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Abstract

Recently, attention has been given to the content of mathematics textbooks and its influence on students' mathematical learning and development. This article extends this line of research and reviews how textbooks can influence teachers' beliefs and practices. In addition to a review of the literature, a study of the implicit views of mathematical learning and development evident in ten algebra level textbooks from the US is reported. The analysis focused on the sequence in which problem-solving activities were presented. It revealed that new topics within algebra were overwhelming presented first through symbolic problem-solving activities, and only later were they applied to verbally presented problems such as story problems or word equations. The presence of this symbol precedence view of mathematical development was more apparent for high school level algebra textbooks than middle school level pre-algebra textbooks from the same publisher. This pattern parallels grade-level differences found among middle school and high school mathematics teachers. Since prior research (e.g., Nathan & Koedinger, 2000) has shown that students at the middle school, high school and college levels have greater facility with verbally presented problems, this raises several issues relevant to mathematics education and research, particularly the ways textbooks institutionalize assumptions about mathematical development that can influence teachers' beliefs and practices, the responsibility of the educational community to test these assumptions, and the implications for putting research-based educational reform into practice.

The international controversy surrounding the portrayal of Japan during World War Two in newly published Japanese history textbooks highlights the importance we place on textbooks and the role we believe them to play in shaping people's views of the world (Prusher, 2001). No one questions that the specific treatment of topics in history textbooks can be highly influential. More recently, attention has been paid to the role the content of mathematics textbooks plays in students' mathematical development (e.g., Brenner Herman, Ho & Zimmer, 1999; Ma, 1999; Meyer, Sims, & Tajika, 1995; Stevenson, Lee, Chen, Stigler, Hsu, & Kitamura, 1990). We extend this line of research and investigate ways that textbooks implicitly portray students' mathematical development through their organizational structures. In turn, we consider how these implicit messages can influence teachers' beliefs about student thinking and development as well as teachers' instructional practices.

In an earlier study (Nathan, Long & Alibali, in press), the organizational structure of a set of ten algebra and pre-algebra textbooks (for the complete list of textbooks see Appendix A) in common use in the US was examined from a rhetorical perspective found within the text processing literature. Here, we review that work, situate it within the larger body of research on mathematics textbooks, and explore some of the implications as they pertain to teacher cognition and mathematics educational reform.

The textbooks chosen by Nathan, Long and Alibali (in press) spanned a 10-year period, from 1986 to 1995, that included the release of some of the seminal documents of mathematics education reform, including the National Council of Teachers of Mathematics Curriculum Standards (NCTM, 1989) and Reshaping School Mathematics (NRC, 1989). These documents called for a greater emphasis on a variety of representational forms beyond just the symbolic, including verbally based quantitative reasoning (NCTM, 1989; NRC, 1989, 1990; OMELC,

1989). While many alternative curricula, such as Connected Mathematics and Mathematics-in-Context, spurred in part by an NSF initiative, reflect approaches that do not necessarily follow the approaches evident in this sample, they make up a relatively small proportion of the commercial curriculum materials that are adopted by schools (Rothman, 1989). The intent of this original investigation was to see whether the sequence of mathematical activities was suggestive of a particular view of mathematical development, and whether that view reflected an accurate picture of students' mathematical development.

Textbook Structure, Teacher Beliefs, and Student Performance: Contrasting Views of Mathematical Development

The textbook analyses reviewed here focused on the sequence by which problem-solving activities for new algebra topics were presented to students. The major hypothesis was that high school-level algebra textbooks would tend to exhibit an organizational structure consistent with the symbol precedence view of mathematical development. As Koedinger and Nathan (1999) define it, under the symbol precedence view it is assumed that learners first develop facility with formal mathematical structures, such as algebraic equations, before they can reason about or solve verbally presented quantitative relationships such as algebra story problems.

Prior research (Nathan & Koedinger, 2000a, 2000b) has shown that high school teachers in the United States tend toward the symbol precedence view of algebraic development. When asked to rank order verbal and symbolic problems (like those in Table 1) that were matched for their underlying mathematical content, the majority of high school teachers in two samples (n_1 = 67, n_2 = 39) chose to rank symbolic problems as easiest, and verbally presented story problems (like P1, Table 1) and word equation problems (P2) as most difficult. High school teachers were

also more likely than their colleagues who taught in middle school and elementary grades to agree with symbol precedence statements in a survey of their views on teaching, students' reasoning, and mathematical development (Nathan & Koedinger, 2000b). In interviews and written comments, high school teachers defended their decisions by describing symbolic problems as simpler forms of the mathematics, more familiar to students, and less complex than verbal problems because of the ability to apply mathematical procedures directly rather than translating the words first to mathematical expressions.

