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SCRUMPTIOUS FUNCTIONS

Graham ODDIE
University of Colorado at Boulder

Summary

The taste of this particular chunk of fresh pineapple, the one which I am just now eating, is scrumptious. That taste is something the chunk has in common with other such chunks, like the one I had a few seconds ago and the one I will have in a few seconds time. The taste of this pineapple chunk is thus a feature, a property, which this and various other chunks of pineapple share. Now, intuitively at least, no purely mathematical entity, like a function, is scrumptious. Hence a property, like the taste of this chunk of pineapple, cannot be any such function.

This kind of argument, if sound, would cut a wide swathe through the field of metaphysics. All those theories which attempt to reduce ontologically suspicious types of entities (properties, propositions, persons, medium-sized dry goods) to mathematical entities (like classes or functions) would fall to the blade. This could well help to reduce the impact of contemporary philosophy on the world's dwindling forests. I save contemporary metaphysics by explaining where the argument goes wrong.

1. *What a property might be*

It was once proposed that we "identify" a property with its own extension – that class of individuals which have the property in fact. Everyone (save perhaps extreme nominalists) countenances the extension of a property. To postulate something over and above the extension is thus ontologically less abstemious than just making do with the extension. The proposal thus has the advantages of parsimony, perspicuity and precision. As is well known, we can "explicate" classes as a certain sort of function. A class of individuals can be "identified with" the function which takes each individual in the class to True and each individual not in the class to False. Thus the extensional proposal is tantamount to the claim that properties of individuals are "reducible to" functions from individuals to truth values.

I place the key terms ("identify", "explicate", "reducible to") in scare-quotes because what is at issue here is the very propriety of explications, identifications and reductions of this sort. Not all reductions are intended to be strict identifications (see Oddie 2000) but here I will focus mainly on those reductions which are intended to be identifications or explications.

Unfortunately for this extensional proposal, there can be distinct properties with the very same actual extension. We can distinguish actually coextensive properties, like *chordate* and *renate*, by considering non-actual possibilities. There are possible circumstances in which some renates do not have a pump mechanism for circulating their blood. They circulate their blood by, say, convection. Since *chordate* and *renate* have different extensions in different possible circumstances, it is natural to wonder whether we cannot make do with the associated functions from possible circumstances to extensions.

A determinate property P is "associated with" or "induces" a unique function P^* from possible circumstances to extensions: the extension of the property in that circumstance. Again I use scare-quotes because I don't want to prejudge the issue of whether P^* is not the *very same thing* as P. I also want to leave it open here how best to construe a possible circumstance. (One candidate for possible circumstances are complete possible worlds. But if a world is a possible *history*, and a dynamic theory of time is an option, then a complete circumstance must specify not just a history but a time as well. The extension of a property may be time-dependent as well as world-dependent. On this view, to specify a complete circumstance one needs to specify a couple consisting of a world and a time.)

Each property P thus uniquely determines a function P^*, which takes each possible circumstance to the class *of things which have the property P* in that circumstance. Since it is undeniable that P determines P^*, the question arises whether P^* also determines P, and if so, whether we need to make a distinction between P and P^*, or whether it suffices to deal with P^* alone. A tradition which can be traced back to Carnap's *Meaning and Necessity* holds that we do not need to countenance two distinct things, the property P in addition to the mapping P^*. This tradition holds that we can simply "identify" the property with the mapping. This functional analysis of properties shares the advantages of the extensionalist proposal (parsimony, perspicuity and precision). Further, it avoids the coextension problem. And in addition, from the point of view of logical analysis P^* looks as though it can do by itself the logical work for us that P^* together with a possibly distinct P can do for us.

2. *The taste of pineapple*

Here, however, is the apparently devastating objection. Consider the taste of pineapple, **T**. This scrumptious taste is something a normal human can experience by the simple procedure of placing one chunk of pineapple in her mouth. That taste is something which one chunk shares with other chunks. They all have the same taste. So the taste is a property of those sundry chunks, as well as anything else with that taste. Consequently the functional theory of properties appears committed to the strange idea that, since **T** is both scrumptious and directly experiencable as such by normal tasters, **T*** – a certain *mapping* from circumstances to mappings from particulars to truth values – is also both scrumptious and directly experiencable as such by normal human beings. Surely that is absurd! One does not consume pineapple chunks to experience a complex mathematical entity – a mapping from possible circumstances to other mappings from particulars to truth values!! Philosophically innocent pineapple lovers would be incredulous to find out that *that* is what they up to. Could a function be *scrumptious* in just the way that the taste of pineapple is?

