Currents: Current is the source of \( B \) fields, (also react to \( B \) fields), so it's very important to clearly define and understand it. It's a measure of the flow-rate of charge. Current counts "how many charges pass by each sec."

\[
|I| = \frac{dQ}{dt}
\]

Consider first a line charge (2 Coulombs/meter) moving steadily with velocity \( \vec{V} \)

\[
\frac{2 \text{ Coul/m}}{1 \text{ m}} \rightarrow \vec{V}
\]

\( \uparrow \) How many coul move past this point in 1 sec?

\( V \cdot t = d, \text{ so } \Rightarrow \) all the charges in the chunk \( V \cdot 1 \text{ sec} \) long "behind" the point will make it past!

That's \( \lambda = \frac{Q}{T} = \frac{2 \cdot \vec{V}}{(1 \text{ sec})} \)

so charge in one sec = \( \frac{Q}{T} = \lambda \cdot \vec{V} \).

So \( \vec{I} = 2 \vec{V} \). \( \vec{I} \) Griffiths convention is to call \( \vec{I} \) a vector, (not everyone does this.)

Note: If \( \lambda \) is negative, current goes "other way".

so \( \vec{V} \rightarrow \) is same current as \( \vec{V} \rightarrow \), both are \( \vec{I} \rightarrow \)
$I$ measured in $\frac{\text{Coul}}{\text{sec}} = \text{Amperes}$.

If the wire has $n_e \frac{\text{charge carriers}}{\text{length}}$, each of which carries $\dot{q}$, then

$$\lambda \frac{\text{Coul}}{m} = n_e \frac{\text{carriers}}{m} \cdot \dot{q} \frac{\text{Coul}}{\text{carrier}}$$

so

$$\vec{I} = n_e \dot{q} \vec{V}$$

Since $F_{\text{mag}} = q \vec{V} \times \vec{B}$ for individual charges, a current feels a force too (each individual charge feels a force, so the "current" feels the sum of those).

For a small piece of my wire (above), $d\vec{l}$ long, there are $(n_e d\vec{l})$ charges, each feeling $\dot{q} \vec{V} \times \vec{B}$, giving

$$d\vec{F}_{\text{on chunk}} = n_e(d\vec{l})\dot{q} \vec{V} \times \vec{B}$$

Since $\vec{V}$ is along the wire, and so is $d\vec{l}$, I can write $d\vec{l} \cdot \vec{V} = V d\vec{l} \cdot \vec{V} \cdot \vec{V}$ (No dot here !)

so

$$d\vec{F} = n_e \dot{q} V d\vec{l} \times \vec{B} = I d\vec{l} \times \vec{B}$$

($I$ is here mag of current, $d\vec{l}$ tells direction)
Summary:

\[ I = \alpha V \text{ along wire } = n \lambda \beta V \]

\[ dF_{\text{chunk}} = I \frac{d\vec{L} \times \vec{B}}{dt} \]

What if charges move throughout a volume, not just along lines?

We define the current passing our little \( da \) as usual,

\[ \text{current} = \frac{\text{total charge}}{\text{sec}} = \frac{dq}{dt} \]

If \( da \) is tiny, we can think of \( dI \), a tiny current passing through.

I will define \( \vec{J} = \text{volume current density} = \frac{d\vec{I}}{da} \)

(and the direction of \( \vec{J} \) will be the direction of \( d\vec{I} \))

But to be careful, \( da \) is really \( da_+ \), here (see next p)
Given a steady \( \vec{J} \) clearly current through either \( da \) or \( da_\perp \) is some here, so to uniquely define \( \frac{\partial \vec{J}}{\partial a} \) we need to pick \( da_\perp \) as the area we mean...

so \( d \vec{I} = \vec{J} \, da_\perp = \vec{J} \cdot d \vec{a} \)

This would also serve to define \( \vec{J} \)!

Just like w. line charges, you can see that in 1 sec, the total current passing through = total charge in a volume that extends back (along \( \vec{J} \)) by distance \( \vec{V} \cdot (1 \text{sec}) \)

If we have \( \rho \) charges in that region, that means

\( \rho [\vec{V} \cdot 1 \text{sec}][da_\perp] \) charges will pass through, so

\[
d \vec{I} = \rho \frac{\vec{V} \cdot 1 \text{sec} \cdot da_\perp}{1 \text{sec}} = \vec{J} \, da_\perp
\]

\[
\text{so } \sqrt{\vec{J}} = \rho \sqrt{\vec{V}}
\]
as before, instead of \( p \), we might use

\[
p = \frac{N}{\text{charge carriers}} \cdot \frac{q}{\text{coulombs}} \cdot \frac{1}{\text{volume}}
\]

so \( \vec{J} = \frac{N \cdot q \cdot \vec{v}}{\text{number density, per unit volume}} \) = current passing area

\[
\text{Volume current density}
\]

\[
\text{Units are } \frac{\text{A}}{\text{m}^2}
\]

so in 3-D situations, for a "chunk" of volume \( d\tau \),

\[
d\mathbf{F} = \frac{N \cdot d\tau \cdot q \cdot \vec{v} \times \vec{B}}{\text{area}}
\]

This is sum of \( q \cdot \vec{v} \times \vec{B} \) for all \( N \cdot d\tau \) charges!

