Homework Set 3, Physics 3320, Fall 2011
Due Wednesday, Sept. 14, 2011 (start of class)

1) A long wire carries a steady current \( I_1 \). Nearby to the long wire is a square loop of wire (side length \( a \)) with resistance \( R \). You apply a force on the loop away from or toward the wire so that the loop maintains a constant velocity \( v \).

(Ignore any possible "self-inductance" effects in this problem, i.e. assume the magnetic field produced by current in the loop is small compared to the field produced by the wire.)

a) At the moment shown (when the left edge is at a distance "\( x \)" away from the wire), find:
   i) the magnetic flux through the loop.
   ii) the EMF around the loop.
   iii) the magnitude of the current circulating around the loop.
      (If \( v \) is to the right, as shown, which way does this current flow?)
   iv) the power dissipated in the loop.

b) Determine the magnetic force on the loop and the power you need to supply to keep the loop moving at a constant velocity, as a function of position \( x \). Show that this power is always positive, independent of whether the applied force is toward or away from the wire. Compare your result here to the result from part (a-iv): does the result make sense?

c) Qualitatively, what in the above parts would change if the velocity \( v \) were parallel to the current \( I_1 \) instead of perpendicular?

2) A square loop with side \( a \) is mounted on a horizontal axis and rotates with a steady frequency \( f \) (rotations/sec.) A uniform magnetic field \( B \) points left to right between the two pole faces. The figure shows the configuration at time \( t=0 \).

a) Find the EMF around this loop as a function of time for this (AC) generator.

b) If the output is connected to a load resistance \( R \), calculate the instantaneous and average power dissipated in the resistor. Compare your results to the mechanical power needed to turn the loop.

c) If the rotation rate is 60 Hz, the loop has area 0.01m\(^2\), and the B-field is 0.01T, about how many turns of wire would you need to produce a standard 120 V (RMS) output?

A common variant is to hold the loop fixed (the stator) with an electromagnet coil (the rotor) rotated around the stator. This configuration is commonly called an alternator.
3) A conducting disk with radius $a$, height $h \ll a$, and conductivity $\sigma$ is immersed in a time varying but spatially uniform magnetic field parallel to its axis: $B = B_0 \sin(\omega t)$ (in the $+z$ direction).

a) Ignoring the effects of any induced magnetic fields, find the induced electric field $E(r,t)$ and current density $J(r,t)$ in the disk. Sketch the current distribution. Compare $E(r,t)$ to the electric field in presented in problem (3) on Homework 1 when $\omega t \ll 1$.

b) If the power dissipated in a resistor is $P = I \cdot V$, show that the power dissipated per unit volume is $\bar{J} \cdot \bar{E}$. Calculate the total power dissipated in the disk at time $t$, and the average power dissipated per cycle of the field.

Extra credit (4 pts) If the disk in question was roughly the size of the solid base of a typical frying pan, and the frequency was 10 kHz, what approximate scale for $B_0$ would you need to significantly heat up the pan (say, 1000 watts of power). Does this seem feasible? Now, use the current distribution in part (a) above to determine the induced magnetic field at the center of the pan. For what range of parameters is the induced magnetic field small compared to the applied field?

Note: For real induction stoves, it turns out that the induced magnetic field is NOT small compared to the applied field, and so this calculation of $E$ and $B$ (which ignored the induced field) is not quantitatively correct. Induction stoves work just fine! We need to learn some more physics to improve this calculation.

(One more problem on next page)
4) Consider the electrical circuit shown below. There is an infinitely long solenoid (shaded, shown end-on in the figure). The current around that solenoid circulates CW and increasing with time, producing a flux (into the page) that is linear in time: \( \Phi(t) = \alpha t \).

Surrounding that solenoid are some "ideal" wires and two resistors, as shown, with ideal voltmeters (that means they have very high internal resistance, and thus negligible current flows through them) connected as shown. (The probes of voltmeter \( V_1 \) are directly across resistor \( R_1 \), and those of \( V_2 \) are across \( R_2 \).)

a) What do the two voltmeters read? (Including signs)
How can it be that the two readings are different? (Look carefully - shouldn't points \( a \) and \( b \) have a unique voltage drop between them? If not... why not?)

b) Consider two of those circuits connected as shown in the figure below. The solenoids are identical, both with the same cross sectional area (\( = 0.1 \text{ m}^2 \)), and the magnitude of the magnetic field inside both is equal and increasing at a constant rate (\( = 1 \text{ T/s} \)). (However, note the direction of the current is opposite in the two solenoids, so the B-fields are in fact increasing in opposite directions.) The resistors have the values shown.

Determine the current passing through each resistor. (Magnitude and direction)