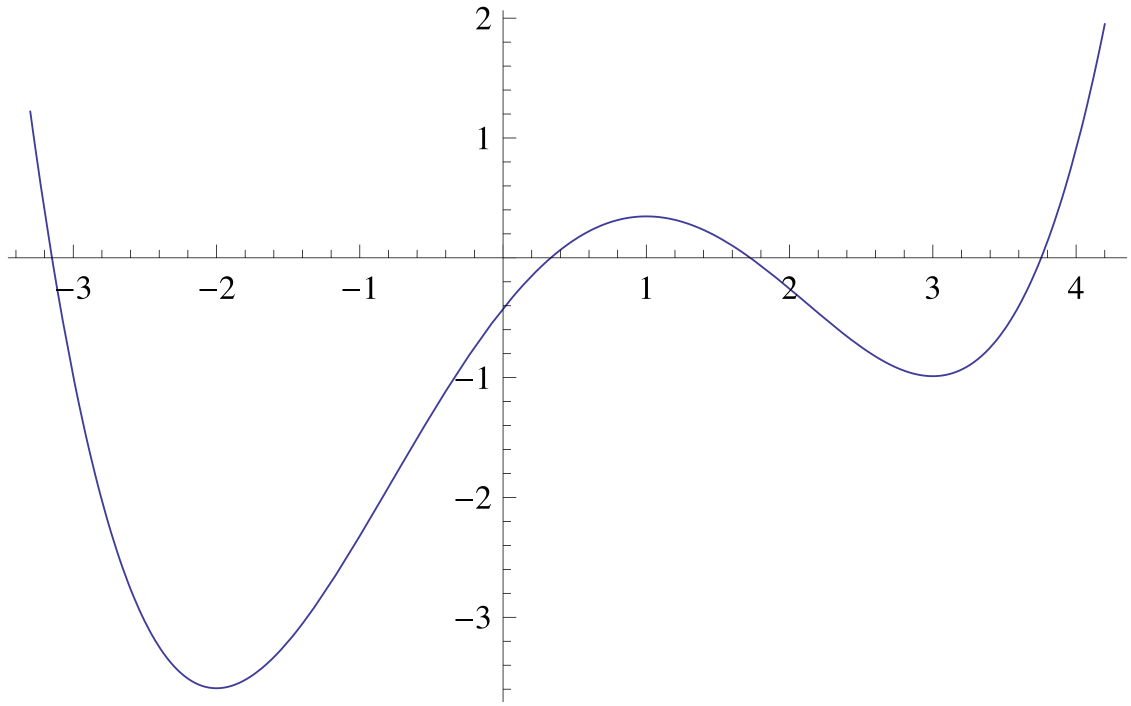
Note: This review worksheet does not cover *all* material that you are expected to know.

1. Consider the following graph of the polynomial $f(x)$.




2. Is the degree of $f$ even or odd? What is the smallest that this degree can be? Please explain.

**Solution:** The degree is even. Polynomials of even degree grow in the same direction (that is, to either $+\infty$ or $-\infty$) at both ends of the $x$-axis, while polynomials of odd degree grow in opposite directions.

According to our text (top of page 40), an $n$th degree polynomial turns around at most $n-1$ times, but there may be fewer turns. The above polynomial turns around three times, so its degree must be AT LEAST four.

