COULOMB’ S LAW, E FIELDS
Class Activities
Coulomb’s Law

Tutorials
Lab 1 – Intro to Electrostatics
Griffiths by Inquiry

Whiteboard
Integral for E for line of charge
Work out the integral for the example of the E from a finite line of charge, with constant lambda. This is practice in constructing “script R”, mostly.

Discussion
Questions for Lecture (from UIUC)
1. Is Coulomb’s force law valid for all separation distances \( r_{ab} \)? Is it valid for \( r_{ab} = 0 \)?
2. What is the physics origin of the \( 1/r^2 \) dependence of Coulomb’s force law? Of the \( 1/r^0 \) dependence? Of the \( 1/4\pi \) factor?
3. What really is electric charge? What is the equivalent of “charge” for any/all of the 4 fundamental forces of nature?
4. Why is electric charge quantized (in units of \( e \))? What operative physics dictates this?
5. What really is negative vs positive electric charge (i.e., \(-e\) vs \(+e\))
6. Why does the Coulomb force vary as the product of two electric charges?
7. What is a macroscopic vector field, such as \( E(r,t) \)?
8. Are electric field lines real? Do they really exist in space and time?
See Activities/Activity resources “Deep Questions” for answers to these questions.

Tutorials
Electric Field due to a Ring of Charge
Oregon State University
In this activity, students working in small groups to write the electric field in all space due to a charged ring. The groups are then asked to expand this potential in a series, either on the axis or the plane of symmetry, and either close to the center of the charge distribution or far away.
Class Activities: Charge Distributions

Kinesthetic activity
Line Charge
Oregon State University
Asked about 6-10 students to stand up. "Make a linear charge density for me, a lambda". Then, lots of discussion and other variations - Make lambda bigger. Is this really a lambda, or an approximation? What if the charges were only on your heads, is that more "lambda-like". In what way(s)? Does it matter that the charges are coming in chunks, does THAT make it "not a line charge"? Make it nonuniform. Make it curl around - is it still a lambda?)

Tutorial
Two-dimensional charge distributions
Paul van Kampen – Dublin University (Tutorials 1-8, page 25)
Two dimensional charge distributions. Practice in integration in polar coordinates. Calculate net charge on a disk, the problem is broken into pieces (find dA, write dQ, write out the integral, evaluate), and then do it again in Cartesian.

Computer visualizations
E-field, Charge distributions, line and surface integrals
Electric field lines of various charge distributions, including the ability to do line and surface integrals to determine circulation and flux:
http://www.falstad.com/mathphysics.html
2D electrostatics: http://www.falstad.com/emstatic/
2D electrostatic fields: http://www.falstad.com/vector2de/
3D electrostatic fields: http://www.falstad.com/vector3de/

Discussion
Deep Questions
Some "deep questions" discussed (e.g., WHY is Coulomb 1/r^2, WHY is it the product of charges. What IS charge anyway? Opened this up to them, let them discuss a little, some questions we can/will address - like 1/r^2 in coulomb. Others, perhaps not, like "what is charge")

Whiteboards
Concept Map of Physics
Passed out white boards - (brief discussion of "how and why") Concept map of physics on white board. Everyone does it on their own, then share with neighbors. What is physics? Think of the BIG ideas, and then organize them (connect the parts that need connecting - perhaps the connections themselves have names...) It can be by courses, topics, ideas, whatever you think represents the whole world of physics on one whiteboard!
Two charges $+Q$ and $-Q$ are fixed a distance $r$ apart. The direction of the force on a test charge $-q$ at $A$ is...

A. Up  
B. Down  
C. Left  
D. Right  
E. Some other direction, or $F = 0$
Two charges +q and -q are on the y-axis, symmetric about the origin. Point A is an empty point in space on the x-axis. The direction of the E field at A is...

A. Up
B. Down
C. Left
D. Right
E. Some other direction, or E = 0, or ambiguous
How is the vector $\mathbf{r}_{12}$ related to $\mathbf{r}_1$ and $\mathbf{r}_2$?

