1. **Poynting Vector of a Solenoid.** Consider a very long solenoid of length $L$, radius $R$, and turns per length $n$. The current $I$ in the solenoid is linearly ramped from $I = 0$ to $I = I_0$ over a period $t_0$ as shown in the graph.

A) Integrate the magnetic field energy density to derive a formula for the total field energy stored in the solenoid at times $t > t_0$.

B) Solve for the electric field everywhere at times $0 < t < t_0$.

C) Solve for the Poynting Vector (direction and magnitude) at $r = R$ (just inside the walls of the solenoid) as a function of time $t$.

D) Show that the total field energy/time passing from the walls of the solenoid into its interior, when integrated from $t = 0$ to $t = t_0$, gives the same total energy as you computed in part (A).

2. **Electromagnetic mass of the electron.** According to Dr. Einstein, energy of any kind is equivalent to mass according to $E = mc^2$. So if you have an empty box and you fill it with energy $U$ of any kind, then the mass of the box will increase by $U/c^2$. Electric and magnetic fields contain energy, so they have an effective mass. It has been suggested that the mass of the electron is due to the energy of the field in the space surrounding the electron. (This turns out to be not quite true.)

A) Assume the electron consists of a uniformly charged spherical shell of radius $r_0$, with total charge $e$. (Assume a thin hollow shell, not a solid sphere.) Compute the total energy in the electric field of the electron by integrating the field energy density over all space. What is the total energy in the field if the electron is a point particle ($r_0 = 0$)?

B) If we assume that the mass $m$ of the electron is due entirely to the field energy, what is the formula for $r_0$? (Write $r_0$ in terms of $m$, $e$, and other constants) This value of $r_0$ is called the Compton radius. What is the
numerical value of $r_0$ for an electron? How big is this compared to the size of an atom? Of a nucleus?

C) A proton is a composite object (meaning it has internal parts) consisting of 3 quarks, plus gluons that hold the quarks together. The radius of a proton is about 0.9 fm (femto-meters). Assume that the proton is uniform sphere of charge (a solid sphere of charge, not a shell of charge). Compute expressions for the field energy both inside the proton sphere and outside the proton sphere. What is the ratio of the field energies $\frac{\text{field energy inside proton}}{\text{field energy outside proton}}$?

Now put numbers into your equations, and determine what fraction of the proton’s mass is due to its field energy. (In a fission atomic bomb, it is field energy which is released in the explosion.)

3. The Feynman Rotator. In volume 2 of the Feynman Lectures on Physics, the author poses this paradox: Two small spheres, each with mass $m$ and positive charge $+Q$, are attached to a light plastic horizontal ring of radius $R$ of negligible mass that can spin without friction around its vertical axis. On the axis is a long, stationary solenoid of length $L$, with $n$ turns per length, and radius $r$, initially carrying current $I$, as shown. When the current in the solenoid is turned off (by an automatic timer), there is an induced E-field at the location of the charges. This E-field pushes the charges, so the ring starts rotating. But wait! While the current was on, everything was stationary. After the current turns off, the ring is spinning. Doesn’t this violate Conservation of Angular Momentum? (Note: the angular momentum due to the moving conduction electrons in the current is completely negligible.)

A) What is the magnitude of the E-field at the location of the charges when the solenoid current is decreasing to zero. Write the answer in terms of $dI/dt$. 
B) If you integrate the force on the charged spheres over the time period that the current decreases to zero, you get the change in momentum of the spheres. How fast are they moving after the field is turned off?

C) What was the magnitude and direction of the total angular momentum in the field before the current was turned off?

4. Inductance per length of a coaxial coil. Consider a standard coaxial cable ("coax") consisting of a very long straight length wire of radius $a$ surrounded by a cylindrical conductor of radius $b$. In real coax, the space between the conductors is filled by plastic, but in this problem, assume the space is vacuum. A current $I(t)$ flows along the inner wire and a corresponding current $I(t)$ flows in the opposite direction uniformly on the outer cylinder.

(A) Find the self-inductance per length of the cable by considering the flux between the inner and outer conductor (in the dotted area in the diagram). Ignore the small amount of flux within the conductors themselves.

(B) Compute the magnetic energy per length in the coax, by integrating the magnetic field energy density. Compare with the energy per length you get by using $U_{\text{mag}} = (1/2) L I^2$ and your result from part (A). Do your results agree?

5. Standing wave.

A) Starting with the complex relation $\exp[i\alpha] \cdot \exp[i\beta] = \exp[i(\alpha + \beta)]$, complete the trig identities: $\cos(\alpha + \beta) = ?$ and $\sin(\alpha + \beta) = ?$

B) If you add two sinusoidal traveling waves, with the same frequency and wavelength, but traveling in opposite directions, you get a standing wave.
Use the results of part (A) to show that the sum

\[ A \cos(kx - \omega t) + A \cos(kx + \omega t) \]

is a standing wave. Make a sketch of this standing wave, showing its shape at two different times, and indicate on the sketch where the nodes and anti-nodes are.