In contrast to the beliefs of high school teachers, the rankings of middle school mathematics teachers demonstrated the symbol precedence view far less often. Middle school teachers predicted that students would find verbally based problems easier to solve than symbolic problems. Middle school teachers were also far more optimistic about the power and efficacy of students' invented solution strategies.

High school students' performance data did not follow high school teachers' expectations, but was more closely aligned with the expectations of middle school teachers. Ninth grade students in two different samples ($n1 = 76$, $n2 = 171$; Nathan & Koedinger, 2000a), each with a year or more of formal algebra instruction, correctly solved fewer than 30% of the symbolic equations but approximately 50% of the mathematically matched verbal problems. Comparisons of the difficulty rankings provided by teachers and the performance ranking from students' problem-solving performances showed that middle school teachers' predictions were significantly correlated with the problem-solving performance of students, $p < .05$ (Nathan & Koedinger, 2000b). Surprisingly, the rank ordering provided by high school teachers was not significantly related to student performance at all, despite the high school teachers' more extensive mathematics education and their traditional role in algebra instruction.

Analyses of solution strategies revealed that students frequently applied informal strategies such as guess-and-test and working-backwards when solving verbally presented story and word-equation problems. As Table 2 shows, verbally based problems tended to elicit informal strategies like unwind (Figure 2a) and guess-and-test (Figure 2b) that were successful about 70% of the time. When solving equations students tended to apply symbol manipulation strategies, such as isolating the unknown variable, which was reliable only about 50% of the time. Symbolic equations also frequently (32% of the time) led to "no response" errors, suggesting that these algebra students struggled to comprehend the abstract formalisms.

Analyses of students' error patterns revealed a developmental trend. Students who could solve verbal problems could not necessarily solve matched symbolic problems. However, students who accurately solved symbolic problems were very likely to solve the matched verbal problems (Nathan & Koedinger, 2000a). This suggested that these algebra students followed a verbal precedence model of mathematical development, whereby verbally based reasoning about quantitative relations (such as mathematically inverting the events of a story problem) precedes symbolic reasoning. Empirically, the vast majority of high school students studied fit this verbal precedence pattern (91%, n1 = 76; 88%, n2 = 171), while a much smaller portion of students (62%, n1 = 76; 46%, n2 = 171) fit the competing symbol precedence pattern.

The prevalence of the symbol precedence view among teachers, despite the tendency of students to follow an alternate trajectory of development, raised the question of which view, if either, was dominant among textbooks. Based on the literature documenting the strong reliance of mathematics teachers on textbooks (e.g., Borko & Livingston, 1989; Borko & Shavelson, 1990; Cooney, 1985; Flanders,1994; Ornstein, 1994) it was expected that teachers' views were probably reflective of the views exhibited by textbooks. Thus, it was hypothesized that textbooks

would be organized in accordance with the symbol precedence model of development. A second expectation was that the documented differences between middle and high school teachers would also be evident in pre-algebra and algebra level textbooks. From this, it was anticipated that algebra textbooks would be more likely than pre-algebra textbooks to exhibit the symbol precedence view. The final expectation concerned changes over the 10-year span covered by the sample of textbooks under study. Since several influential reform documents came out in 1989 (NCTM, 1989; NRC, 1989, 1990; OMELC, 1989) emphasizing the need to go beyond symbolic formalisms, it was expected that the symbol precedence organization would become less common in textbooks published after 1990.

As predicted, textbooks generally followed the symbol precedence view. (The reader interested in the methodological details of the coding of the textbooks and the analyses of the data is referred to Nathan, Long & Alibali, in press.) Problem-solving activities for new topics most typically started with symbolic problems and then went on to verbal problems. Of the textbook sections that contained both verbal and symbolic activities, the symbol precedence pattern was used 64% to 90% of the time (Table 3). Eight of the 10 textbooks in this study exhibited this pattern significantly more often than chance expectations. The remaining two textbooks, both published by the University of Chicago School Mathematics Project (UCSMP), were developed within an academic rather than a commercial setting. Rarely were activities introduced with verbally presented problems and directed toward symbolic reasoning, despite evidence that verbal reasoning abilities may precede more formal reasoning (e.g., Case & Okamoto, 1996; Kalchman & Case, 1998; Nathan & Koedinger, 2000a).