Once again, \( \vec{v} \) and \( \vec{J} \) point in same direction locally,

so \( N \cdot q \cdot \vec{v} = \vec{J} \), and

\[
d\mathbf{F} = (\vec{J} \times \vec{B}) d\tau
\]

We jumped for line currents to volume currents.

But many times charges live on surfaces, so we can also define a surface current density.
Surface current density $\mathbf{\dot{I}}$:

Here the current passing a line segment $dl$ is defined as

$$ dl = \frac{dQ}{dt} $$

I will define $\mathbf{\dot{I}} = \text{surface current density} = \frac{d\mathbf{I}}{dl}$

again, direction of $\mathbf{\dot{I}} = \text{direction of } d\mathbf{I}$

(As before, for uniqueness I put $dl$ in the definition.)

This one is slippery! It's a ribbon of current, and $\mathbf{\dot{I}}$ tells how much current passes by a unit length perpendicular to flow!

Just as in prev 2 cases, we can quickly get

$$ \mathbf{\dot{I}} = \sigma \mathbf{V} \quad \text{with} \quad \sigma = \frac{\text{Coulombs}}{m^2} = \text{surface charge density} $$

$$ \sigma = ns q \mathbf{V} \quad \text{with} \quad ns = \# \text{ of charge carriers per } m^2 $$

Example: Could write $\mathbf{\dot{I}} = \mathbf{K} \delta(t) \mathbf{V}$. Think about this!

Units of $\mathbf{\dot{I}} = A/m$, it's current passing unit length + $\mathbf{F}$ little piece of ribbon = $(\mathbf{\dot{I}} \times \mathbf{B}) da$, also as before...
Conservation of current (+ charge)

Total charge is conserved \( \implies \) exp'rol fact.

which means, if you pick any volume,

total inflow of charge = growth of net charge inside

total outflow of charge = loss of net charge inside.

Since \( \mathbf{J} \cdot d\mathbf{a} = \frac{dQ}{dt} \) flowing out through area \( d\mathbf{a} \) (p.10)

**Total outflow** = \( \iiint \mathbf{J} \cdot d\mathbf{a} \) \( \left( \text{this is Count \frac{\text{charge}}{sec}, it's rate of loss} \right) \)

Since \( Q \) inside = \( \iiint p \cdot d\mathbf{r} \),

then rate of loss of charge = \( -\frac{d}{dt} \iiint p \cdot d\mathbf{r} \)

so, if \( Q \) decreasing, loss is +.

so \( \iiint \mathbf{J} \cdot d\mathbf{a} = -\iiint -\frac{\partial p}{\partial t} \, d\mathbf{r} \) \( \implies \) Just conservation of charge

<table>
<thead>
<tr>
<th>Div. theorem</th>
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<tbody>
<tr>
<td>( \iiint (\nabla \cdot \mathbf{J}) , d\mathbf{r} = \iiint -\frac{\partial p}{\partial t} , d\mathbf{r} ) ( \implies ) True for any volume, remember!</td>
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so \( \nabla \cdot \mathbf{J} = -\frac{\partial p}{\partial t} \) \( \implies \) Continuity equation
Continuity eqn is basic statement of charge conservation:

$$\nabla \cdot \vec{J} + \frac{\partial \rho}{\partial t} = 0$$

"out flow of current" + "increase in local charge" must cancel!

and, to summarize:

$$\vec{J} = \rho \vec{v} = \text{volume current density} = \text{Amps passing at}$$

$$\vec{K} = \sigma \vec{v} = \text{surface current density} = \text{Amps passing at}$$

$$\vec{I} = I \vec{v} = \text{line current} = \text{Amps passing point}$$

and

$$\vec{J} = N_{\text{Vol}} q \vec{v}$$

$$\vec{K} = N_{\text{Surf}} q \vec{v}$$

$$\vec{I} = n_{\text{Line}} q \vec{v}$$

and, when you need to sum (e.g. finding forces):

$$\iiint \vec{J} \, d\tau \leftrightarrow \iint \vec{K} \, d\alpha \leftrightarrow \oint \vec{I} \, dl \leftrightarrow \sum q \vec{v}_i$$

won't use much for now, 'cause

In magnetostatics, (by definition), charges don't pile up anywhere, $\frac{\partial \rho}{\partial t} = 0$ + $\nabla \cdot \vec{J} = 0$ ← spatial

[Ohm's Law: $\vec{J} \propto \vec{E}$. In conductors, obeying Ohm's law ⇒ magnetostatics!]