Conductors: Perfect conductance is idealization (though realizable w. superconductors) but metals \((\text{Cu, Al, etc})\) are excellent approximations!\

\(\rightarrow\) Charges are free inside: can respond (instantly \(+\text{no loss}\)) to forces. Consequences in e-static situations (¡¡)

\(\mathbf{E} = 0\) inside. See Griffiths p. 97,

but clearly if \(\mathbf{E} \neq 0\), \(\mathbf{F} = q \mathbf{E} \neq 0\), charges will move! Keep moving until \(\mathbf{E} = 0\). (That's "static", then)

\(\mathbf{p} = E_0 \nabla \cdot \mathbf{E} = 0\) inside

Excess charge must live on outside edge. See ¡¡!

\(\Delta V(\mathbf{a} \rightarrow \mathbf{b}) = -\int_{\mathbf{a}}^{\mathbf{b}} \mathbf{E} \cdot d\mathbf{L} = 0\) if \(a, \mathbf{b}\) are both in or on conductor. So \(\Rightarrow\) Equipotential throughout. (Even if charged!)

\(\mathbf{E} \perp\) surface at edges. (If there was any \(\mathbf{E}_n\), then surface charges would move.) (Or, see next page)
Consider \( \oint \mathbf{E} \cdot d\mathbf{l} \), with \( h \to 0 \). Must be 0!

So get \( \text{O inside + tiny leg} + E_{\|} \cdot L + \text{tiny leg} = 0 \).

This is a formal proof that \( E_{\|} \) (outside) = 0.

Also, consider Gauss pillbox, with tiny height \( h \).

\[
E_{\text{out}} \cdot A + E_{\text{in}} \cdot A \quad + \text{tiny walls} = \frac{Q_{\text{enc}}}{E_0} = \frac{\sigma \cdot A}{E_0}
\]

So \( E_{\text{out}} = \frac{\sigma}{E_0} \), pointing normal.

Interesting, it's not \( \frac{\sigma}{2E_0} \) like an isolated sheet of charge gives.

(There must be other charges superposing to give us this \( E \) field)

Many many consequences!

- Conductors polarize in presence of external \( \mathbf{E} \)'s.
  - Have to, to ensure \( \mathbf{E} = 0 \) inside.

- Makes it harder to solve for \( V(r) \) (or \( \mathbf{E} \)), since no longer know "a-priori" \( \rho \), it will adjust itself!

  (So e.g. \( V = \frac{1}{4\pi\epsilon_0} \int \frac{\rho}{r} \; d\mathbf{r} \) is still true... but what is \( \rho \) now?)
Cavities & "Shielding".

If conductor has hole, what's going to happen? $\vec{E} = 0$ in metal region, but what about in hole?

Gauss for dashed line says $\int \vec{E} \cdot d\vec{A} = \frac{Q_{enc}}{\varepsilon_0}$

But $\vec{E} = 0$ on dashed line! So $Q_{enc} = 0$.

- If there is a $q$ in the hole, this says $-q$ appears at "inside edge".
- If there is no $q$ in the hole, I claim $\vec{E} = 0$ in hole too, why?

Arg #1: Suppose you start with solid conductor. We know $E = 0$ & $\rho = 0$ throughout. Now remove hole material. Note removing any Q's so not changing $\vec{E}$ anywhere! (It's Q's that create $\vec{E}$'s, after all.)

Arg #2: If $\vec{E} \neq 0$, then this loop shows $\int \vec{E} \cdot d\vec{L} = 0$

But it's 0 for any line, in either direction! No way (unless $\vec{E} = 0$ everywhere.)
Arg #3: We'll soon learn "uniqueness theorem":

Once you find a sol'n for \( V \) (or \( E \)) throughout space that is consistent with "boundary conditions", there is no other sol'n.

\( E = 0 \) is consistent, so it's unique, so that's what it is.

This is "Faraday cage" effect: Inside a conductor, \( E = 0 \), (even in cavities, even with \( Q \)'s outside, even if conductor has \( Q \).)

- What if put \( Q \) in there?
  - We know you polarize, (putting \(-Q\) on inside wall...)
  - Charge conservation puts \(+Q\) on outside wall.

\( E = 0 \) inside the cavity anymore.