A) $\mathbf{r}_{12} = \mathbf{r}_1 + \mathbf{r}_2$

B) $\mathbf{r}_{12} = \mathbf{r}_1 - \mathbf{r}_2$

C) $\mathbf{r}_{12} = \mathbf{r}_2 - \mathbf{r}_1$

D) None of these
Coulomb's law: $\vec{F}(\text{by 1 on 2}) = \frac{kq_1q_2}{2^{\frac{1}{12}}}\hat{\vec{a}}_{12}$

In the fig, $q_1$ and $q_2$ are 2 m apart. Which arrow *can* represent $\hat{\vec{a}}_{12}$? 

D) More than one (or NONE) of the above

E) You can't decide until you know if $q_1$ and $q_2$ are the same or opposite signed charges
What is \( \overrightarrow{r} \) ("from 1 to the point r") here?

\[ \overrightarrow{r_1} = (x_1, y_1) \]

\[ \overrightarrow{r} = \overrightarrow{r_1} - q \]

\[ \hat{A} = \overrightarrow{\hat{A}} / |A| \]

\[ r = (x, y) \]

A) \( (x, x_1, y, y_1) \)  
B) \( (x_1, x, y_1, y) \)  
C) \( \frac{(x, x_1, y, y_1)}{\sqrt{(x - x_1)^2 + (y - y_1)^2}} \)  
D) \( \frac{(x_1, x, y_1, y)}{\sqrt{(x - x_1)^2 + (y - y_1)^2}} \)  

E) None of these
Is the answer to part 1- iii
A) A sum?
B) An integral over dy?
C) An integral over something else?
Only after you finish Part 2, what is  in part 2-iv?

A) \((x \ x', y \ y', z \ z')\)  

B) \((x' \ x, y' \ y, z' \ z)\)

C) \[\sqrt{(x-x')^2 + (y-y')^2 + (z-z')^2}\]

D) \[\sqrt{(x-x')^2 + (y-y')^2 + (z-z')^2}\]

E) None of these!
5 charges, q, are arranged in a regular pentagon, as shown. What is the E field at the center?

A) Zero
B) Non-zero
C) Really need trig and a calculator to decide
1 of the 5 charges has been removed, as shown. What’s the E field at the center?

A) \(+\left(\frac{kq}{a^2}\right) \mathbf{j}\)
B) \(-\left(\frac{kq}{a^2}\right) \mathbf{j}\)
C) 0
D) Something entirely different!
E) This is a nasty problem which I need more time to solve
To find the E-field at $P=(x,y,z)$ from a thin line (uniform linear charge density $\lambda$):

$$E = \frac{1}{4} \left[ \frac{1}{2} \right] \mathbf{dl}'$$

What is $\mathbf{dl}'$?

A) $x$  
B) $y'$  
C) $\sqrt{dl'^2 + x^2}$  
D) $\sqrt{x^2 + y'^2}$  
E) Something *completely* different!!
\[ \vec{E}(\vec{r}) = \frac{\text{dl}'}{4} \left[ 0, 0, \frac{1}{3} \text{dl}' \right] \rightarrow \]

\[ r' = (0, y', 0) \]

\[ \vec{E}(\vec{r}) = l \text{ dl}' \]

\[ \int_{0}^{x} \frac{\text{dl}'}{4} \left[ 0, 0, \frac{1}{3} \text{dl}' \right] \rightarrow \]

\[ P = (x, 0, 0) \]
\[ \vec{E}(\vec{r}) = \frac{dl'}{3} \rightarrow \]

A) \[ \frac{dy' x}{x^3} \]

B) \[ \frac{dy' x}{(x^2 + y'^2)^{3/2}} \]

C) \[ \frac{dy' y'}{x^3} \]

D) \[ \frac{dy' y'}{(x^2 + y'^2)^{3/2}} \]

E) Something else

so

\[ E_x(x,0,0) = \frac{4}{0} \]

\[ \begin{array}{c}
\sum y \\
\end{array} \]

\[ P = (x,0,0) \]

\[ \vec{r'} = (0,y',0) \]

\[ \text{Diagram showing } \vec{r}' \text{ and } \vec{r} \text{ with } dl' \]
To find the E-field at P from a thin ring (radius R, uniform linear charge density λ):

$$E = \frac{1}{4\pi} \frac{1}{2} \int_0^1 dl'$$

what is $E = \frac{1}{4\pi} \frac{1}{2} \int_0^1 dl'$?

