Appendices from

Has Science Found God?

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Note that the published version contains several minor errors that do not affect any of the conclusions in the book. These have hopefully all been corrected. I have also tried to make the arguments clearer added a reference that was inadvertently omitted. So this can be regarded as a "second edition."

Appendix A: The Planck Length

By international agreement, the distance or length, L, between two points is defined as the time, t, it takes for light to travel between the points in a vacuum, multiplied by a constant, c:

(A1) $L = ct$

While c is called the speed of light in a vacuum, it is an arbitrary number that simply sets the units of distance. For example, if t is measured in seconds and you want L in meters, then $c = 3\text{x}10^8$ by definition. Physicists and astronomers often work in units where $c = 1$, for example, when t is in years and L in light-years.

In order to measure t we need a clock with an uncertainty, Δt, no larger than t. The time-energy uncertainty principle says that the product of Δt and the uncertainty in a measurement of energy in that time interval, ΔE, can be no less than $\hbar/2$, where $\hbar = h/2\pi$ and $h = 6.63\text{x}10^{-34}$ Joule-sec is Planck's constant.

(A2) $\Delta E\, t \geq \Delta E\, \Delta t \geq \hbar/2$

Thus,

(A3) $\Delta E \geq \hbar/2t \geq \hbar c/(2L)$

This energy equals the rest energy of a body of mass m,

(A4) $\Delta E = mc^2$

Let L be the radius of a sphere. Within a spherical region of space of radius L we cannot determine, by any measurement, that it contains a mass less than

(A5) $m = \hbar/(2cL)$

Now, let us consider the special case where the gravitational potential energy of a spherical body of mass m and radius R equals half its rest energy,

(A6) $mc^2/2 = Gm^2/R$

so that,

(A7) $R = 2Gm/c^2$

This is called the *Schwarzschild radius*. According to general relativity, any body of mass m with radius less than R is a black hole. Suppose that $L = R$. Let us call that special case L_{PL}. Then, from (A5) and (A7),

$$\text{(A8)} \quad L_{PL} = (\hbar G/c^3)^{1/2}$$

which is called the *Planck length*, $L_{PL} = 1.6 \times 10^{-35}$ meter. We can see that it represents the smallest length that can be operationally defined, that is, defined in terms of measurements that can be made with clocks and other instruments. If we tried to measure a smaller distance, the time interval would be smaller, the uncertainty in rest energy larger, the uncertainty in mass larger, and the region of space would be experimentally indistinguishable from a black hole. Since nothing inside a black hole can climb outside its gravitational field, we cannot see inside and thus cannot make smaller measurement of distance.

Similarly, we can make no smaller measurement of time than the *Planck time,*

$$\text{(A9)} \quad t_{PL} = L_{PL}/c = (\hbar G/c^5)^{1/2}$$

which has the value $t_{PL} = 5.4 \times 10^{-44}$ seconds. Also of some interest are the *Planck mass,*

$$\text{(A10)} \quad m_{PL} = \hbar/(c\, L_{PL}) = (\hbar c/G)^{1/2}$$

which has a value of $2.2\text{x}10^{-8}$ kilogram, and the Planck energy, (A11) $E_{PL} = m_{PL}c^2 = (\hbar\, c^5/G)^{1/2}$

which has a value of $2.0\text{x}10^9$ Joules or $1.2\text{x}10^{28}$ electron-volts. These represent the uncertainty in rest mass and rest energy within the space of a Planck sphere or within a time interval equal to the Planck time.

Appendix B: The Lifetimes of Stars

The minimum lifetime of the class of stars larger than the Sun that can end their lives as supernovae can be calculated from

(B1) $t_s = (\alpha^2/\alpha_G)\,(m_p/m_e)^2\,\hbar\,(m_p\,c^2)^{-1}$

where $\alpha = e^2/4\pi\varepsilon_o h/\,c$ is the dimensionless strength of the electromagnetic force, e is the unit electric charge, ε_o is the permittivity of free space (Standard International Units are being used), and

(B2) $\alpha_G = G\,m_p^2\,(\hbar\,c)^{-1}$

is the dimensionless strength of the gravitational force, m_p is the mass of the proton and m_e is the mass of the electron.[1] In our universe, $\alpha = 1/137$, $m_p = 1.67 \times 10^{-27}$ kilogram, $m_e = 9.11 \times 10^{-31}$ kilogram, $\alpha_G = 5.9 \times 10^{-39}$, and $t_s = 680$ million years. Actually, most stars, like our Sun, have much greater lifetimes; larger stars evolve more rapidly. The shorter-lived stars we are considering here generate the heavy elements from which planets and life later can evolve.

