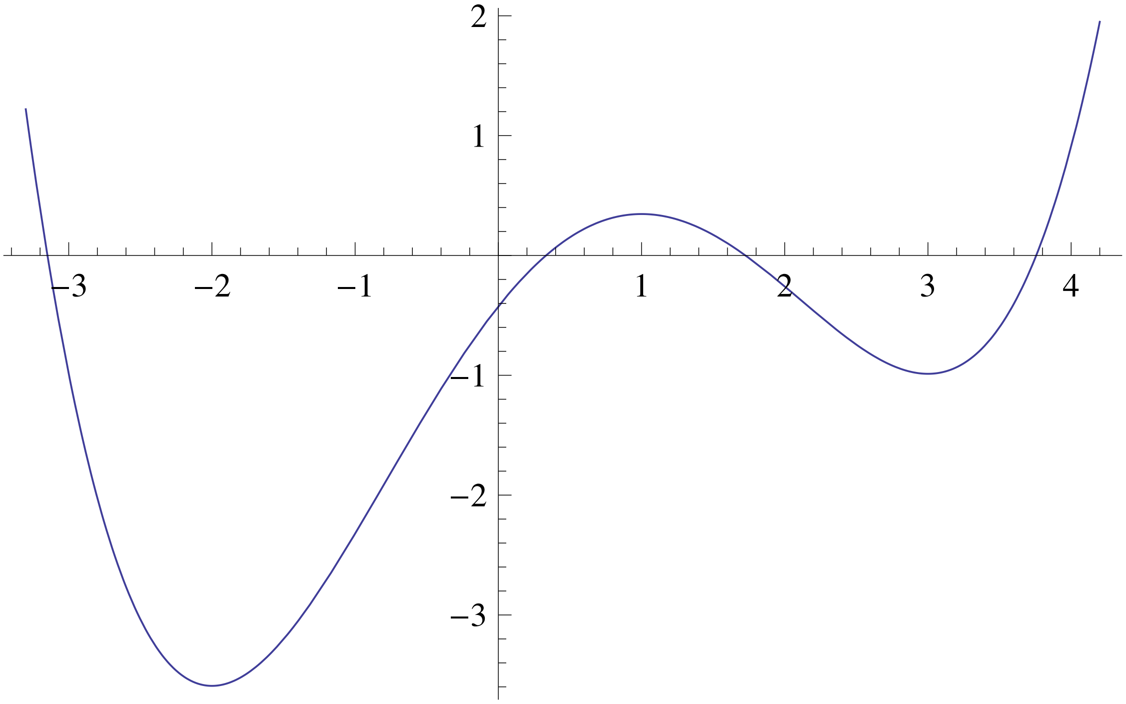
1. Consider the following graph of the polynomial $f(x)$.



(a) Is the highest power of x in the polynomial even or odd? What is the smallest that this power can be? Please explain.

Solution: The power is even. Polynomials with even power grow in the same direction (that is, to either $+\infty$ or $-\infty$) at both ends of the x-axis, while polynomials with odd power grow in opposite directions.
According to our text (top of page 40), an nth power polynomial turns around at most $n-1$ times, but there may be fewer turns. The above polynomial turns around three times, so it must have power AT LEAST four.