George Bealer makes the following objection to a closely related functional account – the identification of properties with functions from individuals to propositions:

> How implausible that familiar sensible properties are functions – the color of this ink, the aroma of coffee, the shape of your hand, the special painfulness of a burn or itchiness of a mosquito bite. No function is a color, a smell, a shape, or a feeling. (Bealer, p. 1)

Bealer doesn't here argue for the implausibility of the claim that sensible properties are functions – he takes it to be more or less self-evident – but the pineapple objection goes some way to fleshing out the bald intuition. The idea is presumably that functions cannot be colors, smells, shapes and feelings – indeed, they are the wrong *kinds* of things – because much of what is true of sensible properties is not just false, but plain *silly*, when ascribed to functions. I have focussed on scrumptiousness as something that we can happily ascribe to a taste, but very unhappily to a function. Analogous features of the other sensible properties mentioned could easily be cited. The itchiness may be ameliorated by spreading camomile lotion on the mosquito bite, but no function is ameliorated by spreading camomile lotion on any bite. The shape of your hand is delightful to me, but no functions are delightful to me, and so on.

Peter Gärdenfors also begins his criticism of the functional account of properties with a similar kind of argument:

> The standard definition of a property within intensional semantics, I contend, leads to a number of grave problems. First of all, it is highly counterintuitive since properties become very abstract things. (Gärdenfors 2000, p. 62.)

However, despite leading the charge against the traditional account with this argument, he appears, a little further on, to discount it.

> The fact that the definition is counterintuitive, however, is not a decisive argument. It may be argued that the abstract character of properties is merely a cosmetic feature of intensional semantics – as long as the semantics produces the right results, the technical form of the semantic concepts is not so important. (p. 63.)

I suspect that Gärdenfors is less than enthusiastic about his leading objection partly because his own theory (briefly: that a property is a convex region of a single dimension of a multi-dimensional conceptual space) might be subject to rather similar criticisms. Indeed, *any* theory at all would probably be so subject. The problem with the rejoinder, as stated, is that it is not at all clear what "the right results" are. Bealer could justifiably point out that the entailment of the existence of scrumptious, noisome and piquant functions is, all by itself, an egregiously wrong result. Nevertheless, I think Gärdenfors's tentative reply to the pineapple objection is along the right lines, and it is the purpose of this paper to elaborate it in some detail and defend it.

3. *Tasting properties, tasting things*

The pineapple objection, if sound, appears to impugn more than the functional account of properties. For example, it appears to impugn any robustly Platonic account of properties – any account according to which properties are necessary beings, abstract objects, not located in space and time. How can you taste a Platonic form – an eternal, necessary, unchanging, abstract entity outside of space and time – just by placing one particular pineapple chunk in your mouth? Or, for that matter, how can you taste a tiny convex region of a single dimension of a multidimensional conceptual space?

Before proliferating the argument we should answer a very basic question: when you put a chunk of pineapple in your mouth what exactly is it that you taste? The natural thing to say is that you taste *that chunk of pineapple*. Tasting is a relation between an individual taster and a tasty *particular*, one the individual happens to be interacting with in a special way, typically through licking or sucking. A properly equipped individual in a normal condition (his tongue is working and appropriately hooked up to a working brain) will, when he places a particular chunk of pineapple on his tongue, have an experience with a certain quality to it, the quality characteristic of pineapple-tasting experiences. In virtue of having that experience our pineapple-taster tastes the chunk of pineapple. It is the *particular chunk of pineapple* that he is tasting – not the property **T** – and hence, according to the functional theory, not the function **T***.

You lick a particular to find out *its* taste. You don't lick properties to find out their tastes. Indeed you cannot lick a property, and the functional theory makes it abundantly clear why not. There is no way to lick a function from circumstances to functions from particulars to truth values. You can lick a particular that *has* an interesting property, like **T**, and when you do, if you are a normal veridical taster, you will experience its taste. But you can no more taste properties than you can lick or suck functions.

