Today & Thursday:
“Classical” wave-view of light & its interaction with matter
• Pre-quantum
• Still useful in many situations.
Thursday:
HW01 due, beginning of class; HW02 assigned

Next week:
Special Relativity ➔ Universal speed of light
Length contraction, time dilation

Maxwell’s Equations: Describe EM radiation
\[
\oint E \cdot dA = \frac{Q_{\text{in}}}{\varepsilon_0} \\
\oint B \cdot dA = 0 \\
\oint B \cdot dl = \mu_0 I_{\text{through}} + \varepsilon_0 \mu_0 \frac{d\Phi_E}{dt}
\]

‘Flux’ of a vector field through a surface:
The average outward-directed component of a vector field multiplied by a surface area.
\[
\int \vec{E} \cdot d\vec{A}
\]
Some surface, A

‘Circulation’ of a vector field around a path:
The average tangential component of a vector field multiplied by the path length.
\[
\oint \vec{B} \cdot d\vec{l}
\]
Some path, \( \vec{l} \)

An example:
Gauss’ Law
Electric flux through a closed surface tells you the total charge inside the surface.
\[
\oiint_{\text{Closed surface}} E \cdot dA = \frac{Q_{\text{in}}}{\varepsilon_0}
\]

If the electric field at the surface of a sphere (radius, \( r \)) is radial and of constant magnitude, what is the flux \( \int E \cdot dA \) out of the sphere?

A) \( E \cdot \pi r^2 \)
B) \( E \cdot 4\pi r^2 \)
C) \( E \cdot \frac{4}{3} \pi r^3 \)
D) Something else
An example:

Gauss’ Law
Electric flux through a closed surface tells you the total charge inside the surface.

\[ \iint_{\text{Closed surface}} E \cdot dA = \frac{Q_{\text{enc}}}{\varepsilon_0} \]

So:

\[ E \cdot 4\pi r^2 = \frac{Q}{\varepsilon_0} \]

\[ E = \frac{1}{4\pi \varepsilon_0} \frac{Q}{r^2} \]

Coulomb’s Law is contained in Gauss’ Law

An example:

Faraday’s Law
Electric circulation around a closed path tells you the (negative of) the time change of flux of \( \mathbf{B} \) through any open surface bounded by the path.

\[ \oint \mathbf{E} \cdot d\mathbf{l} = -\frac{d\Phi_B}{dt} \]

\[ \Phi_B(t) = \oint \mathbf{B} \cdot d\mathbf{A} \]

Consider the following configuration of field lines. This could be...

A) ...an E-field
B) ...a B-field
C) Either E or B
D) Neither
E) No idea

Consider the following configuration of field lines. This could be...

A) ...an E-field
B) ...a B-field
C) Either E or B
D) Neither
\[ \int \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt} \]
\[ \int \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{through}} + \varepsilon_0 \mu_0 \frac{d\Phi_E}{dt} \]

Maxwell's Equations in Vacuum

The words "in vacuum" are code for "no charges or currents present"

The equations then become:

\[ \int \vec{E} \cdot d\vec{A} = 0 \quad \int \vec{B} \cdot d\vec{A} = 0 \]
\[ \int \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt} \]
\[ \int \vec{B} \cdot d\vec{l} = \varepsilon_0 \mu_0 \frac{d\Phi_E}{dt} \]

\[ \varepsilon_0 = 8.85 \times 10^{-12} \text{ farad/meter} \]
\[ \mu_0 = 4\pi \times 10^{-7} \text{ henry/meter} \]

Maxwell's Equations: Differential forms

\[ \int \vec{E} \cdot d\vec{A} = 0 \quad \Rightarrow \quad \vec{V} \cdot \vec{E} = 0 \]
\[ \int \vec{B} \cdot d\vec{A} = 0 \quad \Rightarrow \quad \vec{V} \cdot \vec{B} = 0 \]
\[ \int \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt} \quad \Rightarrow \quad \vec{V} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \]
\[ \int \vec{B} \cdot d\vec{l} = \varepsilon_0 \mu_0 \frac{d\Phi_E}{dt} \quad \Rightarrow \quad \vec{V} \times \vec{B} = \varepsilon_0 \mu_0 \frac{d\vec{E}}{dt} \]

Each of these is a single partial differential equation.

Each of these is a set of three coupled partial differential equations.

Maxwell's Equations: 1-Dimensional Differential Equations

\[ \frac{\partial E_y(x,t)}{\partial x} = -\frac{\partial B_z(x,t)}{\partial t} \]
\[ -\frac{\partial B_z(x,t)}{\partial x} = \mu_0 \varepsilon_0 \frac{\partial E_y(x,t)}{\partial t} \]

...with the requirement that:

\[ \frac{\partial^2 E_y}{\partial x^2} = \frac{1}{c^2} \frac{\partial^2 E_y}{\partial t^2} \]
\[ \frac{\partial^2 B_z}{\partial x^2} = \frac{1}{c^2} \frac{\partial^2 B_z}{\partial t^2} \]

1-Dimensional Wave Equation

\[ \frac{\partial^2 E_y}{\partial x^2} = \frac{1}{c^2} \frac{\partial^2 E_y}{\partial t^2} \]
\[ \Rightarrow \quad \frac{2\pi}{\lambda} \quad \frac{2\pi}{T} \]

Solutions are sines and cosines:

\[ E_y = A \sin(kx - \omega t) + B \cos(kx - \omega t) \]

...with the requirement that:

\[ k^2 = \frac{\omega^2}{c^2} \quad \text{or} \quad c = \frac{\lambda}{T} \]
Sinusoidal waves:

Wave in time: \( \cos(2\pi ft/T) = \cos(\omega t) = \cos(2\pi ft) \)

Wave in space: \( \cos(2\pi x/\lambda) = \cos(kx) \)

\( k \) is spatial analogue of angular frequency \( \omega \).