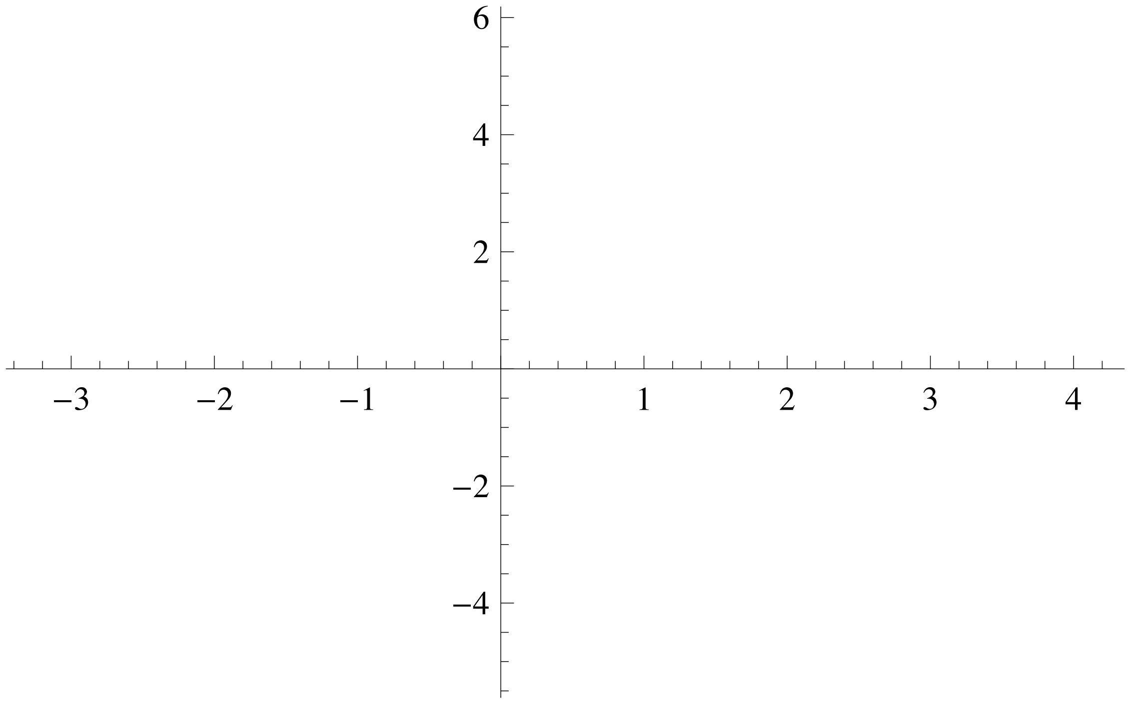
(b) Using the above graph, estimate the slope of the tangent line to the graph of f at each of the points $x = -3, -2, -1, 0, 1, 2, 3$, and 4. Use your estimates to fill in the following table.

x	-3	-2	-1	0	1	2	3	4
slope of the tangent line of f at x								

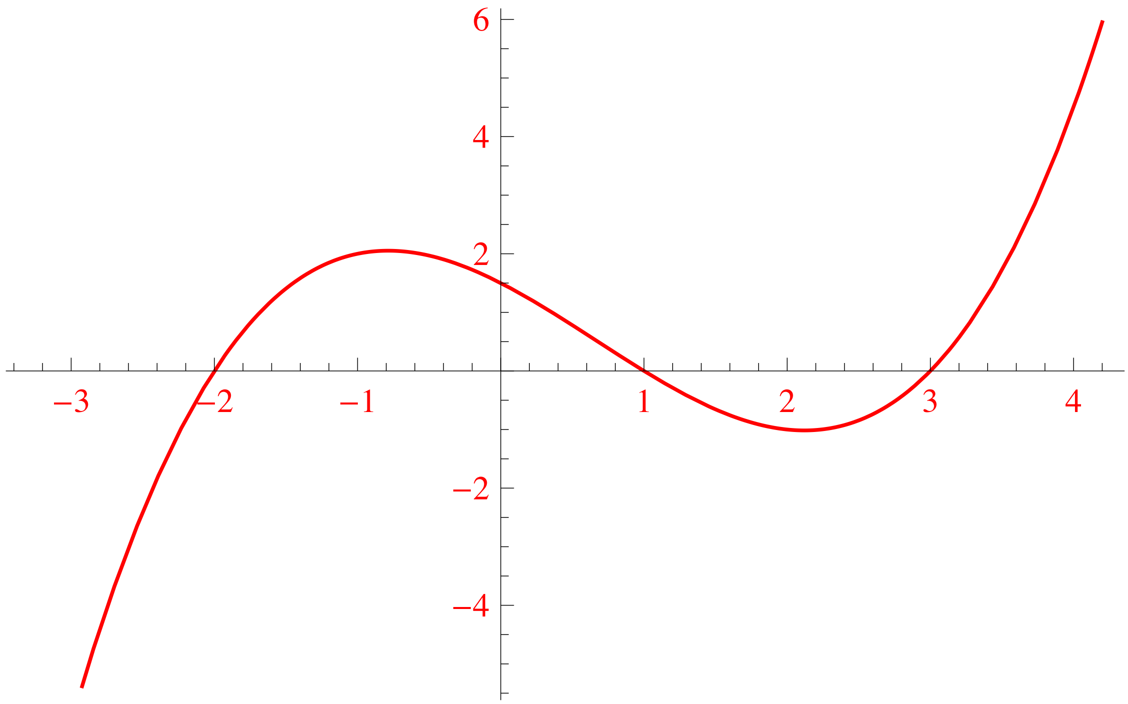
Solution:

x	-3	-2	-1	0	1	2	3	4
slope of f at x	-6	0	2	1.5	0	-1	0	4.5

(c) Using the table from part (b) above, draw a graph from the table with x on the horizontal axis. Connect the dots with a continuous curve.