If this is right then the argument might be surreptitiously trading on an absurdity of which the functional theory is not guilty. The taste of pineapple may well be a property, but you do not taste *the taste of pineapple*. You taste *a pineapple chunk*.

Unfortunately a closely related objection can be rescued from this rejoinder. We do talk freely about *experiencing* the taste of pineapple. You can *experience* **T** even if it is not right to say that you *taste* **T**. Interestingly, you can experience **T** even if there is no particular pineapple chunk on your tongue. You can hallucinate a gustatory interaction with a scrumptious-tasting pineapple chunk, or you can have a vivid dream of such, or have it called to mind by reading Proust. So what it is that you experience in such a case cannot be a particular, since there is no particular present. What you experience must be something other than a particular, and one candidate is the property **T**, a suitable relatum even if it is not instantiated in the vicinity of your tongue, even if it is not instantiated at all.

Since you can experience the property **T** simply by licking a particular, according to the functional theory you can experience the function **T*** by the same mundane procedure. Isn't that a tad counterintuitive? *Almost* as counterintuitive as your tasting a scrumptious function?

4. *Reduction*

This pineapple objection shares features with familiar objections to other reductive analyses.

Consider the Russellian reduction of numbers to classes of classes. Much of arithmetic can be deduced and explained by the reduction. But the reduction also entails that the singleton of this pineapple chunk is a member of the number one. Doesn't *that* sound a tad absurd? For any number whatsoever, this pineapple chunk is a member of something which is a member of that number. Are the children in grade school aware that when they learn the sequence of numbers they are learning about things which contain things which contain this pineapple chunk? And if they don't know that, then aren't they missing what the Russellian reduction deems to be a fundamental truth about the number one? Aren't these grade-school intuitions blatantly incompatible with one of the greatest contributions to twentieth century philosophical logic by perhaps its most preeminent exponent?

Or consider the physicist's explication of *acceleration* as the second derivative of distance with respect to time. A kid is putting his father's new Mustang through its paces. He knows when the vehicle is accelerating and when it isn't. He has some pretty good intuitions about acceleration. As he steps on the gas the Mustang kid experiences the thrill of rapid acceleration. Put this to the Mustang kid and he grunts in affirmation. Predictably, the kid slept through his 7.30 am calculus classes, and so he has no idea that as he steps on the gas he experiences the thrill of a high value for the second derivative of distance with respect to time. Put that to the Mustang kid and he gives you the blank stare. Does the Mustang kid's ignorance of calculus refute the physicist's explication of acceleration? (Things are even worse for the identification of *observables* with Hermitian operators in Hilbert space!)

In each of these cases, if the untutored intuitions alone were sufficient to kill the explications, that would put an end to the enterprise of philosophical and scientific explication. But it can hardly be a requirement on an explication that it leave absolutely *everything* the same, for then it would seem pointless to indulge in it in the first place. We should just stick to repeating the original platitudes and intuitions, neither supplementing them nor tampering with them in any way. At most we should devote a little time to denouncing explicators and reducers for abusing our prephilosophical intuitions.

This seems to have been the later Wittgenstein's official and dreary view of the enterprise. I cannot resist repeating Russell's retort in his review of Urmson's *Philosophical Analysis*: "The earlier Wittgenstein , whom I knew intimately, was a man addicted to passionately intense thinking, profoundly aware of difficult problems of which I, like him, felt the true importance, and possessed (or at least so I thought) of true philosophical genius. The later Wittgenstein, on the contrary, seems to have grown tired of serious thinking and to have invented a doctrine which would make such an activity unnecessary." (Russell 1959, pp 216-7.)

This raises an interesting methodological question. What exactly is the *point* of a reductive analysis, of "identifying" something with what, at the outset at least, appears (intuitively) to be something *else*.

Either something is deeply wrong with the enterprise of reduction/explication/identification, or else more needs to be said about the enterprise, about the point of the enterprise and about the relationship between a reduction and the intuitions which motivate and ground it.

