1. **Classical instability of the H-atom:** In the Bohr Model of the H-atom, an electron orbits the nucleus in a circle of radius $r_0$. One serious problem with this model is that a charged particle moving in a circle is accelerating, and an accelerating charge radiates away EM energy according to the Larmor formula (Griffiths 11.70). So the H-atom will lose energy and the electron will quickly spiral into the nucleus, according to classical electrodynamics and mechanics.

A) Derive a formula for the total energy $[U(r) = KE + PE]$ of an electron in circular orbit at radius $r$. (As usual, let’s set the zero of PE at $r = \infty$.) Start with $F = ma$ to get the KE. You can use your PHYS1120 knowledge to write down the formula for the PE.

B) Using your result from (A) and the Larmor formula for radiated power, derive the differential equation that describes $r(t)$.

C) Derive an expression for $r = r(t)$. *Hint: Your formula for $r(t)$ should be of the form $r(t) = (r_0^3 - C \cdot t)^{1/3}$, where $C$ is a constant that you will figure out.*

D) Assume that the initial radius $r_0$ is the Bohr radius ($\approx 0.05$ nm); put in numbers and solve for the time for the electron to crash into the nucleus. How does this compare to the period of the electron orbit in the Bohr model (in the ground state, $r_0 \approx 0.05$ nm)?

2. **Radiation Resistance** In the Pollock lecture notes, you can find the expression for the radiation resistance $R_{rad} = P/I_{rms}^2$ of the little electric dipole, for the case of electric dipole radiation. Now, let’s think about applying it to your cell phone. Assuming we can continue to use that “small electric dipole approximation” all the way up to the scale of wires in the phone, how does the radiative resistance compare to the ohmic resistance of the same wire? (Compute both, in ohms, and compare them.) Cell phone operate at a frequency of about 1000 MHz. Assume the length of the radiating antenna wire is roughly the size of your phone. *(Note that the assumptions we’re making are not necessarily realistic for a real antenna, but this is the best we can do working with this section of Griffiths!)*
3. Health effects of EM radiation. A pirate radio station has been set up behind a residential home in South Boulder. At the top of a tall ham radio tower of height $H$ is a small electric dipole antenna, of length $d$, with its axis oriented vertically ($d \ll H$). The antenna broadcasts at angular frequency $\omega$, with total time-averaged radiated power $P$. A neighbor living nearby is a friend of yours. Although they don't want to turn the station in to the authorities, your friend is worried that he is being “irradiated”, with possible health effects. Knowing that you’re a physics major with access to equipment, he asks for your help.

A) In terms of variable names given above, find a formula for the time-averaged intensity (watts/m$^2$) of EM radiation at ground level a distance $R$ away from the base of the tower.

B) You want to identify the “worst case” spot. Where should you measure the intensity if you want it to be as large as it can get at ground level? Use your formula from part A and rewrite it at this spot, simplifying as much as possible.

C) Let’s put in some plausible numbers. Suppose the pirate station is broadcasting at 100 MHz (FM), and putting out a total EM power of 10 kW. Their electric dipole antenna is 5 cm tall, and the tower is 50 m tall. According to FCC information, 100 mW/cm$^2$ is where human bodies can start to heat up (!), and in some circumstances intensity levels down to 1-10 mW/cm$^2$ are potentially harmful (e.g. to eyeballs). What do you conclude – does your friend need to worry about her health?

D) Do I have to worry about EM radiation producing a cancer-causing mutation in my skin cells? Quantum mechanics says that EM waves can only be absorbed by atomic systems, like molecules, in discrete “packets”, called photons. What is the energy of a 100 MHz photon, in electron-volts (eV)? To cause a mutation in a cell, I have to break a chemical bond. Bond energies in molecules are about 0.1 eV. Can radio-wave EM waves cause cancer?
4. Star Wars with Relativity
A) Space probe #1 passes very close to earth at a time that both we (on earth) and the onboard computer on Probe1 decide to call t=0 in their respective frames. The probe moves at a constant speed of 0.6c away from earth. When the clock aboard Probe1 reads t=60 sec, it sends a light signal straight back to earth.
- At what time was the signal sent, according to the earth's rest frame?
- At what time in the earth's rest frame do we receive the signal?
- At what time in Probe1's rest frame does the signal reach earth?

B) Space probe #2 passes very close to earth at t=1 sec (earth time), chasing Probe1. Probe2 is only moving at 0.3c (as viewed by earth). Probe2 launches a proton beam (which moves at v=0.31c relative to Probe2) directed at Probe1. Does this proton beam strike Probe1? Would the proton beam hit Probe1 if Galilean Relativity was correct, rather than Special relativity?

5. Origin of Magnetism. One of the best texts for EM is Purcell’s advanced freshman text. In it, he shows a wonderful application of the Lorentz contraction, to begin to understand how magnetism and electricity are related to one another. Let’s step through the basics! (This exercise comes from an AAPT talk by Dan Shroeder of Weber State.)

![Diagram of a wire with current flowing to the right](image)

Shown above is a model of a wire with a current flowing to the right. To avoid minus signs we take the current to consist of a flow of positive charge carriers, each with charge +q, separated by an average distance of \( l \). The wire is electrically neutral in the lab frame, so there must also be a bunch of negative charges, at rest, separated by the same average distance \( l \) in this frame. Be aware that charge is Lorenz invariant: a charge \( Q \) has the same value in every inertial frame.

A) Using Gauss’s law, what is the electric field outside this wire in the lab frame?
Suppose there is a test charge +Q outside the wire, a distance \( R \) from the center of the wire, moving to the right (For simplicity, let’s say the velocity is the same as that of the moving charges in the wire, i.e. \( v \), as shown in the figure)
- Given your answer for the E field, what is the electrostatic force on this charge, in this frame?
- Using Ampere’s (and Lorentz force) law - what is the magnetic force on the moving test charge \( Q \)?
- Put it together, what is the direction of the net force on the test charge, and what “causes” it?
B) Now consider how all this looks in the reference frame of the test charge, where it's at rest.

- In THIS frame, what is the magnetic force on the test charge \( Q \)?
  
  In this frame, it's the negative charges in the wire that are moving to the left. Because they're moving, the average distance between them is length-contracted. Meanwhile the positive charges are now at rest, so the average distance between them is now longer than \( l \).

- What is the average distance \((l_a)\) between the positive charge carriers in this frame?
  
  Both of these effects give the wire a non-zero charge density.

- Compute the charge density (charge per length) in this frame, with correct overall sign.

- Use Gauss’s Law to compute the electrostatic force on the test charge.

- In THIS frame, what is the magnitude and direction of the force on the test charge, and what “causes” it?

C) For normal currents, \( \beta = v/c \) is about \( 10^{-13} \). (Drift velocities are small!)

Given this, show that the forces you computed in parts (B) and (C) are the same size. Hint: Taylor Series expand.