Solution:



(d) Assume that the graph in part (c) above looks is a polynomial. Make a conjecture about the highest power of this polynomial.

Solution: Certainly the degree is odd, because the graph goes off in opposite vertical directions for large values of $|x|$.

The degree of this polynomial must therefore be AT LEAST THREE (if the degree were one, we'd just have a straight line).

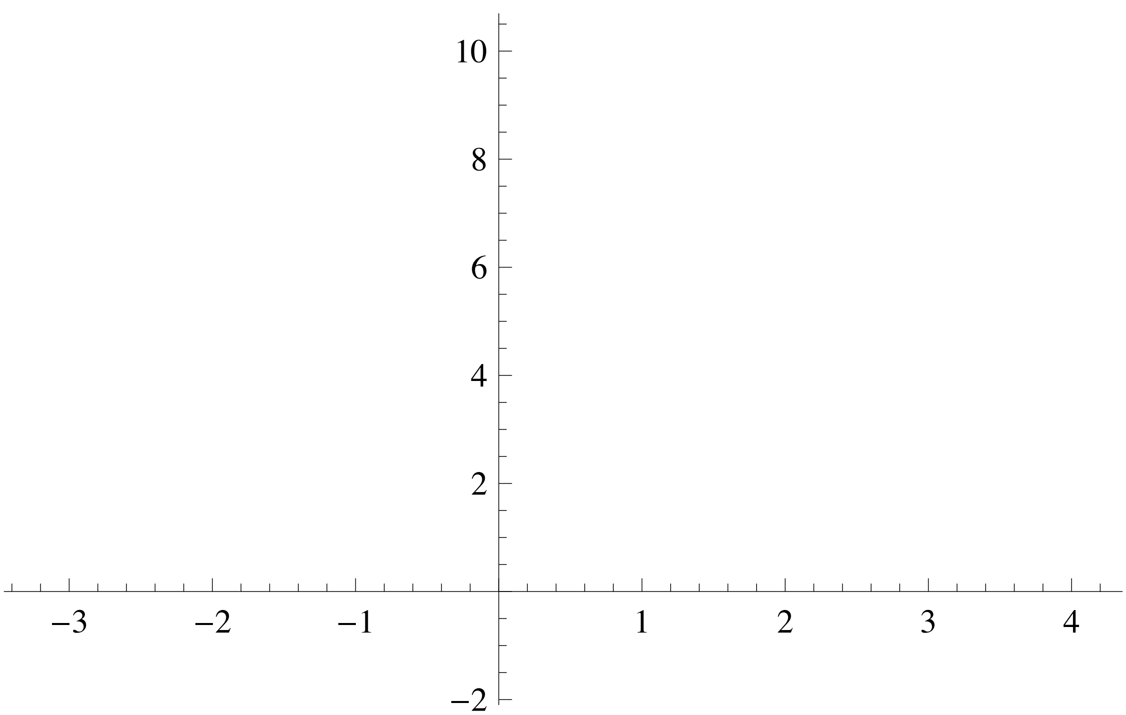
(e) Denote the function you graphed in part (c) above by $g(x)$. Using the graph in part (c), estimate the slope of g at $x = -3, -2, -1, 0, 1, 2, 3$, and 4. Use your estimates to fill in the following table. (Make sure to work with your group and share your results so you don't have to do this entire table by yourself.)