As we saw in Appendix A, c is an arbitrary number that simply determines the units we use to measure physical quantities. The same can be said for $\hbar$ and G.[2] Indeed, each can be set equal to unity without changing any observational results. This does not mean that the strength of the gravitational force is arbitrary, since α_G depends on m_p, as seen in (B2). Thus, just three parameters determine the value of t_s: α, m_p, and m_e. We can choose any two to be whatever value we wish and then find the range of values for the third parameter that will give t_s equal to or greater than some value, say the value in our universe.

For example, suppose we are interested in a particular t_s and pick arbitrary values of m_p and m_e. Then, with the help of (B2), we can rewrite (B1)

(B3) $\alpha = (Gm_p\, c\, t_s)^{1/2}\, m_e/\hbar$

to find the value of α needed. Note that no fine tuning is required to produce a universe with long-lifetime stars. We can always find a value of α for any value of t_s, equal to or larger than its value in our universe, regardless of the masses of the proton and electron. While the values of the three parameters may not yield some form of life for another reason, that reason is not insufficient time for stars to cook up the elements needed for life.

Appendix C: The Entropy of the Expanding Universe

Entropy is defined by

(C1) $S = k \ln\{\text{number of states}\}$

where k is Boltzmann's constant. For N particles of the same type,

(C2) $S = k \ln\{(\text{number of states per particle})^N\}$

$= kN \ln \{\text{a not-too-big-number}\}$

$\approx kN$

$\approx N$

in units where $k = 1$.

Just after the initiation of the big bang, we can treat the universe as a sphere of radius R containing an expanding relativistic gas of total energy E. Its maximum entropy was equal to the maximum number of particles,

(C3) $S_U^{\max} = E/\varepsilon_{\min} = E\ \lambda_{max}/(hc) = 2\pi RE/hc = RE$

where $\varepsilon_{\min} = hc/\ \lambda_{\max}$ is the minimum energy of the relativistic particles, and $\lambda_{\max} = 2\pi R$ is their corresponding maximum wavelength. I have assumed units where $\hbar = h/2\pi = c = 1$, which greatly simplifies the calculations. Note that, in these units, the Planck length given in (A8) is simply $L_{PL} = \sqrt{G}$. This equals the Planck time given in (A9), while the Planck mass and energy, from (A10) and (A11) are both equal to $1/\sqrt{G}$.

Equation (C3) is not to be taken as the entropy of the universe for all times, which is likely to be dominated by black holes. It applies for the early universe.

The maximum entropy of a black hole of radius R and mass M, or rest energy Mc^2, is given similarly to (C3) by[3]

(C4) $$S_{BH}^{\max} = Mc^2/\varepsilon_{\min} = E\,\lambda_{\max}/hc = 2\pi R\, Mc^2/hc = RM$$

From (A7), $M = R/G$ and from (A8), $G = L_{PL}^2$. Thus,

(C5) $$S_{BH}^{\max} = R^2/L_{PL}^2$$

Now let us consider the situation when the universe was a Planck sphere, $R = L_{PL.}$ At that time, the universe is a black hole. Its maximum energy in that case would be the Planck energy (A11), $E_{PL} = 1/\sqrt{G} = L_{PL}$ and so, from (C3),

(C6) $$S_U^{\max} = R/\mathrm{L_{PL}}$$

which equals (C5) when $R = L_{PL}$. So, we see that, as the universe expands, $S_U^{\max}$ increases linearly with R while $S_{BH}^{\max}$ increases quadratically. Thus, although the universe starts out with maximum entropy, its maximum entropy becomes less than its maximum allowable entropy, that of a black hole of the same size, leaving increasing room for order to form.

Notes

[1] E. E. Salpeter, "Accretion of Interstellar Matter by Massive Objects, " *Astrophysical Journal* 140 (1964): 796-800.

[2] For a further discussion of this point, see my *Timeless Reality: Symmetry, Simplicity, and Multiple Universes,* (Amherst, N.Y.: Prometheus Books, 2000), chap. 15.

[3] Frautschi, Steven, "Entropy in an Expanding Universe," *Science* 217, no. 4560 (1982):593-99.