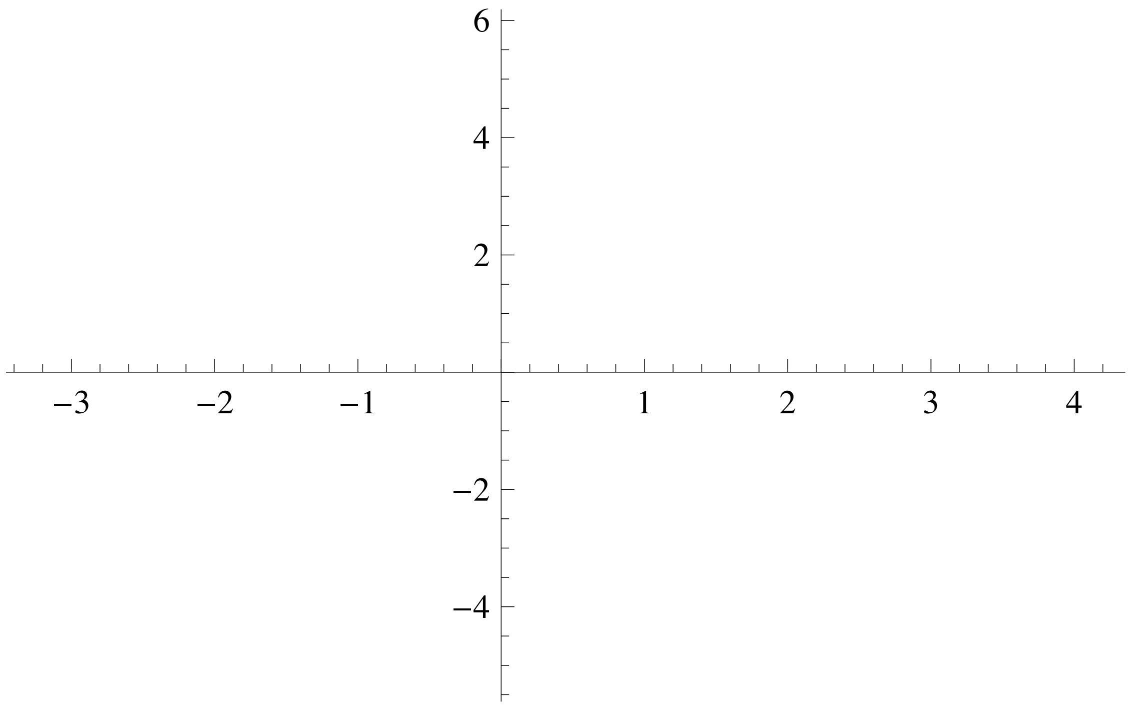
3. Using the above graph, estimate the slope of $f$ at $x = -3, -2, -1, 0, 1, 2, 3,$ and $4$. Use your estimates to fill in the following table.

| $x$ | $-3$ | $-2$ | $-1$ | $0$ | $1$ | $2$ | $3$ | $4$ |
|---|---|---|---|---|---|---|---|---|
| slope of $f$ at $x$ | | | | | | | | |

