INSTRUCTORS MANUAL: TUTORIAL REVIEW 1

Coulomb’s Law, Delta Function, Gauss, Conductors

Goals:
To revisit the topics covered in the previous 4 weeks of tutorials

Reflections on this tutorial:

Part 1  Students spent a lot of time struggling with the curly R vector. They had no trouble labeling them on the diagram, but there was a lot of confusion when it came to writing down its actual components. Students weren't sure which set of coordinates to switch into cylindrical. Many students assumed that they were going to have to solve on just the z-axis and chose to ignore the x and y components.

Part 2  This was very straightforward for everyone. A few students struggled with the units of the delta function.

Part 3  The first three questions were quick for most students. A few of them had some trouble finding the fields inside the objects, but some thoughtful questioning from the instructor helped them on their way. Most of the students got stuck on the Path Integrals in questions (iv) and (v). The difficulty came when finding V inside the shell. Most students forgot that they have to integrate from infinity to the outside shell and then from the outside shell to the point inside the shell (as opposed to integrating from the shell to the inside point).

Part 4. Very few students made it this far. Those that did make it this far had very little trouble as this was the material from the previous week. The only difficulties arose in question (ii) about what the outside charges would do on the outer cylinder. Some students still thought that they would move to counter the internal charge arrangement.

In the future, it may be prudent to tell the students to do this tutorial in the order they feel would be most beneficial to them. Part 1 and 3 take a lot more time than the other sections.
Part 1 – Finding E with Coulomb’s Law

In the year 2240, a bicyclist, named Thomas, gets lost east of Boulder, and he gets a flat tire. Thomas pulls off the tire and then consults his Tricorder to find out what life forms are nearby. However, the flat tire has somehow been charged with uniform charge density $\lambda$. The Tricorder complains that the electric field from the tire is very annoying.

Your goal is to calculate the electric field produced by the electrically-charged tire (ring of line-charge density $\lambda$).

i. The origin is at the center of the tire. Label the diagram with points $(x,y,z)$ and $(x',y',z')$ (that is, define your coordinate system!)

ii. Now label the three vectors: $\vec{r}$, $\vec{r}'$, and $\vec{R}$ (where $\vec{R}$ is Griffiths’ “script r”).
iii. Find the explicit integral expression for \( \vec{E} \) at the position of the tricorder; you do not need to evaluate the integral, but it should be in a form that it could be solved with a computer. This means using the variables defined in this particular problem and explicit limits on the integrals.

Part 2 - Delta Functions for 3D Distributions

i) What is the physical situation represented by this volume charge density? Make a 3-D sketch of the charge distribution:

\[ \rho(x, y, z) = b\delta(y - 3). \]

What are the units of \( b \)?

ii) Determine a purely mathematical expression for the volume charge density, \( \rho \), of an infinitely long cylindrical shell of charge, with radius \( R \) and uniform surface charge density \( \sigma \).
Part 3 – Using Gauss’ Law on shells of charge

i) Derive the electric field from a uniformly charged infinite \textit{cylindrical shell} with radius \(R\) and surface charge density \(\sigma\). Use Gauss’ law to find \(\vec{E}\) everywhere in space. Show the Gaussian surfaces that you used. Make a sketch of the magnitude of the field as a function of the radial distance \(s\) from the central axis of the cylinder.

ii) Make a vector field sketch of the electric field.
iii) Derive the electric field (everywhere) from a uniformly charged spherical shell with total charge $Q$ and radius $R$. Make a sketch of the magnitude of the field as a function of the radial distance $r$ from the center of the sphere.

iv) Using the definition of voltage (potential), $V = -\int \vec{E} \cdot d\vec{L}$, find $V$ for this distribution, in all regions of space ($r<R$ and $r>R$) with the choice of zero voltage at $r = \infty$. Sketch $V$ as a function of $r$. Compare your sketches of $E(r)$ and $V(r)$; how should they be related?

v) Now using the origin as the point of zero voltage, sketch this $V$ as a function of $r$.

vi) Find the energy of this spherical shell.
Part 4 - Conductors

A coax cable is essentially one long conducting cylinder surrounded by a conducting cylindrical shell (the shell has some thickness). The two conductors are separated by a small distance. (Neglect all fringing fields near the cable’s ends)

i) Draw the charge distribution (little + and – signs) if the inner conductor has a total charge +Q on it, and the outer conductor is electrically neutral. Be precise about exactly where the charge will be on these conductors, and how you know. Also draw a rough sketch of the E field lines.
ii) Consider how the charge distribution would change if the inner conductor is shifted off-center, but still has +Q on it, and the outer conductor remains electrically neutral. Draw the new charge distribution (little + and – signs) and E field lines and be precise about how you know.