- Outside: field \( \neq 0 \) (because \( Q_{enc} = +Q \))

Field outside is same as if had same conductor with no cavity, but no charge \(+Q\). (Again, that "uniqueness theorem".)
Forces: Consider a sheet (conductor or not) with surface charge $\sigma$. Apply an $\vec{E}$ field… what force would a patch (area $dA$) feel? Well, $d\vec{F} = dq \vec{E} = (\sigma \cdot dA) \vec{E}$. But what is $\vec{E}$? If you're on a surface, $\vec{E}$ is not continuous, $E_{\text{above}} - E_{\text{below}} = \frac{\sigma}{\varepsilon_0}$

Answer: Use $\vec{E}$ from all $q$'s except the patch, 'cause nothing exerts a force on itself.

Superposition: $\vec{E}_{\text{total}} = \vec{E}_{\text{external}} + \vec{E}_{\text{patch itself}}$

$\vec{E}_{\text{external}}$ will be continuous (!) so $\vec{E}_{\text{above}} = \vec{E}_{\text{ext}} + \frac{\sigma}{2\varepsilon_0}$

$\vec{E}_{\text{below}} = \vec{E}_{\text{ext}} - \frac{\sigma}{2\varepsilon_0}$

Thus $\vec{E}_{\text{ext}} = \frac{1}{2} (\vec{E}_{\text{above}} + \vec{E}_{\text{below}})$. Use the average real fields.

For conductor, $\vec{E}_{\text{above}} = \frac{\sigma}{\varepsilon_0}$, $\vec{E}_{\text{below}} = 0 \implies d\vec{F} = \sigma \cdot dA \cdot \frac{\vec{E}}{2\varepsilon_0}$ (our)

Thus, there is an outward pressure $\frac{d\vec{F}}{dA} = \frac{\sigma^2}{2\varepsilon_0}$ outward.
CAPACITANCE: Any pair of conductors will have a well-defined \( \Delta V \) (cause each one is an equipotential)

\[ \Delta V_{bar} \]

\[ \Delta V_{bar} \]

\[ \Delta V \]  
\[ = \int \vec{E} \cdot d\vec{L} \]

\( \Delta V = -\int \vec{E} \cdot d\vec{L} \)

But Gauss' law says \( \vec{E} \propto \vec{Q} \), so \( \Delta V \propto \vec{Q} \), so

Define \( C = \frac{Q}{\Delta V} \) \[ \text{[depends on objects, config, shape,]} \]

\[ \text{but not on } \vec{Q} \text{ or } \Delta V! \]

\( 1 \text{C} / \text{V} = 1 \text{ Farad} \)

Some situations are easy to calculate, if known \( \vec{E} \) field

**Ex 1:**

\[ \begin{array}{c}
+Q \\
\downarrow E \\
\hline
L \\
\hline
-\alpha
\end{array} \]

\( \vec{E} = \frac{\sigma}{\varepsilon_0} \) between, 0 elsewhere

\( \Delta V = -\int \vec{E} \cdot d\vec{L} = +\frac{\sigma}{\varepsilon_0} \cdot L \)

So \( C = \frac{Q}{\Delta V} = \frac{Q}{Q/A \cdot \varepsilon_0} = \frac{\varepsilon_0 \cdot A}{L} \)

\[ \text{(Do you see why it's +?)} \]
Ex 2:

Here \[ E = \frac{1}{4\pi \varepsilon_0} \frac{Q}{r^2}, \]

\[ \Delta V = \int_a^b \mathbf{E} \cdot d\mathbf{l} = \frac{1}{4\pi \varepsilon_0} Q \cdot \frac{1}{r} \bigg|_a^b \]

and \[ C = \frac{Q}{\Delta V} = 4\pi \varepsilon_0 \left( \frac{1}{a} - \frac{1}{b} \right) \]

[If \( b \to \infty \) this is \( 4\pi \varepsilon_0 a \)]

**Energy stored**: Could compute \( \frac{E_0}{2} \int E^2 \, dV \),

but can also ask "how much external work needed" to charge it up to \( Q \)?

Each \( dq \) that gets moved takes work

\[ dW_{\text{to move}} = dq \cdot \Delta V, \]

\[ \int dq \text{ over } \]

\[ q = Q \]

\[ W_{\text{tot}} = \int \Delta V \cdot dq = \int_0^Q \frac{Q}{C} \, dq = \frac{1}{2} \frac{Q^2}{C} \]

(\( \Delta V = Q/C \) depends on \( q \) we've built up so far.)

\( q = 0 \)

(Circle, our sphere has stored energy \( \frac{1}{2} \frac{Q^2}{(4\pi \varepsilon_0 \cdot q)^2} \))