Let *A* be a partly understood but still problematic domain, a domain we wish to understand better. (So, for example, *A* might be the natural numbers.) What I will call *A-claims* are propositions over *A*, whether true or false, known or unknown. (For example, the claim that the sum of 2 and 2 is 4, or that multiplication is associative, or that there is a largest prime.) *A-intuitions* are *A*-claims, concerning *A*-objects, *A*-properties and *A*-relations, which are known *a priori*. *A-counterintuitions* are *A*-claims which are known *a priori* to be false.

Let *B* be some reductive base, the domain of entities to which objects in *A* are to be reduced together with properties and relations of those. *B*-claims are typical propositions over *B*. For example, in the case where *A* is the domain of natural numbers, *B* might be the set-theoretic hierarchy over the domain of particulars. *B*-claims are typical propositions over *B* – for example, that the empty set of particulars is a subset of every set of particulars. Any adequate explication of the entities in *A* will, of course, have to deliver the *A*-intuitions (the "core" intuitions) and exclude the *A*-counterintuitions.

A reduction maps the *A*-domain into the *B*-domain, and *A*-claims into *B*-claims, in systematic ways. The point is to clarify, unify and explain the *A*-phenomena by appealing to well-established and well-understood features of the *B*-domain. The reduction will succeed just to the extent that it does that. From the naive standpoint entities in the domain of *A* (like the numbers themselves, but also addition and multiplication) present themselves as metaphysical "atoms". Further, the intuitions we have about the way they fit together present themselves to us as brute, albeit necessary, facts. That these facts are both brute and necessary is often puzzling. The point of a reduction is to show that these apparently brute but necessary facts are really the consequence of intelligible necessary relations holding between "non-atomic" entities.

Of course, this is the goal of the reductionist, and maybe it is more honored in the aspiration than the actual achievement. Many attempted reductions – like the reduction of causation to regularity – are well known to suffer what are no doubt fatal flaws. And others – like the reduction of material objects to four-dimensional worms – are still hotly contested. Still, the reductive enterprise can chalk up a few successes, and in the next section I outline a striking example using the functional theory of intensions in general.

5. *An example: the reduction of intensions to functions*

The functional theory of properties is part of a more general reduction of intensions to functions. This begins with the domain of propositions, properties, relations, and other intensions (call this the *I*-domain) and proposes an identification of these entities with functions from possible circumstances to appropriate extensions (the *F*-domain). A proposition, for example, is identified with a function from possible circumstances to truth values; a property of individuals with a function from possible circumstances to classes of individuals (themselves functions from individuals to truth values); and an *n*-place relation with a function from circumstances to an *n*-argument function from individuals to truth values.

A proposition is something which may or may not be believed. One might be inclined to think that of any given proposition it should be at least *logically* possible for one to believe it. But this principle is no sooner stated than it elicits counterexamples. Consider the unique belief (if there is one) to which I am currently attending – my *occurrent belief*. It might be the proposition that the world is oblate, or that I am not very comfortable, or that seven plus five is twelve. It doesn't matter what it is, but whichever it is, it is the sort thing which bears a truth value, truth or falsehood. (In certain circumstances it may fail to bear a truthvalue. The possibility of truthvalueless propositions complicates but doesn't undermine the ensuing argument.) Now consider the proposition *B* – *that my occurrent belief is false*. In some circumstances *B* is true (those in which what I occurrently believe in that circumstance is true) and otherwise it is false. Now here is something strange. *B cannot be my occurrent belief.* I *cannot* – *logically* cannot – take the occurrent-belief attitude to *B*, regardless of whether or not I am rational. For, by a standard liar-type argument, that *B* is my occurrent belief entails a contradiction. (Note that truthvaluelessness doesn't block the derivation of a contradiction. Suppose *B* is truthvalueless. Since it is false that *B* is false, it is false that *my occurrent belief is false*. So *B* is false: contradiction.)