The expected difference between pre-algebra and algebra level textbooks was also found. Algebra textbooks intended for high school and advanced middle school students showed the

symbol precedence pattern significantly more often than pre-algebra textbooks aimed principally at the middle school level. This pattern parallels grade-level differences found among middle school and high school teachers. It also establishes that high school teachers are more likely to work with curricular materials on a daily basis that model the symbol precedence view of mathematical development.

Finally, the analyses revealed that newer textbooks, those published after the NCTM (1989) and NRC (1989) reform documents appeared, contained significantly fewer sections that were exclusively filled with symbolic equations than did older textbooks. This is consistent with the prescriptions from the mathematics educational community that called for greater student use of alternative forms of reasoning and representation, including verbal problem-solving methods. While symbol use was generally reduced, newer textbooks still introduced a major portion of topics using symbolic problem-solving activities (Table 3). Thus, even though there was a demonstrable change overall in the content of these textbooks, there was still a strong, implicit reliance on symbolic precedence as the basis for algebraic development.

Implications for Educational Reform

The findings reported here on the organizational structure of mathematics textbooks and the beliefs held by the teachers who use them raise several issues related to teaching and learning mathematics. We consider 3 areas: the ways textbooks, as agents for institutionalizing assumptions about mathematical development, influence teachers' beliefs and practices; the responsibility of the educational community to test these assumptions; and implications for putting educational reform into practice.

How Textbooks Institutionalize Assumptions About Mathematical Development: Influences on Teachers' Beliefs

Beliefs are powerful mediators of decision-making and action. Beliefs that contradict reality can guide a teacher toward instructional decisions that hinder student learning. For example, Shepard (1991) reviewed how teachers' beliefs about student tracking and grade retention led to instructional decisions and practices that actually obstructed students' intellectual and social development.

While the textbook—teacher belief link is still only circumstantial in the studies reviewed here, investigators have documented the influence that mathematics textbooks have on teaching practices and subsequent student learning. Mathematics typically textbooks serve as the primary resources for teachers' instructional practices and selection of content (Borko & Livingston, 1989; Borko & Shavelson, 1990; Cooney, 1985; Flanders, 1994; Johnsen, 1993; Ornstein, 1994). Teachers also tend to expect student learning to follow the patterns found in their textbooks. For example, Flanders (1994) showed that teachers' expectations for student success was highest for test items (taken from the Second International Mathematics Study, SIMS) covered by the classroom textbooks, even though students also practiced solving SIMS items that were not covered in the textbooks. Teachers had greater confidence in textbook-based curricula than lessons they themselves constructed. These teachers also relied heavily on textbook organization when sequencing topics for their classes.

More than just affecting curriculum, these findings suggest that textbooks may directly influence the very beliefs about student learning and development that shape teacher practices. It would be inappropriate to overstate the nature of the causal relationship between textbooks and teachers' practices or beliefs. It is far more likely that textbooks are only one component in a rich

mix of current and historical social influences on beliefs and practices that include personal learning experiences both as a student and a pre-service teacher (e.g., Ball, 1988). Still, one need only look at the strong parallels between the expectations of middle school and high school teachers and the pre-algebra and algebra textbooks they use respectively to suspect that such a link exists. As teachers use textbooks it is reasonable to consider that they may come to internalize some of the practices that textbooks advocate, even when they are implicit. Clearly, more work needs to be done in this area to determine what influences textbooks play on teachers' decision-making processes and instructional practices.