E) NONE of the arrows shown correctly represents...
To find the E-field at P from a thin ring (radius a, uniform linear charge density $\lambda$):

$$E = \frac{1}{4} \int_{0}^{1} \frac{1}{2} dl'$$

what is $\text{?}$?

A) $\sqrt{a^2 + z^2}$

B) $a$

C) $\sqrt{dl'^2 + z^2}$

D) $z$

E) Something completely different!!
Griffiths p. 63 finds $E$ a distance $z$ from a line segment with charge density $\lambda$:

$$\vec{E} = \frac{1}{4} \frac{2}{\int_{0}^{2} z \sqrt{z^2 + L^2}} \hat{k}$$

What is the approx. form for $E$, if $z \gg L$?

$$E = \frac{2}{4} \frac{L}{\int_{0}^{2}} \times (\ldots)$$

A) 0  B) 1  C) $1/z$  D) $1/z^2$

E) None of these is remotely correct.
Griffiths p. 63 finds $E$ a distance $z$ from a line segment with charge density $\lambda$:

$$\vec{E} = \frac{1}{4} \frac{2L}{z_0 \sqrt{z^2 + L^2}} \hat{k}$$

What is the approx. form for $E$, if $z << L$?

$$E = \frac{2}{4} \times (...)$$

A) 0    B) 1    C) $1/z$    D) $1/z^2$

E) None of these is remotely correct.
To find \( \mathbf{E} \) at \( P \) from a negatively charged sphere (radius \( R \), uniform volume charge density \( \rho \)) using what is \( \frac{1}{4\pi\varepsilon_0} \) (given the small volume element shown)?

\[
\mathbf{E} = \frac{1}{4\pi\varepsilon_0} \frac{1}{r^2} \hat{r} \int \mathbf{d}^3 \rho
\]

D) None of these
\[ E = \frac{1}{4\pi \varepsilon_0} \frac{1}{\sqrt{\left[(X-x)^2 + (Y-y)^2 + (Z-z)^2\right]}} \]

A) \[ \int \frac{(X,Y,Z)}{\left[(X-x)^2 + (Y-y)^2 + (Z-z)^2\right]^{3/2}} \rho \, dx \, dy \, dz \]

B) \[ \int \frac{(X,Y,Z)}{\left[(X-x)^2 + (Y-y)^2 + (Z-z)^2\right]^{3/2}} \rho \, dx \, dy \, dz \]

C) \[ \int \frac{(X-x,Y-y,Z-z)}{\left[(X-x)^2 + (Y-y)^2 + (Z-z)^2\right]^{3/2}} \rho \, dx \, dy \, dz \]

D) \[ \int \frac{(X-x,Y-y,Z-z)}{\left[(X-x)^2 + (Y-y)^2 + (Z-z)^2\right]^{3/2}} \rho \, dx \, dy \, dz \]

E) None of these
\[ E = \frac{1}{4\pi\varepsilon_0} \int \frac{(X,Y,Z)}{\left( (X-x)^2 + (Y-y)^2 + (Z-z)^2 \right)^{3/2}} \rho \, dx \, dy \, dz \]

\[ \text{A)} \int \frac{(X,Y,Z)}{\left( (X-x)^2 + (Y-y)^2 + (Z-z)^2 \right)} \rho \, dx \, dy \, dz \]

\[ \text{B)} \int \frac{(X,Y,Z)}{\left( (X-x)^2 + (Y-y)^2 + (Z-z)^2 \right)^{3/2}} \rho \, dx \, dy \, dz \]

\[ \text{C)} \int \frac{(X-x,Y-y,Z-z)}{\left( (X-x)^2 + (Y-y)^2 + (Z-z)^2 \right)^{3/2}} \rho \, dx \, dy \, dz \]

\[ \text{D)} \int \frac{(X-x,Y-y,Z-z)}{\left( (X-x)^2 + (Y-y)^2 + (Z-z)^2 \right)^{3/2}} \rho \, dx \, dy \, dz \]

\[ \text{E) None of these} \]