This is a puzzling fact about propositions. Why cannot I believe whatever I like? It's tempting to think that this has something to do with the connection between belief and truth, that believing involves commitment to truth, and that believing the proposition *B* in particular violates that constraint. But the phenomenon is much more widespread. Any propositional attitude will do. Consider the relation of *contemplating* a proposition. Contemplating a proposition, unlike believing it, does not involve a commitment to its truth. So surely you can contemplate any proposition you like. Consider the following perfectly proposition *C*: *that the proposition you are currently contemplating is false.* The truth condition for *C* is plain enough. It is true if the truth value of the proposition you are contemplating is false,and false otherwise. On the face of it, not only *can* you contemplate *C*, it appears as though you are, in fact, currently contemplating *C*. For I have just brought *C* to your attention and, if (dear Reader) you are following my words with understanding, you must be successfully contemplating the proposition to which I am rather persistently adverting. But, of course, you *cannot* be – again by a standard liar-type argument. For what would the actual current truth value of *C* be? If *C* is true then it is false, and vice-versa. (And if *C* is truthvalueless then it is false that *C* is false, so it is false that the proposition you are currently contemplating is false. That is to say, *C* is false, not truthvalueless.)

Take any propositional-attitude based property *A* – being contemplated by you, entertained by you, favored by you, whatever – or, indeed, *any* property that propositions might have or lack. Then there is a proposition P^A: *that the A is false*. By a liar-type argument we can show that P^A cannot itself be the unique proposition with the feature *A*.

Thus there are a multitude of brute facts about propositions which are also necessary, and hence somewhat puzzling. What light can the reduction of propositions to functions throw on this? As it turns out, rather a lot. Let ω be the number of possible circumstances. Given the reduction, propositions are functions from possible circumstances to the two truth values, so the number of propositions is 2^ω (or 3^ω if we countenance partial functions to accommodate truth-value gaps) and it is a consequence of Cantor's theorem that 2^ω is vastly greater than ω, the number of possible circumstances. Since in any given circumstance at most one proposition can be *the* proposition you are contemplating in that circumstance (or *the A*) there are at most ω propositions that it is logically possible for you to contemplate. Thus there are $2^\omega – \omega$ (or $3^\omega – \omega$) propositions which it is logically *impossible* for you to contemplate. That is to say, *most* of them. So we have shown that, as a matter of logical necessity, most propositions cannot be entertained by you (or me, or the Mustang kid, or even God for that matter). The reduction of propositions to functions delivers this result immediately from well-known features of the general theory of functions.

But why especially should the particular proposition *C* be one of those one cannot contemplate? Indeed, it *appears* that that is precisely the proposition that you are currently contemplating. (If not, dear Reader, then *listen up*!) The general theorem – most propositions are not contemplatable by you – although it makes it very likely that any given proposition, like *C*, cannot be contemplated by you, does not tell us why *C in particular* is not contemplatable by you. That brute but necessary fact remains unexplained, and hence puzzling.

Consider an application of the classic diagonalization procedure which Cantor used to prove the theorem on which all of this rests. For each circumstance *w*, let P^w be the unique proposition which in *w* you are contemplating (and, if there is none, let P^w be some well known contemplatable proposition – say the tautology). It is logically possible for you to contemplate a proposition *Q* if there is some circumstance *w* in which $Q = P^w$. Call those the propositions *contemplatable by you*. To show, by a classic Cantorian diagonalization, that there is at least one proposition which is not contemplatable by you consider the following diagonal function *C**: one which takes circumstance *w* to true if P^w takes *w* to false, and takes *w* to false otherwise (including the case where P^w is truthvalueless). We have specified a function, *C**, from circumstances to truth values which differs in its value from every contemplatable *Q*. (A contemplatable *Q* is P^w, for some *w*, and *C** differs from P^w in the value it assigns to *w*.) So C^* isn't one of the *Q*'s contemplatable by you. Now the diagonal function C^*, so defined, is the very function from circumstances to truth values induced by the proposition *C* itself – they take the same values at the same arguments. Consequently, according to the functional reduction, C^* just *is* the proposition *C*. The reduction of propositions to functions thus explains in a very direct way the otherwise brute, necessary, and hence puzzling fact that *C* cannot be contemplated by you. It is just one more instance of well-understood Cantorian facts. (For a discussion of related limitations imposed by logical space, see Tichý 1988, ch. 12.)

