Homework Set 1, Physics 3320, Spring 2012
Due Wednesday, January 25, 2012 (start of class)
In general, each lettered part of a problem will be worth 4 points.
In all problems, show your work and explain your thinking. Correct answers for which we cannot follow the work are worth no credit.

1. A positive point charge of \(+Q\) sits at the center of a hollow, neutral, spherical metal shell of inner radius, \(a\), and outer radius, \(b\).

   a) Determine the electric field \(E\) in all space. You may use whatever method you think best, but be sure to explain your reasoning clearly.

   Sketch the magnitude of the electric field as a function of distance from the center, commenting briefly on any interesting features of the sketch.

   b) Describe how charge is distributed on the metallic shell. Be specific: where does the charge reside and how is it distributed? Explicitly calculate any charge densities that are non-zero.

   c) Find the voltage \(V\) everywhere in space, using the usual assumption that \(V(\text{r} = \infty) = 0\). Sketch the potential as a function of distance from the center and comment briefly on any interesting features.

2. Consider a thin rod of length \(2a\), negligible diameter, and uniform charge per length \(\lambda\). The rod is centered on the origin, oriented as shown in the diagram.

   a) Compute the voltage at position \(y\) on the \(y\)-axis. As usual, set the zero of voltage at infinity.

   b) From the voltage, compute the electric field at position \(y\) on the \(y\)-axis. Simplify the result as much as possible.

   c) Write down expressions for the \(E\)-field in the limits \(y >> a\) and \(0 < y << 1\). (Note that the limit \(y >> a\) does not mean \(y = \infty\). At \(y = \infty\), the \(E\)-field is zero. We want an expression for how \(E\) approaches zero, as \(y\) goes to infinity.)

   d) Argue that the limits you wrote down in part (c) make sense, and that you could have figured out the limits directly without first computing the exact expression for \(E\).
3. Consider a circular ring of charge in the xy plane, centered on the origin, with radius a and uniform charge per length $\lambda$.

a) Derive an integral expression for the voltage $V(r)$ outside the ring in the xy plane, at a distance $r$ from the origin, where $r > a$. Don’t attempt to solve the integral – it’s a messy one – just write it in a form that’s ready to plug into Mathematica.

b) Now consider the voltage $V_{pt\text{Charge}}$ due to a point charge at the origin, that has the same total charge as the ring in part (a). Derive an expression for the ratio of the voltage due to the ring to that due to the point charge: $V(r)/V_{pt\text{Charge}}(r)$, for $r > a$. Use Mathematica to evaluate this ratio at $r = 1.1a$, $r = 2a$, and $r = 10a$.

4. Suppose an electric field in a region of space is given by

$$\vec{E} = \frac{B_0}{2\tau} (-y\hat{x} + x\hat{y}) \quad (1)$$

where $\tau$ is a constant with units of time and $B_0$ is some given constant.

a) What are the units of the constant $B_0$?

Sketch the electric field in the xy plane (by hand), and then check yourself using Mathematica (Hint: VectorPlot[...] is a built-in command!) Please include your MMA plot with your homework.

b) Calculate the closed line integral

$$\oint_L \vec{E} \cdot d\ell \quad (2)$$

where $L$ is a circle of radius R, parallel to the x-y plane, centered at $(0,0,z_0)$. Integrate counterclockwise as viewed from "above". Calculate this line integral directly; do not use Stokes’ Thm to compute the integral in this part.

c) Calculate the closed line integral equation (2) again, this time where $L$ is a rectangle (sides of length a and b) oriented parallel to the x-y plane. (Again, integrate counterclockwise as viewed from "above")

Clearly describe in words how the value of equation (2), here and in part (b), depends on $B_0/\tau$ and the location and geometry of $L$.

d) Calculate $\nabla \times \vec{E}$ and describe the resulting vector field in words. Show that the closed line integral values in 3(b) and 3(c) are equal to the corresponding surface integrals of $\nabla \times \vec{E}$. Relate this to Stokes’ theorem!

e) Determine the scalar potential (the voltage) that gives equation (1) or explain why no such potential exists.
Finally - describe how you could use such a static electric field to make a lot of money. Does the electric field in this problem violate any of Maxwell's equation for a static situation?

*Does this "mathematical exercise" strike you as a physically unrealistic problem? We'll come back to this field soon, there's more to it than meets the eye!*

5. B-fields due to currents in your house.
   a) Use Ampere's Law to compute the magnetic field around a long straight wire carrying a current \( I \). Show your steps clearly.

The wires in the walls of your house carry currents as high as 15 A (if higher than 15 amps, the circuit breaker will trip, stopping the current). Estimate the magnitude of the magnetic field at a distance of 10 cm from a long straight wire carrying a current of 10 A. Compare this field to that of the earth's magnetic field.

b) The wires in your house create negligible magnetic fields because the wires always come in pairs, carrying the same current in opposite directions. Consider two long, straight parallel wires, separated by a distance \( d \), and carrying current \( I \) in opposite directions. Compute the magnetic field at a distance \( r \), where \( r >> d \). (We want the 1st-order term in the Taylor Series expansion for the answer, not the exact answer.)

Use your answer to compute the size of the B-field at distance \( r = 10 \) cm, if \( d = 2 \) mm, \( I = 10 \) m.