1. Last week, you found an AJP article, downloaded the pdf, and uploaded it to our D2L dropbox. This week, read the paper (carefully! Take your time, make a serious effort). If you really despise the paper you picked, just find another one (and upload the new one to our dropbox). Your assignment is twofold – summarize and review the article. These are two separate things - keep it all under one page, this is not me ant to be a big paper! You do not have to turn it in this week (but you are encouraged to). Final due date will be next week. Please type this up. Print a copy to turn in with your homework, but also cut and paste what you wrote and put it up on the D2L “Discussion” board (I’ll set a forum up for this specific purpose – look for it). If you use equations, you might upload the pdf, but it’s easier for us all to read each other’s work if it’s just text on the discussion board.

Start your report with your name (!) and the full formal reference to the paper, including title. Keep your summary brief – half a page is fine. What was it about, can you rephrase the abstract and conclusions in your own words? For the “review” part, I’d like to know how hard the paper was to follow, whether you found it instructive, or interesting? Basically, should we (course staff, and/or your fellow students) read it? Help us decide!

2. Consider a 3D electromagnetic plane wave in vacuum, described in usual complex form by \( \mathbf{E}(r,t) = \tilde{E}_0 e^{i(k \cdot r - \omega t)} \), in which \( \tilde{E}_0 \) is a constant vector equal to \( E_0 \mathbf{\hat{x}} \), with \( E_0 = E_0 e^{i\pi/2} \).

Assume \( k \) is the wave vector \( k \mathbf{\hat{y}} \), \( \omega \) is the angular frequency. As usual, the real field is \( \mathbf{E} = \text{Re}[\mathbf{E}] \)

A. Describe in words what this mathematical expression represents physically. You may use sketches, but if you do, they should be well described.
- In which direction the wave is moving?
- What is the speed, wavelength, and period of the wave? (What does that phase of \( \pi/2 \) in \( \tilde{E}_0 \) do?)
- Sketch the real field \( \mathbf{E}(x=0,y,z=0,t=0) \) (a 2D plot with y as the horizontal axis) and \( \mathbf{E}(x=0,y=0,z=0,t) \) (a 2D plot with t as the horizontal axis).
  Clearly indicate the direction of the field and the scale of both your axes.
- How is the field at \( x=a \), i.e. \( \mathbf{E}(x=a,y,z=0,t=0) \), different from the case at \( x=0 \)?

B. Why is this called a plane wave (where is (are) the plane(s))? Sketch or represent this in 3D.
- Describe how the direction of the electric field changes in time. If \( \mathbf{E} \) always points in the same direction, the wave is said to be linearly polarized. Is this wave linearly polarized?

C. Find the associated magnetic field \( \mathbf{B}(r,t) \) for this plane electric wave.
- Sketch the magnetic fields, \( \mathbf{B}(x=0,y,z=0,t=0) \) and \( \mathbf{B}(x=0,y=0,z=0,t) \) indicating field direction. (As above, be clear about your axes) A 3D sketch of B would be helpful here too, what’s the simplest way to draw it?
- Describe in words how \( \mathbf{B} \) compares/contrasts with \( \mathbf{E} \).

D. Calculate the energy density \( u_{EM} \), Poynting vector \( \mathbf{S} \), and momentum density for these fields. Interpret the answers physically (Make sense of them, including units, signs, directions, etc!)
E. Calculate the angular momentum density \( \ell_{EM} \) (see Griffiths’ p. 358) about the origin (0,0,0). If you integrate this density over a cube of centered at the origin at one instant in time, would the angular momentum in that cube be zero or non-zero? Briefly, discuss.

F. Suppose now that we add two plane waves, \( \mathbf{E}_1 \) and \( \mathbf{E}_2 \), (superposition still works!) to find the total electric field. Let \( \mathbf{E}_1(r,t) = \mathbf{E}_1 \cos(\mathbf{k} \cdot \mathbf{r} - \omega t + \delta_1) \) and \( \mathbf{E}_2(r,t) = \mathbf{E}_2 \cos(\mathbf{k} \cdot \mathbf{r} - \omega t + \delta_2) \) so in this simple case the waves propagate in the same direction. Let’s say the amplitudes are \( \mathbf{E}_1 = E_1 \mathbf{z} \) and \( \mathbf{E}_2 = E_2 \mathbf{z} \). Use complex notation (taking the real part only at the very end) to find \( \mathbf{E}_T(r,t) = \mathbf{E}_1(r,t) + \mathbf{E}_2(r,t) \) in the form \( \mathbf{E}_T(r,t) = \mathbf{E}_T \cos(\mathbf{k} \cdot \mathbf{r} - \omega t + \delta_T) \), giving expressions for the total amplitude and phase shift in terms of those from \( \mathbf{E}_1(r,t) \) and \( \mathbf{E}_2(r,t) \).

- Explicitly check your answer in the special case \( \delta_1 = \delta_2 \).

G. Let’s examine one more situation, this time

\[
\mathbf{E}(r,t) = E_1 \cos(\mathbf{k} \cdot \mathbf{r} - \omega t) + E_2 \cos(\mathbf{k} \cdot \mathbf{r} - \omega t + \pi / 2),
\]

in which \( \mathbf{E}_1 \) is a constant vector equal to \( E_0 \mathbf{\hat{z}} \), \( \mathbf{E}_2 \) is a constant vector equal to \( E_0 \mathbf{\hat{x}} \), \( \mathbf{k} \) is the wave vector \( \mathbf{k} \mathbf{\hat{y}} \) (as before,) and \( \omega \) is the angular frequency.

Find the total \( \mathbf{E}(r,t) \).

- Describe how the direction and magnitude of \( \mathbf{E} \) changes in time. Is this wave linearly polarized?
- Consider all points in space where \( \mathbf{k} \cdot \mathbf{r} = 0 \) (in this case, how would you describe such a set of points in words?), and describe in words or pictures what your \( \mathbf{E} \) field looks like. Does this help you describe the polarization state? (If you look down the axis with the wave approaching you, is the \( \mathbf{E} \) vector circling CW? CCW? Or, something else?)

3. **Radiation pressure**

A. On earth, the time-averaged flux of electromagnetic energy (\( \langle \mathcal{S} \rangle \)) from the sun is 0.14 watt/cm². Consider steady sunlight hitting 1 m² of earth: picture an imaginary box (containing streaming sunshine) striking this area, with a “box height” of 1 light-second. There is a certain amount of momentum stored in that box, and in one second, ALL that momentum will strike the 1 m² area. Assuming the EM wave is absorbed (not reflected), what force does that work out to? How does the radiation pressure from this light compare to atmospheric air pressure, Comment! If the earth reflected the sunlight, how would that affect the radiation pressure (qualitatively)?

B. What is the net force on the earth from this radiation pressure, assuming the earth absorbs all the EM energy? Compare this force to the gravitational force of the sun on the earth, and comment.

C. If I made a 100 kg spacecraft with a 10,000 m² large black sail to absorb the sunlight and propel me away from the sun, what would its acceleration be? (This is called a solar sail, and there are serious proposals to build such a craft!) What are some advantages and disadvantages over conventional spacecraft?

**Extra Credit: CHOOSE just one (4 points max):**

Either A) **Play with the stress tensor!** Use Griffiths 8.19 and write out the full 3x3 stress tensor for the traveling wave of problem 2A. It’s surprisingly simple! Interpret the result physically. In particular, use it to check your answer to problem 3A.

Or, B) **Do what I think is the coolest problem in Griffiths:** Griffiths 8.12 p.362. (Oh, it’s magical – read his footnote!) *(But, the integral involved in this problem is challenging.)*