Name: ______________________________

Economics 3070
Intermediate Microeconomic Theory
Fall 2008
Prof. Barham

Midterm 1 Examination
October 16th, 2008

You have an hour and 15 minutes for this exam. The exam is out of 55 points. Good luck!

The breakdown of points is as follows, plan your time appropriately.

Multiple Choice Section: 17 questions, worth 17 points in total.
Problem Section: Starts on Page 5

Question 1: 4 points
Question 2: 6 points
Question 3: 8 points
Question 4: 16 points
Question 5: 4 points
Problems: Point values for each question are indicated at the end of each question. If you need more space use the back of the page but mark it well.

1. (4 points)
Suppose demand for good A is given by \( Q_A^d = 500 - 15P_A + 2P_B + 0.70I \) where \( P_A \) is the price of good A, \( P_B \) is the price of some other good B, and I is income. Assume that \( P_A \) is currently $10, \( P_B \) is currently $5, and I is currently $100.

a. (2 points)
What is the cross-price elasticity of the demand for good A with respect to the price of good B at the current situation? Explain in one sentence what the elasticity you calculated means.

First we need to figure out what \( Q_A \) is a current prices:
\[
Q_A = 500 - 15*10 + 2*5 + .70*100 = 430
\]

\[
\varepsilon_{Q_A,P_B} = \frac{\partial Q_A}{\partial P_B} \cdot \frac{P_B}{Q_A} = 2(\frac{5}{430}) = 0.023
\]

So, if price of good B increase by 10 percent, the quantity demanded of good A will decrease by .23 percent. So these goods are substitutes.

b. (2 points)
What is the income elasticity of the demand for good A with respect to the price of good B at the current situation? Explain in one sentence what the elasticity you calculated means.

\[
\varepsilon_{Q_A,I} = \frac{\partial Q_A}{\partial I} \cdot \frac{I}{Q_A} = .7(\frac{100}{430}) = 0.16
\]

So if the income increase by 10 percent, the quantity demanded of good A will increase by 1.6 percent.
2. (6 points)
List the 3 basic assumptions that consumer preferences must obey if the consumer is a utility maximizer (or that underlie the theory of consumer choice). Give the name of the assumption, and a brief explanation of what this assumption means.

1. Preferences must be Complete: Preferences are complete if a consumer can rank any two baskets.

2. Preferences must be Transitive: For preferences to be transitive if a consumer who prefers basket A to basket B, and basket B to basket C, must prefer basket A to basket C.

3. Preferences must be Monotonic (More is Better): Preferences are monotonic if a basket with more of one good and no less of the other good is preferred to the original basket. This assumes we do not dislike a good
2. 3. (7 points)
David is at the grocery store choosing between two beverages: Super Power Aid (S) and lemonade (L). David may choose to purchase one or both of these beverages. Super Power Aid costs $4 (Pₜ) per quart and lemonade costs $2 (Pₗ) per quart. David has $10 (I) to spend on beverages. David’s utility as a function of these two beverages can be represented as \( U = S + L \).

a) (1 point)
What is the objective function for this problem?

\[ \underset{S,L}{\text{Max}} U(S,L) = S + L \]

a) (1 point)
What is the constraint?

\[ 4S + 2L = 10 \]

b) (2 points)
Which variables in this model (Pₜ, S, Pₗ, L, I) are exogenous? Which are endogenous? Explain

S and L are endogenous since they are determined within the model (i.e. they are what David has to choose). All other variables are exogenous because they are given to David (i.e. they are determined outside the model).

c) (3 points)
Calculate David’s utility maximizing amounts of lemonade and super power aid.

This is a case of perfect substitutes. Both the indifference curve and the budget constraint are linear so we will be at a corner point solution (i.e. either consume only Lemonade or only Superaid). S and L both provide the same amount of utility so you would consumer the cheaper one, which is Lemonade. More formally lets compare:

\[ \frac{MU_{LL}}{P_L} = \frac{1}{2} \]
\[ \frac{MU_{SS}}{P_S} = \frac{1}{4} \]

Comparing the “bang for the buck”, drinking lemonade give more utility per $ spent than super power aid so David will only buy lemonade. Using the budget constraint we see that David will consume 5 units of lemonade.

\[ L = 5 \text{ and } S = 0 \]
4. (16 points)

Lou’s preferences over pizza (X) and other goods (Y) are given by \( U(x,y) = XY \). His income is $120. We are going to figure out Lou’s income and substitution effect from a price change but do it in steps.

a. (1 points)
If \( P_x = $4 \) and \( P_y = 1 \) what is Lou’s maximization problem. Indicate what is the objective function and what is the constraint.

\[
\begin{align*}
\text{Max } U(x,y) & = XY \\
\text{subject to } 4X + Y = 120
\end{align*}
\]

b. (3 points)
Using the method we learnt in class and that is shown on homework solutions, calculate Lou’s optimal basket of X and Y when \( P_x = $4 \) and \( P_y = 1 \). Call this bundle A.

