Magneto静电学

New topic. Until 1800's, different distinct force of nature!

Lodestones attract + repel. Is it electric? No.

→ Think of how you convince yourself Kitchen magnets are not electrical in nature!

→ Magnets don't attract or repel charges!
→ Magnets don't fade away
→ Charged rods never repel magnets

Compass needles are simple indicators of presence (+ direction) even

1568 Mercator: Needles point terrestrial source.

1600 Gilbert: Earth is a big lodestone

90 yrs before Newton realizes earth gravitates!

1820 Oersted observed that currents produces magnetic effects on compasses.

\[ \vec{F} = m \vec{a} \]

\[ \vec{F} = q \vec{E} + q \vec{v} \times \vec{B} \]

Purely from 

Magnetic field

Units: 1 Tesla = \( \frac{1 \text{ N}}{\text{C} \cdot \text{m/s}} \)

Big 1 = \( 10^4 \) gauss

(Earth field ~ \( \frac{1}{4} \) gauss)
Order of investigation:

1) General features + sources of $\vec{B}$ (still static/steady, no time dep. yet!)

2) Consequences of $\vec{E} = \nabla \times \vec{B}$

3) Current = source of $\vec{B}$ fields (surface current; volume current)

4) Biot-Savart: "Coulomb's law for magnetism"
   (How current creates $\vec{B}$).

5) Ampere's Law: "Like Gauss' Law: How currents $\Rightarrow \vec{B}$ fields"

$\vec{B}$ fields are both very different + also very similar/analagous to $\vec{E}$. Look for parallels + connections, some are very deep!

Statics:

$$\nabla \cdot \vec{E} = \rho / \epsilon_0$$ → $\rho$ is "source of $\vec{E}$" which diverges

$$\nabla \times \vec{B} = \mu_0 \vec{J}$$ → $\vec{J} =$ "current" is "source of $\vec{B}$" which curls.

$$4\pi \cdot 10^{-7} \text{ N/A}^2$$ with $1 \text{ A} = 1 \text{ C/sec}$

(Exacts! Defines the Amp $\Rightarrow$ the Coulomb!)

$$\nabla \cdot \vec{B} = 0$$ → No magnetic monopoles
2) Since \( \vec{F} = q \vec{v} \times \vec{B} \), \( \vec{F} \cdot d\vec{x} = q (\vec{v} \times \vec{B}) \cdot (\vec{v} \, dt) = 0 \).

\[ \text{\( \uparrow \perp \) to } \vec{v}! \]

\( \vec{B} \) fields do not work. (They "bend" trajectories) (We'll return to this)

\[ \text{cyclotrons} : \quad \vec{F} = \frac{d\vec{p}}{dt} = q \vec{v} \times \vec{B} \quad \text{Suppose } \vec{B} = B_0 \hat{z} \]

\[ \text{so } F_z = 0, \text{ any motion in } z \text{-direction is unaffected) "drifts"} \]

\( \text{so let's set } V_z = 0. \)

\[ \text{No work } \Rightarrow |p| \text{ won't change. Force is } \uparrow \perp \text{ motion,} \]

\[ \text{constant speed } \Rightarrow \text{uniform circular motion.} \]

\[ \vec{F} = m \vec{a} \Rightarrow q \vec{v} \vec{B} = m \frac{v^2}{R} \]

\[ \text{Circular motion with } \quad R = \frac{mv}{qB} = \frac{|p|}{qB} \]

\[ \text{Frequency } = \frac{\text{cycles}}{\text{sec}} = \frac{1}{(\text{sec/revolution})} \quad \text{but } \quad V \cdot T = \pi R \]

\[ \Rightarrow T = \frac{2\pi R}{v} \]

\[ \text{so } \quad f_{\text{cyclotron}} = \frac{V}{2\pi R} = \frac{qBR/m}{2\pi R} = \frac{qB}{2\pi m} \]

\[ \text{Independence of radius or initial speed!} \]

\[ \text{If } V_z \to 0, \text{ superpose "z drift" } \Rightarrow \text{helix} \]
Inject low $\vec{p}$ electron, fixed time $T = \frac{1}{2f} = \frac{8B}{4nm}$ to go "half circle", then turn on $\vec{E}$ to accel across gap (capac!)