6. *Reduction and the clash with intuition*

A reduction of the *A*-domain to the *B*-domain will, of course involve not only *A*-claims and *B*-claims, but also mixed *A+B*-claims. The reduction captures *A*-intuitions by showing them to be a species of *B*-truths, and excludes *A*-counterintuitions by showing them to be *B*-falsehoods. But the reduction of necessity involves and entails a collection of mixed *A+B*-claims, claims about which either we will have few pretheoretical intuitions, or which sound so odd that they clash with our prephilosophical dispositions. The reduction must deliver the *A*-intuitions, but there is no reason why it should be required to agree with prephilosophical dispositions to accept or reject *A+B*-claims. Thus it is that we should not expect every consequence of a reductive analysis to be something we (or the Mustang kid) would instantly embrace. The Mustang kid is happy with the claim that accelerating is exciting, and maybe even with the claim that a higher value for the second derivative of distance with respect to time will tend to produce a much higher value for the first derivative as well. But he will be surprised when told that the higher the value for the second derivative, the greater the excitement.

The reduction of the *I*-domain to the *F*-domain, of intensions to functions, also entails mixed *I+F*-claims. For example, in conjunction with platitudes about pineapple chunks, it entails: *one can directly experience the scrumptiousness of a function from possible circumstances to functions from individuals to truth values.* Prior to reduction this claim is probably not one we would have been inclined to embrace. It would have been puzzling to us, much as the claim mixing derivatives with platitudes about acceleration may be puzzling, even unintelligible, to the mathematically challenged Mustang kid. But if the kid is taken gently through a course in calculus, and is shown how the explication of acceleration captures his pretheoretical *acceleration*-intuitions, then at the end of the course he will no longer be thrown by the mixed claims. They might still sound a little odd to him, but he will recognise that their oddness is partly a result of mixing and partly a result of their newness.

The reductive analysis itself is, of course, a mixed claim, and hence will itself sound odd, even startling, at the outset. That properties are functions sounds odd when one first hears it. But it begins to sound more plausible as puzzling truths about properties are rung out of truisms about functions.

7. *Experiencing functions*

These reflections on reduction should be sufficient to dispel any residual discomfort with the very idea of experiencing a function. But if not then it may prove worthwhile to confront this discomfort directly. What exactly is the *problem* with experiencing a function?

You can certainly *think about* functions. You can think about, for example, the addition function – a particular function from pairs of numbers to numbers. You can also think about **T***, the object with which the functional theory identifies the taste of pineapple **T**. Thinking about something may be a way of experiencing that thing, in which case there is no problem *per se* with experiencing functions. At least, not without spreading the problem too widely for most people's tastes (save, perhaps, those of the extreme nominalist). However, experiencing something through thinking about it is not entirely like experiencing it *via* sense. (Note that the passage from Bealer emphasizes the counterintuitiveness of identifying *sensible properties* with functions.) Thinking is not necessarily sensory, but tasting, seeing, hearing and so on are. To sense something is, presumably, to experience it in a particular way – one which involves sensual components peculiar to a certain sense modality. The real problem with experiencing **T***, if there is one, must be that it is the experiencing of an entity by means of sense. It involves a sensory component.

Consider the Mustang kid again. Suppose the Mustang is a technologically advanced vehicle, with computer gadgets installed precisely for the purpose of illuminating the mathematics of acceleration for otherwise mathematically challenged, auto-obsessed youngsters. It does this by displaying computer graphs of various quantities against time elapsed: distance travelled, velocity, and acceleration. So the Mustang kid gets to actually *see* the ways these functions relate to each other. In other words, he sees pictorial representations of functions which he is otherwise sensing – for example, through kinesthetic sensations. Isn't the Mustang kid thinking about the function, visually experiencing it *via* its pictorial representation on the screen, and kinesthetically experiencing it *via* sensations of pressure and displacement? This one thing, which he experiences in three different ways, is nothing other than a function from times to numbers.

8. *"Something's left out"*

Often a reduction is met with the complaint that "*something* is left out", something (usually "something I know not what") is not fully captured. This is the converse of the complaint that the reduction gives us too much (the *A+B* claims). Here the complaint is that it gives us too little. This kind of complaint, rather thin in itself, can be fleshed out a little.