Testing Assumptions About Learning and Development

The analyses reviewed here revealed that textbooks tend to follow the symbol precedence view of development (Nathan et al., in press). New topics tend to be introduced through symbolic problem-solving activities and elaborated upon using verbal problems. The symbol precedence pattern has also been documented in mathematics textbooks throughout the past century (e.g., Smith, 1926). One author's advice was to present the symbolic form of equations prior to verbal ones, because, in his opinion, "in algebra it is easier to translate from algebra to English than from English into algebra, and such practice should be given first as a preparation for the more difficult work to follow " (Overman, 1923, p.217). This suggests that students tend to face tasks like story problem solving late and relatively infrequently if they have difficulty with equation solving. This appears to be reasonable because it logically follows from the desire to spare struggling learners with challenges believed to be developmentally beyond their reach. However, this result is ironic and potentially quite disturbing since these students are likely to need the most encouragement and help developing a conceptual base for their mathematical reasoning.

The problem with the symbol precedence approach is that it hinges on the self-evident, but erroneous, assumption that story problems are harder for students. People hold this assumption because they believe that one must first construct a symbolic representation of a problem before one can solve it. This assumption is pervasive, and has been even documented among researchers who specialize in mathematics education (Nathan & Koedinger, 2000a), as well as among teachers and textbooks.

The assumption falls down, however, because we find that students are poor at using symbolic solution methods in these early stages of their mathematical development and are most successful when they use informally acquired but meaningful strategies that rely heavily on verbal reasoning and arithmetic. Story and word equation problems are more likely to elicit these more reliable strategies, and so exhibit significantly higher levels of performance. Findings like this reinforce the need for the educational research community to be skeptical of untested assumptions, no matter how old they are or how self-evident they appear to be. It is the responsibility of the research community to scientifically challenge such claims, especially when they have such far-reaching effects on curriculum and instruction.

Implications For Putting Research-Based Educational Reform into Practice

One final implication of this work is for assessing the promise of producing substantive educational change through research-based reform. Never before has educational policy been based upon so much empirical research. Documents like the NCTM (2000) Curriculum Standards are rich with citations of laboratory and classroom studies that serve as a guide for future education. Among these recommendations is the call to encourage students to use more varied forms of reasoning to represent and solve problems in mathematics.

In the study by Nathan and colleagues (Nathan et al., in press), newer textbooks published shortly after the dissemination of a number of influential reform documents showed a discernable shift in their organization. Specifically, these textbooks showed less focus on symbolic problem-solving activities in general as a means to illustrate and teach new topics in algebra. This should be taken as a sign of the positive and fairly rapid impact of current reform documents on commercial materials.

However, the results also suggest that the changes that took place were rather shallow. The number of textbook sections that were exclusively symbol-oriented were reduced. But post-reform textbooks still exhibited the basic symbol precedence pattern at levels comparable to pre-reform editions. One interpretation is that textbook developers made changes that they believed to be consistent with the calls for reform, but that demanded the least cost and effort. Large publishing houses make enormous investments in their textbooks. Eliminating symbol-only sections is far easier than examining and revising developmental patterns that pervade whole books and even entire textbook series. And, as publishing companies consolidate, there is greater inertia that favors minimal change[1]. Thus, we should acknowledge that pragmatics and economics are powerful forces that may be at odds with research-based recommendations. This is particularly apparent when we compare the 8 commercial textbooks to the two volumes published by the University of Chicago School Mathematics Project (UCSMP; McConnell et al., 1990; Usiskin et al., 1995). UCSMP textbooks stand out as the only ones that do not exhibit the symbol precedence pattern at greater than chance levels (Table 3).

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[1] As Rothman (1989, p. 1) points out, "The top 10 textbook companies control about 70 percent of the market."

However, even if textbook developers are inclined toward reform, there is bound to be resistance toward alternative curriculum designs. Firstly, well-entrenched beliefs are highly resistant to change, even when overwhelming counter-evidence is presented (e.g., Dole and Sinatra, 1994). Secondly, calls for reform tend to be vague and open to interpretation (Wilson & Ball, 1996). There is a tendency to reinterpret one's changes as consistent with those being called for when what one really needs to do is make fundamental changes[2]. This tension between preserving one's old view and being responsive to new evidence and new ideas can lead to superficial change.

One promising avenue for initiating and sustaining substantive change in teachers' beliefs and practices is through the use of reform-based curricula (e.g., Ball & Cohen, 1996). Curricula like Connected Mathematics (Lappan et al., 2002), Mathematics in Context (NCRMSE & Freudenthal Institute, 1999), and The Adventures of Jasper Woodbury (CTGV, 1997) fundamentally reconceptualize mathematical practice, pedagogy, and assessment. These new curricula can renew a teacher's sense of the struggle and wonderment of learning (Lloyd & Frykholm, 2000).