Rewriting the budget constraint and subbing back into the objective function the problem becomes:

\[
\begin{align*}
\text{Max } U(X) & = X \left( \frac{120 - 4X}{1} \right) \\
\frac{\partial U}{\partial X} & = 120 - 2 \frac{4X}{1} = 0 \\
X & = \frac{120}{8} \\
X & = 15
\end{align*}
\]

So optimal amount of X when \( P_x=4 \) is 15.
So optimal amount of Y when \( P_y=1 \) is 60.

c. (1 point)
Calculate how much utility Lou gets from bundle A.

\[
U = 15 \times 60 \\
\text{Total Utility is } 900.
\]
d. (3 points)
Now suppose the price of pizza decreases to $3. Set up the new maximization problem and calculate the optimal basket of X and Y. Call this bundle C.

\[ \text{Max } U(x, y) = xy \]
\[ \text{s.t. } 3x + y = 120 \]

Rewriting the budget constraint and subbing back into the objective function the problem becomes:

\[ \text{Max } U(x) = x \left( \frac{120 - 3x}{1} \right) \]
\[ \frac{\partial U}{\partial x} = 120 - 2 \frac{3x}{1} = 0 \]
\[ x = \frac{120}{6} \]
\[ x = 20 \]

So optimal amount of X when Px=3 is 20.
So optimal amount of Y when Py=1 is 60.
Total Utility is 1200.
e. (3 points)
On a well labeled graph with pizza on the horizontal axis and all other goods on the vertical axis, clearly show the income and substitution effect for Pizza consumption. Show where consumption bundle A and C are on the graph. (You don’t have to be exact with the numbers for the axis, just get the general shape of the indifference curve).

Where A is the tangency point before the price change. C is the tangency point after the price change and B is the decomposition bundle. The substitution effect is \(X_b - X_a\), and the income effect is \(X_c - X_b\).
f. (5 points)
Calculate the income and substitution effects of the price change of pizza on pizza consumption.

There are a number of way to do this. Looking at the graph you drew you can see that the decomposition bundle is that bundle that would be chosen when you are constrained to the original level of utility (900 which we figured out in part A), but have the new price ratio. One way to do this is to use duality. The constraint is that we have to be on the indifference curve where utility is 900 (the original utility curve). So we can look at this problem by asking ourselves, what is the minimum expenditure I can make, using the new prices, such that I reach a utility level of 900.

\[
\begin{align*}
\text{Min} I & = 3X + 1Y \\
\text{s.t.} & 900 = XY \\
\text{Min} I & = 3X + 1 \frac{900}{X} \\
\frac{\partial I}{\partial X} & = 3 - \frac{900}{X^2} = 0 \\
X^2 & = \sqrt{300} \\
X & = 10 \times \sqrt{3} = 17.3 \\
\end{align*}
\]

Using the utility function this means

\[
Y = \frac{900}{10\sqrt{3}} = \frac{90}{\sqrt{3}} = 51.9
\]

Calculate the income and substitution effects of the price change of pizza on pizza.

Income Effect is calculate by comparing how much Pizza is consumed at bundle C and B = 20 – 17.3 = 2.7

Substitution Effect is calculate by comparing how much Pizza is consumed at bundle B and A = 17.3 - 15 = 2.3
Sky shops at the super target each week and has to decide how much of his 150 weekly budget he should spend on books ($B$) and CDs ($C$). Sky may choose to purchase one or both of these goods, but Sky likes both goods. Books cost $P_B$ and CDs $P_C$. Sky’s utility as a function of these two goods can be represented as

$$U(C,B) = CB + 10B$$

a) (2 points)
Find $MU_B$ and $MU_C$. State if they are they both positive if $B$ and $C$ are positive?

$$MU_c = \frac{\partial U}{\partial C} = B$$

$$MU_b = \frac{\partial U}{\partial B} = C + 10$$

b) (2 points)
Write the $MRS_{C,B}$. As $C$ increases is the $MRS_{C,B}$ diminishing? Explain your answer using the MRS that you determined.

$$MRS_{C,B} = \frac{MU_C}{MU_B} = \frac{B}{C + 10}$$

If both marginal utilities are positive indifference curve must be downward sloping. So as $C$ increases, $B$ must decrease. So, yes there is a diminishing marginal rate of substitution because as $C$ increase the denominator gets bigger, and $B$ decreases so the numerator gets smaller, therefore MRS will decrease.