WORKSHEET: Logarithmic Functions **(SOLUTIONS IN RED)** **MATH 1300**

Goal: To review the basics of logarithmic functions, and their relations to exponential functions.

The logarithm function $y = \log_b(x)$ (pronounced "y equals log base b of x") is the inverse of the exponential function $y = b^x$. In other words, $\log_b(x)$ answers the question, "b to what power is equal to x?"

For example, $\log_5(25) = 2$ because $5^2 = 25$.

A special base that you will see frequently in this class is log base e: we will write $\log_e(x)$ simply as "$\ln(x)$" (also called "natural log of x"). In addition, log base 10 is usually written as just log.

1. Find the following logarithms:

(a) $\log_2(32) = 5$ (because $2^5 = 32$)

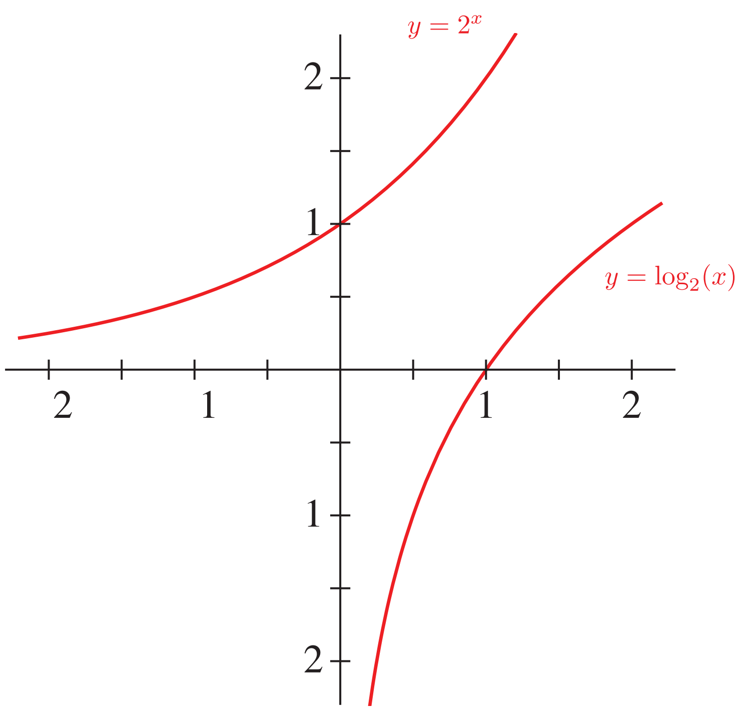
(b) $\log_4\left(\frac{1}{2}\right) = -\frac{1}{2}$

(c) $\log_3(-9) =$ undefined ($3^{\text{any real number}}$ is positive, so it can never equal -9)

(d) $\log(1000) = 3$

(e) $\ln(\pi) =$ there's no simple way to write this. Numerically, the answer is about 1.14473, since $e^{1.14473} \approx \pi$.

2. On the axes below, first sketch the graph of $y = 2^x$. Next, use what you know about graphs of inverse functions to graph $y = \log_2(x)$, on the same set of axes. What are the domain and range of $y = \log_2(x)$?



Domain of $y = \log_2(x)$: $(0, \infty)$
Range of $y = \log_2(x)$: $(-\infty, \infty)$

3. Properties of Logarithms.

(a) Let $x = 10$ and $y = 100$. Compare $\log(x)$ and $\log(y)$ with $\log(xy)$.

$\log(x) = \log(10) = 1$; $\log(y) = \log(100) = 2$; $\log(xy) = \log(1000) = 3$. Note that, in **this** case anyway, $\log(xy) = \log(x) + \log(y)$.

(b) Make a conjecture about what you found in (a) for any x and y (within the domain, of course).

Wow, could it be that $\log(xy) = \log(x) + \log(y)$ for **ALL** x and y in the domain of the function $y = \log(x)$ (that is, for all $x, y > 0$)?? DUDE!!

(c) Using algebra and properties of exponential functions, explain why your conjecture in (b) is true. Hint: write $a = \log(x)$ and $b = \log(y)$. Now express $\log xy$ in terms of a and b.

Let $a = \log(x)$ and $b = \log(y)$. Then $x = 10^a$ and $y = 10^b$, so

$$\log(xy) = \log(10^a \cdot 10^b) = \log(10^{a+b}) = a + b = \log(x) + \log(y),$$

which PROVES our conjecture in part **b**!! DUDE!!!

See p. 25 for the rest of the logarithm rules. You will be expected to know all of them!