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[2] This kind of superficial change is not unique to mathematics education, but appears to be pervasive. Oakes and her colleagues (2000) points out the tendency to rework the status quo in the name of reform. Many such efforts, including those done under the guise of the 1989 Carnegie Foundation Report Turning Points, were introduced in schools as procedural changes rather than changes that fundamentally reconceptualized the educational environment and its practices.

The research reviewed here suggests that the curricula embodied in textbooks can and does have demonstrable impact on teachers' views and practices. This gives us reason to believe that textbook curricula can be effective agents for entrenching old and untested ideas about mathematical development, or for implementing research-based educational reform that can promote substantive teacher change. As such, textbook designers and publishers must be looked upon as allies for instituting and propagating educational reform.

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Appendix A: The ten algebra textbooks analyzed in Nathan, Long, and Alibali (in press).

Publisher	Pre-Algebra Text	Algebra Text
Harcourt Brace Jovanovich	Pre-Algebra: Skills/ Problem Solving/ Applications (Brumfiel et al. 1986)	Introductory Algebra 1 (Jacobs, 1988)
Houghton/ Mifflin	Pre-Algebra: An Accelerated Course (Dolciani et al., 1988)	Algebra: Structure and Method, Book 1 (Brown et al., 1990)
UCSMP	University of Chicago School Mathematics Project: Transition Mathematics (Usiskin et al., 1995).	University of Chicago School Mathematics Project: Algebra (McConnell et al., 1990)
McDougal, Littell	Gateways to Algebra and Geometry: An Integrated Approach. (Benson et al., 1994)	Algebra I: An Integrated Approach (Benson et al., 1991)
Glencoe	Merrill Pre-Algebra: A Transition to Algebra (Price et al., 1992)	Merrill Algebra 1: Applications and Connections (Foster et al., 1995)

Tables and Figures

Table 1. The problems given to students to solve and teachers to rank, organized by the presentation type.

Table 2. The relative use (%) of each strategy by 9th grade students (n = 76) as a function of the presentation type, and its probability of success.

Table 3. Summary of textbook data, including publisher, publication year, curricular focus and proportion of symbolic-to-verbal (SV) pattern.

Table 1. The problems given to students to solve and teachers to rank, organized by the presentation type.

Presentation type →	Verbal problems		
Unknown value ↓	Story	Word Equation	Symbol Equation
Algebra	P1. When Ted got home from his waiter job, he multiplied his hourly wage by the 6 hours he worked that day. Then he added the $66 he made in tips and found he earned $81.90. How much per hour does Ted make?	P2. Starting with some number, if I multiply it by 6 and then add 66, I get 81.9. What number did I start with?	P3. Solve for X: $X * 6 + 66 = 81.90$

Table 2. The relative use (%) of each strategy by 9th grade students (n = 76) as a function of the presentation type, and its probability of success.

Strategy	Unwind	Guess-and-Test	Symbolic	No Response	Answer Only	Unknown	Total
Presentation Type							
Story	50	7	5	12	18	8	100
Word Equation	22	23	11	13	19	12	100
Equation	13	14	22	32	11	8	100
Probability of Success	.69	.71	.51	0	.40	.24	

Table 3. Summary of textbook data, including publisher, publication year, curricular focus and proportion of symbolic-to-verbal (SV) pattern.

Publisher	Publication Year	Curriculum	Textbook Age	SV Proportion
HBJ[a]	1986	Pre-algebra	Older	.82*
HBJ	1988	Algebra	Older	.88*
Houghton	1988	Pre-algebra	Older	.72*
Houghton	1990	Algebra	Older	.90*
UCSMP[b]	1990	Algebra	Older	.53
McDougal	1991	Algebra	Newer	.74*
Glencoe	1992	Pre-algebra	Newer	.86*
McDougal	1994	Pre-algebra	Newer	.64*
Glencoe	1995	Algebra	Newer	.81*
UCSMP	1995	Pre-algebra	Newer	.40
Total				**.73**

[a]HBJ stands for Harcourt Brace Jovanovich.

[b]UCSMP stands for University of Chicago School Mathematics Project.

* Significantly different from chance, $p < .05$.