1. In Griffiths (and in class), we derived equations for EM waves in conductors, assuming a “good conductor”. Interestingly, it turns out that the formulas we got can be pushed a good deal further than you might naively expect, into regimes where e.g. $\sigma$ is not so large (“poor conductors”). In this case, you will need to use Griffiths more careful results (9.126) for the real and imaginary parts of the k vector and work with those.

A) Based on the above comments, show that the skin depth for a “poor conductor” (i.e., $\sigma<<\varepsilon\omega$) is $d \approx ?/\sqrt{\varepsilon/\mu}$, independent of frequency or wavelength. Work out what the “??” is in this equation. (Also, check the units of your answer explicitly, please!) Then, show that the skin depth for a “good conductor” (i.e., $\sigma>>\varepsilon\omega$) is $d \approx \lambda/2\pi$, where $\lambda$ is the wavelength in the conductor.

B) For biological tissues (like skin), $\varepsilon$ and $\sigma$ depend on frequency, you can’t use their free space values. ($\mu$ on the other hand is close to its free space value) At microwave frequencies (say, about 2.5 GHz), their values are $\varepsilon=47\varepsilon_0$, and $\sigma=2.2 \ \Omega^{-1} \ \text{m}^{-1}$. Is this the “good conductor” or “poor conductor” case, or neither? Evaluate the skin depth for microwaves hitting your body. If such an EM wave (e.g. from a radar station) hits your body at this frequency, roughly what fraction of the incident power do you absorb? (Hint: think about “R”, and then get “T”)

C) We mentioned radio contact with submarines in class. For radio waves (say, 3 kHz) evaluate the skin depth in the sea, and comment on the feasibility/issues of such radio communication. (What is the wavelength of this same radio wave in free space, by the way?) (Hint: You can use given values from part C where needed, although as mentioned above, in reality they’ll be different at this very different frequency. Gold star if you can find more appropriate values, and give us the reference!)

2. Dispersion in hydrogen gas:

A. In Ch 4, Griffiths estimated static polarization of atoms. We’re now pushing this to Ch 9, with time dependent fields. First, let’s model hydrogen atoms as a point nucleus (charge $+e$ and mass $m_p$) surrounded by a sphere of uniform charge density of radius $r_H$ and total charge $-e$. In the absence of any external field, the equilibrium position of the nucleus is at the center of sphere. Find the force on the nucleus if it is displaced from the center by a distance $d$ and use this to find an “effective” spring constant $k$. Now, using this $k$ (but assuming it is the electron which does the oscillating while the proton stays put, because the proton is so much heavier), find the natural frequency of oscillation $\omega_0$ for the hydrogen atom in this model. Putting in the actual values for the variables (the radius will be approximate of course, what is appropriate for hydrogen?), where in the electromagnetic spectrum (see Table 9.1, Griffiths’ p. 377) does this frequency lie? (Does your answer seem reasonable, based on other physics you know?)

B. For EM waves with frequencies far from the region of anomalous dispersion, we can ignore damping and use the simplified formula (Eq. 9.173) to predict the coefficients of refraction and dispersion for hydrogen. Using #’s from part A, estimate these coefficients for hydrogen gas at 0°C and 1 atm pressure, and compare them with the measured values of $A=1.36 \times 10^{-4}$, $B=7.7 \times 10^{-15} \text{m}^2$. (Hint: what is N in Eq. 9.173? You will need to resurrect a little freshman chemistry!) Comment briefly on your result, how’d you do with this extremely crude model?
3. In section 9.4.3 of Griffiths we get a (slightly nasty looking?) formula for $k(\omega)$ (Eq. 9.170)
I claimed in class (and showed it in my lecture notes) that the “group velocity” of a traveling pulse is given by $d\omega/dk$. Using 9.170, and **assuming negligible damping** (i.e. $\gamma=0$) show that this group velocity is always less than the speed of light, even though the phase velocity, (i.e. $\omega/k$ might sometimes come out greater than $c$ for certain frequencies, but $d\omega/dk$ won’t)
(Hint: $d\omega/dk = 1/(dk/d\omega),$ this makes the algebra a bit easier)

4. At the start of Ch 10, Griffiths walks us through the reasoning behind Eq 10.2 and 10.3, which are the key formulas relating $E$ and $B$ to $V$ and $A$.
A. Substitute these relations into Maxwell’s equations and show that the general equations for the potentials $V(r,t)$ and $A(r,t)$ (without using any particular gauge condition) are

$$\nabla^2 V + \frac{\partial}{\partial t}(\nabla \cdot A) = -\frac{\rho}{\varepsilon_0}$$

$$\nabla^2 A - \mu_0\varepsilon_0 \frac{\partial^2 A}{\partial t^2} - \nabla \left( \nabla \cdot A + \mu_0\varepsilon_0 \frac{\partial V}{\partial t} \right) = -\mu_0 J$$

*Note that I DO this in my lecture notes for you, as does Griffiths. The idea here is for you to do it yourself, explain your work, convince yourself and the grader!*

The equations in a particular gauge can be obtained from these general equations by substituting in the gauge condition. Derive the differential equations for $V$ and $A$ in the Lorentz
gauge and in the Coulomb gauge by this method.

*(Note: In the Lorentz gauge $\nabla \cdot A + \mu_0\varepsilon_0 \frac{\partial V}{\partial t} = 0,$ whereas in the Coulomb gauge $\nabla \cdot A = 0.$ Again, this is done in the lecture notes and the text, we want you to go through it for yourself!)*

B. Consider a point charge at rest at the origin. You know $V(r, t) = q / (4 \pi \varepsilon_0 r)$, and $A(r, t)=0$. (Right? Convince yourself!) Explicitly check to see if we are in the Coulomb gauge, the Lorentz
gauge, or perhaps, both, or maybe neither!

Now, introduce a gauge transformation $A_{new}=A+\nabla f$, $V_{new}=V-\partial f/\partial t$, using the particular choice $f(r,t) = q\ t/(4 \pi \varepsilon_0 r)$. Find the new $V$ and $A$. Briefly discuss your results: are the potentials "static" or time dependent? Does this represent the same physical situation, or is something
different here? Are we in the Coulomb gauge, or the Lorentz gauge now? (Do we have to be in one of those to solve physics problems?) What has changed, what is the same?

5. Say the potentials throughout space and time are $V(r, t) = 0$ and $A(r, t) = A_0 \cos(k(z-ct)) \hat{x}$, where $A_0$ and $k$ are given constants, and $c$ is the speed of light.
- Find the $E$ and $B$ fields everywhere in space and time, and comment on the physics here, what have we got going on?
- Are we in the Coulomb gauge, the Lorentz gauge, both, or neither?
- Is it possible, in principle, to find a different gauge for this problem (following the general
procedure used in question 4B above) in which $A(r,t)=0$? (If your answer is yes, you don’t have to necessarily find the particular gauge transformation here – I’m just asking if it’s possible **in principle**, and how you know?)