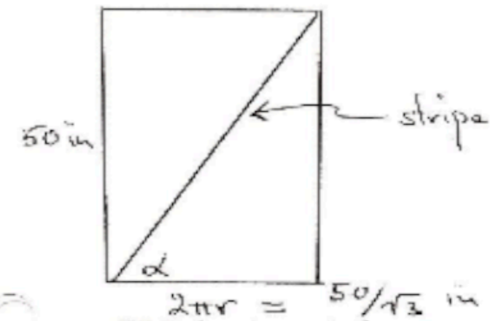
WORKSHEET **MATH 1300** **February 17, 2011**

Goal: To review some information concerning trigonometric functions and to begin using the basic trigonometric functions in so-called "applied problems".

1. The red stripe on a barber pole makes one complete revolution around the pole. If the pole is 50 inches long and with the amazingly, precise radius of $\dfrac{25}{\pi\sqrt{3}}$ inches, what angle does the stripe make with base of the pole, and how long is the stripe? (Assume the stripe is a thin line.)

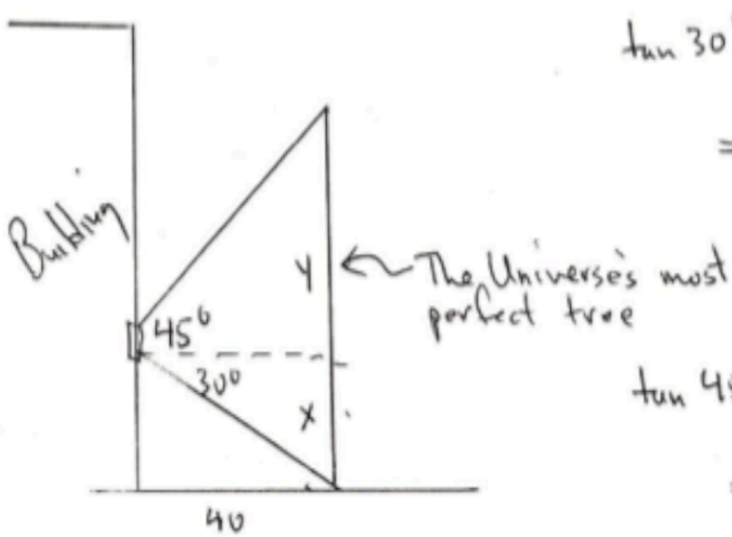
Imagine that the barber pole is a hollow cylinder that we slice from top to bottom and roll out flat. This will produce a rectangle (not drawn to scale):



We see from this drawing that

$$\tan\alpha = \frac{50}{\frac{50}{\sqrt{3}}} = \sqrt{3}. \quad \text{So } \alpha = 60^\circ.$$

2. While leaning out of your apartment's window, you notice a) that is is a warm and sunny day, b) that the line of sight to the top of a nearby tree makes an angle of 45 degrees above the horizontal, and c) that the line of sight to the base of the tree makes an an angle of 30 degrees below the horizontal. Taking advantage of a), you go outside and discover that it is 40 feet from the building to the base of the tree. How tall is the tree?



$$\tan 30^\circ = \frac{x}{40} \ \& \ \tan 30^\circ = \frac{\sqrt{3}}{3}$$

$$\Rightarrow \frac{x}{40} = \frac{\sqrt{3}}{3} \Rightarrow x = 40\frac{\sqrt{3}}{3}$$

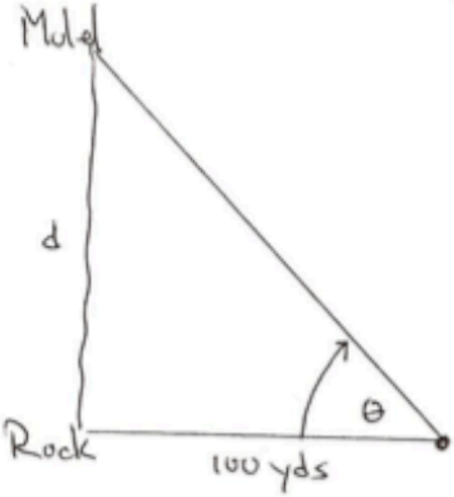
$$\tan 45^\circ = \frac{y}{40} \ \& \ \tan 45^\circ = 1$$

$$\Rightarrow \frac{y}{40} = 1 \Rightarrow y = 40$$

So the height of the tree is $x+y$

$$= \frac{40}{3}\sqrt{3} + 40 \text{ ft}$$

3. A lighthouse stands 100 yards offshore; on the shore at the spot closest to the lighthouse sits the notorious mermaid rock. Due north of mermaid rock is the exclusive That's-No-Mermaid-It's-A-Whale Motel. The lighthouse light rotates twice a minute. If the beam of light from the lighthouse takes 5 seconds to travel along the shore from mermaid rock to the motel, how far is the motel from the rock?

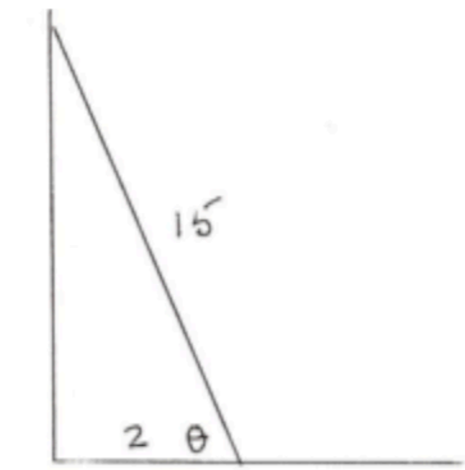


The light rotates $720°$/minute, so in 5 seconds it rotates $\frac{720}{12} = 60°$. Therefore $\theta = 60°$.

As $\tan 60° = \sqrt{3} = \frac{d}{100}$ we have

$$d = 100\sqrt{3} \text{ yds.}$$

4. A 15-foot long ladder is leaning against a wall with its base 2 feet from the wall. What can you say about the angle the ladder makes with the floor? (If you cannot calculate the angle at least describe it using mathematical terms.)



(Not to scale)

θ is the angle having

$$\cos\theta = \frac{2}{15}$$

In other words (symbols)

$$\theta = \cos^{-1}\left(\frac{2}{15}\right)$$

or

$$\theta = \arccos\left(\frac{2}{15}\right)$$

5. The bottom of the ladder in problem 4 starts to slide away from the wall at the constant rate of 1 foot per second.

a. When will the ladder make a 60° angle with the ground?

$\Theta = 60°$ when $\cos\Theta = \frac{1}{2}$.

So $\frac{\text{distance to wall}}{15} = \frac{1}{2} \Rightarrow$ distance to wall $= 7\frac{1}{2}$ ft

Thus take $5\frac{1}{2}$ seconds.

b. When will the ladder make a 45° angle with the ground?

$\Theta = 45°$ when $\cos\Theta = \frac{\sqrt{2}}{2}$

So $\frac{\text{distance to wall}}{15} = \frac{\sqrt{2}}{2} \Rightarrow$ distance to wall $= 7.5\sqrt{2}$ ft

Thus take $7.5\sqrt{2} - 2$ seconds (or about 8.6 seconds)