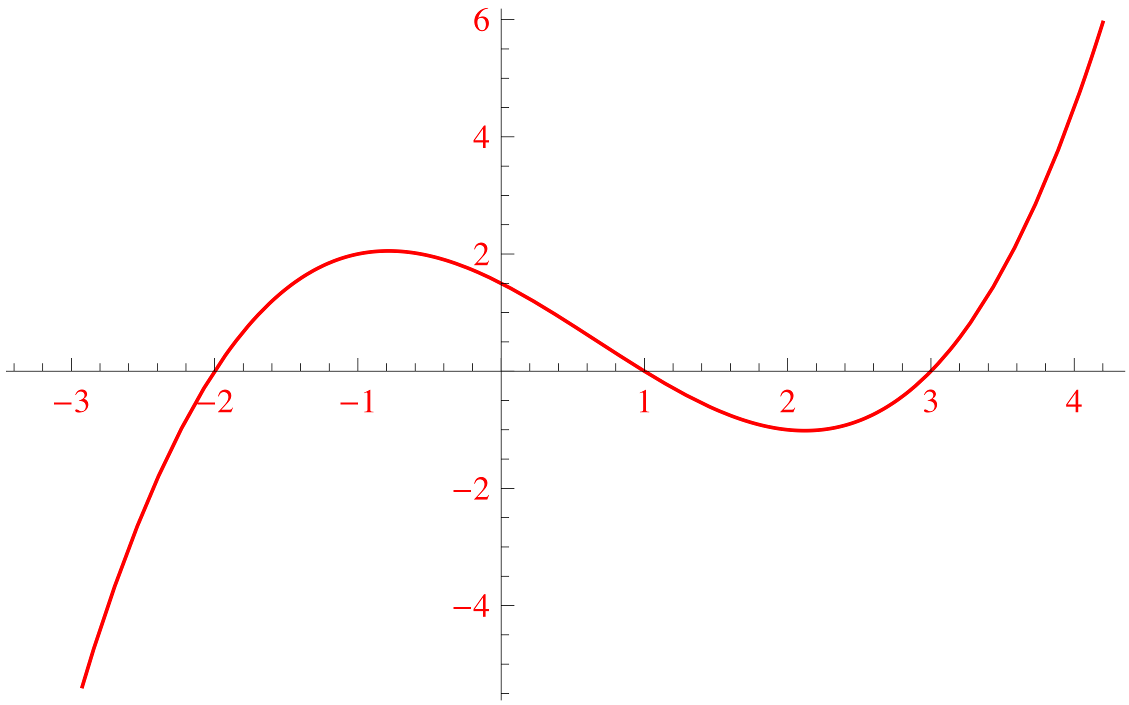
**Solution:**

| $x$ | $-3$ | $-2$ | $-1$ | $0$ | $1$ | $2$ | $3$ | $4$ |
|---|---|---|---|---|---|---|---|---|
| slope of $f$ at $x$ | $-6$ | $0$ | $2$ | $1.5$ | $0$ | $-1$ | $0$ | $4.5$ |

4. Using the table from part (b) above, draw a graph with $x$ on the horizontal axis and the corresponding slope of $f$ on the vertical axis. Connect the dots with a smooth curve.




**Solution:**



5. Note that the graph in part (c) above looks like a polynomial. Make a conjecture about the degree of this polynomial.

   **Solution:** Certainly the degree is odd, because the graph goes off in opposite vertical directions for large values of $|x|$.

   The degree of this polynomial must therefore be AT LEAST THREE (if the degree were one, we'd just have a straight line).

6. (a) Find all critical points of $f(t) = at^2e^{-bt}$, assuming $a$ and $b$ are nonzero constants.
   (b) Find the values of the parameters $a$ and $b$ so that $f$ has a critical point at the point $(5, 12)$.

   (c) Identify each critical point as a local maximum or minimum of the function.

7. Find the points of intersection for each of the following pairs of functions:

   (a) $y = 2x^2 + 2$ and $y = -3x^2 + 22$ **Solution:** $2x^2 + 2 = -3x^2 + 22$, so $5x^2 = 20$. Therefore $x = 2$ and $x = -2$

   (b) $y = \frac{x-1}{x+1}$ and $y = \frac{1}{x}$ **Solution:** we cross multiply the equation $\frac{x-1}{x+1} = \frac{1}{x}$ to obtain: $x + 1 = x^2 - x$., so $x^2 - 2x - 1 = 0$. By the quadratic formula we obtain two solutions $x = 1 \pm \sqrt{2}$.

   (c) $y = \sqrt{x}$ and $y = x^2$**Solution:** We start with $\sqrt{x} = x^2$ and square both sides: $x = x^4$. Therefore we obtain: $0 = x^4 - x = x(x^3 - 1) = x(x - 1)(x^2 + x + 1)$. The only real solutions to this equation are $x = 0$ and $x = 1$ (DO YOU KNOW WHY?)

8. Find $\dfrac{dy}{dx}$ if

   (a) $y = \frac{x^2 - x^4}{x^3}$

   (b) $y = \int_1^{x^2} \frac{1}{t+1} dt$

   (c) $y = \frac{ax+b}{cx+d}$

9. Evaluate the following integrals.

   (a) $\int x\cos(ax^2+b)\,dx$ **Solution:** $u = ax^2 + b$, so $du = 2ax\,dx$, so $x\,dx = du/2a$, so

$$\int x\cos(ax^2+b)\,dx = \frac{1}{2a}\int \cos(u)\,du = \frac{\sin(u)}{2a} + C = \frac{\sin(ax^2+b)}{2a} + C.$$

   (b) $\displaystyle\int_0^1 t^2 e^{(t^3-1)}\,dt$ **Solution:** $u = t^3 - 1$, so $du = 3t^2\,dt$. So $t^2\,dt = du/3$. Also, when $t = 0$, $u = -1$, and when $t = 1$, $u = 0$. So

$$\int_0^1 t^2 e^{(t^3-1)}\,dt = \frac{1}{3}\int_{-1}^0 e^u\,du = \left.\frac{e^u}{3}\right|_{-1}^0 = \frac{1-e^{-1}}{3}.$$

(c) $\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \frac{\cos x}{\sin^3 x}\, dx$

**Solution:** $u = \sin x$, so $du = \cos x\, dx$. Also, when $x = \pi/6$, $u = 1/2$, and when $x = \pi/2$, $u = 1$. So

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \frac{\cos x}{\sin^3 x}\, dx = \int_{1/2}^{1} u^{-3}\, du = \frac{u^{-2}}{-2}\Bigg|_{1/2}^{1} = \frac{1-4}{-2} = \frac{3}{2}.$$