Suppose the Mustang kid releases the clutch at 12.00 sharp and the Mustang accelerates uniformly throughout the next ten seconds, at the rate of five meters per second per second, reaching a velocity of 50 meters per second (or 180 kilometers per hour) at the end of the ten second interval. Acceleration here is a constant mapping which takes us from numbers in the interval between 0 and 10 to the number 5. But is this what the Mustang kid *experiences* as his head is thrust back? Isn't something missing? Isn't the account too "thin" to be credible.

A number of things are indeed missing. For a start, this isn't *all* that the Mustang kid experiences. He experiences his body pressing against the seat, he experiences a rush of adrenaline, and so on. But that doesn't imply that a certain constant function is not among the panoply of items he experiences during this interval. Of course, pressure on his body throughout the interval is also a mapping from times to numbers, as is a rush of adrenaline. But there will be as many distinct experienced functions as the kid has distinct experiences.

Still, there are other missing elements. For example, the Mustang kid could have accelerated at a different rate, say at four meters per second per second. There are a range of possible values for *the acceleration of the Mustang*. The acceleration of the Mustang is something which associates different circumstances with different functions of this sort. One reason the bare mapping from times to numbers looks too thin to be the acceleration is the same reason that the actual extension of a property is too thin to be a property. We need to invoke the intensional entity – a mapping from possible circumstances to such functions. If the Mustang kid is in a drag race then his opponent in the Chevy is experiencing the acceleration of the Chevy. That is something different from the acceleration of the Mustang even if the two are neck and neck throughout the interval. They have the same extension (the constant function) but they need not have. They are distinct mappings from possible circumstances to functions.

Clearly, the extensionalist does not have enough entities in the reductive base to do justice to all the distinctions which can be made legitimately at the level being reduced. By adding the class of possible circumstances, and functions over it, the intensionalist overcomes this difficulty, expanding the reductive base exponentially. However, this leads us to another objection – that there are *too many* entities in the reductive base, rather than too few.

9. *"Too many candidates"*

The number which is the actual value of the acceleration function at various moments depends on a convention: the unit of measurement. We can measure acceleration in meters per second squared, or in kilometers per hour squared. If the former, then the function the Mustang kid experiences takes 5 as its value throughout the interval. If the latter, then the function the Mustang kid experiences takes 18 as its value throughout the interval. Does he experience both functions? If so, then given the infinitely many different possible conventions, every constant function will be experienced by the Mustang kid. But that seems absurd. The acceleration of the Mustang is just one thing, whereas the associated functions are legion. One cannot be identical to many. Since no particular convention is privileged, the acceleration of the Mustang cannot be identical to any of them. At best we can think of the different functions as so many different *representations* of the acceleration.

There is, of course, a way of evading this objection, singling out one entity without privileging any convention. We can simply add an argument place for conventions, making the extension of acceleration dependent not only on circumstances but also on convention. That dissolves the many-one problem, and allows us to view the convention-dependent functions as representatives (relative to a convention) of the convention-independent entity which is acceleration itself.

But there seem to be other apparently arbitrary elements in the selection of a function which are not so easily eliminated. For example one could, in the Carnapian tradition, take a property to be a function from circumstances to extensions (themselves functions from individuals to truth values). Or one could, in the Russellian tradition, take a property to be a propositional function – a function from individuals to propositions. Now if one takes propositions in turn to be functions from circumstances to truth values, then a proposition takes individuals to functions from circumstances to truth values.

Call these respectively the C-proposal and R-proposal for propositions:

C-proposal: circumstances → (individuals → truth values)
R-proposal: individuals → (circumstances → truth values)

Is there any way of deciding which of these two proposals is preferable? If not, don't we again have too many equally acceptable distinct candidates for what is clearly just one thing? (This is close to Benacerraf's well known 1965 objection to the related project of reducing numbers.)

There are two possibilities here. One is that further data (linguistic, logical, metaphysical) will settle which of these proposals is the more acceptable. If so, the purported problem dissolves. The other is that the issue will remain unsettled even when all possible data have been gathered. Underdetermination of theory by data is, of course, always a worry, here as elsewhere. But how much of a worry is it here? Not much, I suspect, because it is very difficult to tell in advance what might turn out to be a datum. Thus it is unlikely that we will be able to prove that two distinct metaphysical theories are "data-equivalent" for all possible future data.