One reason we use \( k \) is because it’s easier to write \( \sin(kx) \) than \( \sin(2\pi x/\lambda) \).

Waves in space & time:

\( \cos(kx + \omega t) \) represents a sinusoidal wave traveling...

A) …to the right (+x-direction).
B) …to the left (-x-direction).

1-Dimensional Wave Equation

\[
\frac{\partial^2 E_x}{\partial x^2} = \frac{1}{c^2} \frac{\partial^2 E_x}{\partial t^2}
\]

The most general solution is:

\( E_x = A_1 \sin(k_x x - \omega_1 t) + A_2 \cos(k_x x - \omega_2 t) \)

A specific solution is found by applying boundary conditions

Complex Exponential Solutions

Recall solution for the Electric field \( E \) from the wave equation:

\( E = A \cos(kx - \omega t) \)

Euler’s Formula says:

\[
\exp[i(kx - \omega t)] = \cos(kx - \omega t) + i\sin(kx - \omega t)
\]

\( E = A \cos(kx) = \text{Re}[A \cos(kx) + iA \sin(kx)] = \text{Re}[A \exp(ikx)] \)

For convenience, write: \( E = A \exp[i(kx - \omega t)] \)
Complex Exponential Solutions

\[ \vec{E}(x,y,z,t) = \vec{E}_0 \exp(i\vec{k} \cdot \vec{r} - \omega t) \]

Constant vector prefactor tells the direction and maximum strength of \( \vec{E} \).

The complex equations reduce Maxwell’s Equations (in vacuum) to a set of vector algebraic relations between the three vectors \( \vec{k} \), \( \vec{E}_0 \), and \( \vec{B}_0 \), and the angular frequency \( \omega \):

\[
\begin{align*}
    ik \cdot \vec{E}_0 &= 0 \\
    ik \times \vec{E}_0 &= i\omega \vec{B}_0 \\
    ik \cdot \vec{B}_0 &= 0 \\
    ik \times \vec{B}_0 &= -i\omega \mu_0 \varepsilon_0 \vec{E}_0 \\
    \frac{\omega^2}{k^2} &= \frac{1}{\mu_0 \varepsilon_0} = c^2 \\
    \left| \frac{\vec{E}_0}{\vec{B}_0} \right| &= c
\end{align*}
\]

A) The direction of the electric field vector
B) The direction of the magnetic field vector
C) The direction in which the wave is not varying
D) The direction the plane wave moves
E) None of these

The electric field for a plane wave is given by:

\[ \vec{E}(x,y,z,t) = \vec{E}_0 \exp\left[i(\vec{k} \cdot \vec{r} - \omega t)\right] \]

The vector \( \vec{k} \) tells you...

A) Direction of the electric field vector
B) Direction of the magnetic field vector
C) Direction in which the wave is not varying
D) Direction the plane wave moves
E) None of these

\[ \vec{E}, \vec{B}, \text{and} \ \vec{k} \] form a ‘right-handed system’, with the wave traveling in the direction \( \vec{k} \), at speed \( c \).
How do you generate light (electromagnetic radiation)?

A) Stationary charges
B) Charges moving at a constant velocity
C) Accelerating charges
D) B and C
E) A, B, and C

Stationary charges →
constant E-field, no magnetic (B)-field

Charges moving at a constant velocity →
Constant current through wire creates a B-field
But B-field is constant

Accelerating charges →
changing E-field and changing B-field
(EM radiation → both E and B are oscillating)

Answer is (C) Accelerating charges create EM radiation.

The Sun
Surface of sun—very hot!
Whole bunch of free electrons whizzing around like crazy. Equal number of protons, but heavier so moving slower, less EM waves generated.

EM radiation often represented by a sinusoidal curve.

Making sense of the Sine Wave

What does the curve tell you?
A) The spatial extent of the E-field. At the peaks and troughs the E-field is covering a larger extent in space
B) The E-field’s direction and strength along the center line of the curve
C) The actual path of the light travels
D) More than one of these
E) None of these.

Correct answer is (B) – the E-field’s direction and strength along the center line of the curve.

What stuff is moving up and down in space as a radio wave passes?
A) Electric field
B) Electrons
C) Air molecules
D) Light ray
E) Nothing

Answer is (E): Nothing
Electric field strength increases and decreases – E-field does not move up and down.
What is moving to the right in space as radio wave propagates?

A) Disturbance in the electric field
B) Electrons
C) Air molecules
D) Nothing  

Answer is (A). Disturbance in the electric field.

Review:
- Light interacts with matter when its electric field exerts forces on electrons.
- In order to create light, we need both changing E and B fields. Can do this only with accelerating charges.
- Light has a sinusoidally changing electric field. Sinusoidal pattern of vectors represents increase and decrease in strength of field, nothing is physically moving up and down in space.

End here - 10/18

Electromagnetic Spectrum

Wavelength (\(\lambda\)) = distance [m] until wave repeats
Frequency (\(f\)) = \# of times per second E-field at point changes through complete cycle as wave passes

\[ c = \frac{\lambda}{f} \]

How far away will this peak in the E-field be before the next peak is generated at this spot?

A) \(c\lambda\)
B) \(c/\lambda\)
C) \(\lambda/c\)
D) \(\sin(c\lambda)\)
E) None of these

Answer is (A):
Distance = velocity \times time
Distance = \(\lambda = \frac{c}{f}\) = wavelength
so \(c/f = \lambda\)