B (via Mirror B)
- C) Both Path A & Path B are possible.
- D) Not enough information.

Experiment Two
If the photon is detected in PMA, then it must have taken:
- A) Path A (via Mirror A)
- B) Path B (via Mirror B)
- C) Both Path A & Path B are possible.
- D) Not enough information.
Quantum Systems

For a quantum system we have to write down **TWO** lists:

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td>PARTICLE</td>
<td>WAVE</td>
</tr>
<tr>
<td>WHICH-PATH</td>
<td>INTERFERENCE</td>
</tr>
<tr>
<td>POSITION</td>
<td>MOMENTUM</td>
</tr>
</tbody>
</table>

• For every characteristic in **List A**, there is a corresponding characteristic in **List B**.
• Knowing a lot about one means we know only a little about the other.

Quantum Systems

These are also incompatible observables, but in a slightly different way than those in the previous list:*  

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td>L_z</td>
<td>L_x, L_y</td>
</tr>
</tbody>
</table>

L_z = angular momentum about the z-axis (think magnetic moments)  
If we know L_z, then angular momentum about any other direction is indeterminate.

*This incompatibility is a consequence of the more familiar constraints placed on position and momentum about which we’ll learn a lot more!

Matter Waves

**de Broglie wavelength:**  
\[ \lambda = \frac{h}{p} \quad H = \frac{m\lambda L}{D} \]

In order to observe electron interference, it would be best to perform a double-slit experiment with:

A) Lower energy electron beam.  
B) Higher energy electron beam.  
C) It doesn’t make any difference.

Lowering the energy will increase the wavelength of the electron.  
Can typically make electron beams with energies from 25 – 1000 eV.

Matter Waves

**de Broglie wavelength:**  
\[ \lambda = \frac{h}{p} \quad H = \frac{m\lambda L}{D} \]

For an electron beam of 25 eV, we expect \( \Theta \) (the angle between the center and the first maximum) to be:

A) \( \Theta \ll 1 \)  
B) \( \Theta < 1 \)  
C) \( \Theta > 1 \)  
D) \( \Theta \gg 1 \)

Use \( D = 5 \times 10^{-4} \) m  
and remember:  
\[ \Theta = \frac{\lambda}{D} \]
\[ E = \frac{p^2}{2m} \]
An electron is described by the following wave function:

\[ \Psi(x) = \frac{\alpha}{\sqrt{L}} x^3 \text{ from } x = -L \text{ to } x = +L \]

\[ \rho(x) = |\Psi(x)|^2 = \frac{\alpha^2 x^6}{L^2} \]

How do the probabilities of finding the electron near (within dx) of a, b, c, and d compare?

A) \( d > c > b > a \)
B) \( a = b = c = d \)
C) \( d > b > a > c \)
D) \( a > d > b > c \)

Plane Waves vs. Wave Packets

\[ \Psi(x,t) = A \exp\left[ i(kx - \omega t) \right] \]

\[ \Psi(x,t) = \sum A_k \exp\left[ i(kx - \omega t) \right] \]

For which type of wave are the position (x) and momentum (p) most well-defined?

A) \( x \) most well-defined for plane wave, \( p \) most well-defined for wave packet.
B) \( p \) most well-defined for plane wave, \( x \) most well-defined for wave packet.
C) \( p \) most well-defined for plane wave, \( x \) equally well-defined for both.
D) \( x \) most well-defined for wave packet, \( p \) equally well-defined for both.
E) \( p \) and \( x \) are equally well-defined for both.