Take the above proposals as an example. The C-proposal can be embedded in a theory which gives a systematic account of the relation between intensions and extensions. An intension is something which, at each possible circumstance, yields an extension of the appropriate sort. Included in the purview of this account are the kinds of intension known as individual concepts, like that of *the King of France* and *the chunk of pineapple in Graham's mouth*, but also particularizing concepts of other kinds of entity. Consider: *the number of pineapple chunks in Graham's mouth*, *the temperature outside* (magnitudes, or number concepts); *George's favorite sensible property*, *the least desirable body shape* (property concepts); *the most likely explanation*, *the proposition you are currently contemplating* (propositional concepts). Each of these kinds of intension are located naturally within an extended C-proposal.

Extended C-proposal:

Properties:	circumstances → (individuals → truth values)
Dyadic relations:	circumstances → (pairs of individuals → truth values)
N-adic relations:	circumstances → (n-tuples of individuals → truth values)
Propositions:	circumstances → truth values

Individual concepts:
(e.g. *the King of France* and *the chunk of pineapple in Graham's mouth*)

circumstances → individuals

Number concepts (magnitudes):
(like *the number of pineapple chunks in Graham's mouth*, *the temperature outside*)

circumstances → numbers

Property concepts (property roles):
(like *George's favorite property*, *the least desirable body shape*)

circumstances → properties
= circumstances → (circumstances → (individuals → truth values)))

Propositional concepts (propositional roles):
(like *the most likely explanation*, *the proposition you are currently contemplating*)

circumstances → propositions
= circumstances → (circumstances → truth values)

Of course, some of the functions with which we identify individual concepts will have to be *partial*. In some circumstances the concept *the King of France* applies to nothing. The function thus returns no value at those arguments. It is a gappy function. This will in turn create gaps elsewhere (for example, in the property role *the favorite taste of the King of France* and in the proposition *the King of France favors the taste of pineapple above all other tastes*.) But these are perfectly natural developments within the extended C-proposal.

If we try to draw up a corresponding table for an extended Russellian view we strike a problem. Individual concepts and the other particularizing intensions do not find a natural place within the schema. Thus it is perhaps unsurprising that one of Russell's major contributions to twentieth century philosophy was his attempt, through his theory of descriptions, to analyze away apparent reference to all such entities. If we can show that the theory of descriptions, while undoubtedly successful over a range of phenomena, does not deliver all the core pretheoretic intuitions, that would be evidence for the C-proposal over the R-proposal. And indeed there are such cases – all those intensional contexts in which a clausal substitute is not forthcoming. (*George is experiencing his favorite property*, *Joe loves God*.) The program for finding such clausal substitutes is degenerating (Forbes 2000).

10. *A modest conclusion*

Naturally, these successes of the functional theory of intensions do not settle the issue in its favor. There are other objections which must be fended off. For example, intensions may turn out to be more fine-grained than this account allows – while *P* certainly singles out the function *P** the converse may not hold (Tichý 1988). Or families of properties may possess a similarity structure which is lost or obscured under the functional account (Gärdenfors 2000). The functional theory will either have to accommodate these objections or somehow explain them away.

What has emerged is something more modest and more general – an instance of a familiar point about the relation between theory and data. A theory, whether scientific or philosophical, is rarely refuted by conflict with a single recalcitrant pretheoretical datum, or even a whole class of such. The apparent motionlessness of the earth doesn't refute heliocentrism – the appearances notwithstanding, the earth is spinning rapidly. In the case of philosophical reduction, the force of a counterintuition depends on its origin. If the reduction is in conflict with a core intuition concerning the original domain, that can indeed be a problem. But mixed claims, those that span the two domains, often sound distinctly odd, at least at the outset. As we become more familiar with the elegance and explanatory power of the reduction, we cheerfully embrace these mixed claims in the teeth of prephilosophical dispositions to the contrary. The beauty of philosophy is precisely that it *never* leaves things quite the same. Some functions really are quite scrumptious.

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