Capac switches sign every $\frac{8B}{4nm}$ sec, (steady freq) $\Rightarrow$ electrons accel each time in $\vec{E}$ field, can build up KE, $\vec{B}$ field "contains" the electrons.

Loss of uses, particle accelerators nowadays, medical beams!

Works up to relativistic energies, then need other tricks

\[ \text{E.g. FermiLab has fixed } R, \text{ increases } \vec{B} \text{ as speed up,} \]
\[ \text{gets protons up to } TeV = 10^{12} \text{ eV} \]
\[ (R \approx 2 \text{ km}) \]
Mass spectrometer

\[ \vec{V} \rightarrow \uparrow \text{beam} \rightarrow \uparrow \text{velocity selector. } \vec{F} = q \vec{E} \uparrow \text{up } \quad \vec{q} \vec{B} \downarrow \text{down} \]

If \( q \vec{E} = q \vec{v} \vec{B} \), i.e. \( \vec{v} = \frac{E}{B} \), then make it through, in straight line...

If know \( q \) (usually \( e \) or \(-e\)) veloc. selector tells you \( \vec{V} \)

\[ R = \frac{|p|}{qB} \text{ tells you } m \vec{v}, \]

so know \( m \Rightarrow \text{identify atoms} \)

what if release \( q \) from rest in the "velocity selector"?
Something different!

\[ \uparrow \vec{E} = E \hat{z} \quad \Rightarrow \vec{F}_x = 0 \text{ so } \vec{v}_x = 0 \]
\[ \uparrow \vec{B} = B \hat{z} \quad \Rightarrow \vec{v} = (0, v_y, v_z) \]

\[ \vec{F} = m(0, v_y, v_z) = q(0, 0, E) + q \vec{v} \times \vec{B} \]

\[ = (0, q v_z B, q (E - q v_y B)) \]

\[ = v_y \vec{B} (\hat{x}) + v_z \vec{B} (\hat{y}) \]

so \( m \dot{v}_y = q v_z B \quad \Rightarrow m \ddot{v}_y = q B \ddot{v}_z \)

Plug this into next one:

\[ m \dot{v}_z = q (E - q v_y B) \Rightarrow m (\dot{v}_y / q B) = q E - q B \dot{v}_y \]
Thus, 
\[ \dot{V}_y = \frac{qB}{m^2} (E - BV_y) \]

This is of the form \[ \ddot{x} = a - bx \] ( \( a = \frac{qB}{m^2} E, \ b = \frac{qB}{m^2} \))

The general sol'n of this \( \ddot{x} \) is
\[ x = C_1 \sin (\sqrt{b}t) + C_2 \cos (\sqrt{b}t) + \frac{a}{b} t \]
(Proof: check it! It has 2 undetermined coeffs...)

But here \( x = V_y = \frac{dy}{dt} \), so integrating gives
\[ y = C_1 \cos (\sqrt{b}t) + C_2 \sin (\sqrt{b}t) + \frac{a}{b} t + C_3 \]

with \( \sqrt{b} = \frac{B}{m} \) and \( \frac{a}{b} = \frac{E}{B} \), see above!

Then \( \dot{V}_x = \frac{qB}{m} \dot{V}_y \) can be integrated, to get
\[ V_x = \frac{qB}{m} t - \frac{qB}{m} \left( \frac{C_1 \cos(\sqrt{b}t) + C_2 (\sqrt{b}t + \frac{E}{b} t + C_3)}{m} \right) \]

Easier!
\[ \dot{V}_x = \frac{m\dot{V}_y}{qB} = \frac{m}{qB} \left( -\frac{qB}{m^2} (C_1 \cos(\sqrt{b}t) + C_2 \sin(\sqrt{b}t)) \right) \]
\[ = -\frac{qB}{m} (C_1 \cos(\sqrt{b}t) + C_2 \sin(\sqrt{b}t)) \]

so \[ Z = \int \dot{V}_x \cdot dt = -C_1 \sin(\sqrt{b}t) + C_2 \cos(\sqrt{b}t) + C_4 \]

(See Griffiths for interp!)