x	-3	-2	-1	0	1	2	3	4
slope of g at x								

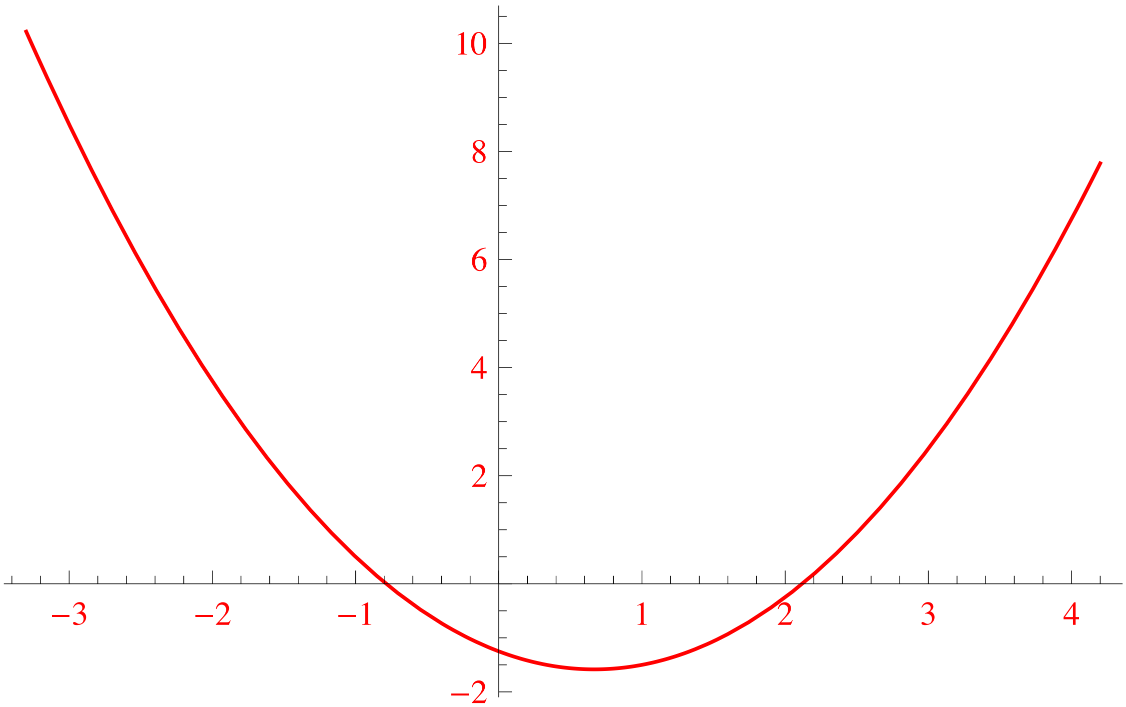
Solution:

x	-3	-2	-1	0	1	2	3	4
slope of g at x	8.5	3.75	0.5	-1.25	-1.5	-0.25	2.5	6.75

(f) Using the table from part (e) above, draw a graph with x on the horizontal axis and the corresponding slope of g on the vertical axis. Connect the dots with a smooth curve.



Solution:



(g) Note that the graph in part (f) above looks like a polynomial. Make a conjecture about the degree of this polynomial.

Solution: It looks like the degree is two (at least).

2. In Section 2.2, you will learn about the derivative of a function at a point. We can now define a derivative **function** f' as follows: for any x, $f'(x)$ equals the slope of f at x.

   (a) Explain how the definition of a derivative function given above relates to the definition
   $$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}.$$

   Solution: Remember that the slope of the tangent line of a function at a point x is defined as the limit, as $h \to 0$, of the average rate of change between x and $x + h$. That is, the slope of the tangent line of f at x is exactly
   $$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h},$$
   which, again, we denote by $f'(x)$.

   (b) Using difference quotients, find the derivative functions f' and g', for $f(x) = 3x$ and $g(x) = x^2$. **Solution:**

   (c) Using difference quotients, find the derivative functions f' and g', for $f(x) = 3x$ and $g(x) = x^2$. We have
   $$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{3(x+h) - 3}{h} = \lim_{h \to 0} \frac{3x + 3h - 3x}{h}$$
   $$= \lim_{h \to 0} \frac{3h}{h} = \lim_{h \to 0} 3 = 3.$$
   Also,
   $$g'(x) = \lim_{h \to 0} \frac{g(x+h) - g(x)}{h} = \lim_{h \to 0} \frac{(x+h)^2 - x^2}{h} = \lim_{h \to 0} \frac{(x^2 + 2xh + h^2) - x^2}{h}$$
   $$= \lim_{h \to 0} \frac{2xh + h^2}{h} = \lim_{h \to 0} 2x + h = 2x.$$