Problem Set 6 Solutions

1. **In a competitive market with no government intervention, the equilibrium price is $10 and the equilibrium quantity is 10,000 units. Explain whether the market will clear under each of the following forms of government intervention:**

   a. **The government imposes a tax of $1 per unit.**

   The market will clear. The tax will alter the equilibrium price and quantity, but there will be no excess demand or excess supply.

   b. **The government pays a subsidy of $5 per unit produced.**

   The market will clear. The subsidy will alter the equilibrium price and quantity, but there will be no excess demand or excess supply.

   c. **The government sets a price floor of $12.**

   The market will not clear. A price floor set above the equilibrium price will create excess supply.

   d. **The government sets a price ceiling of $8.**

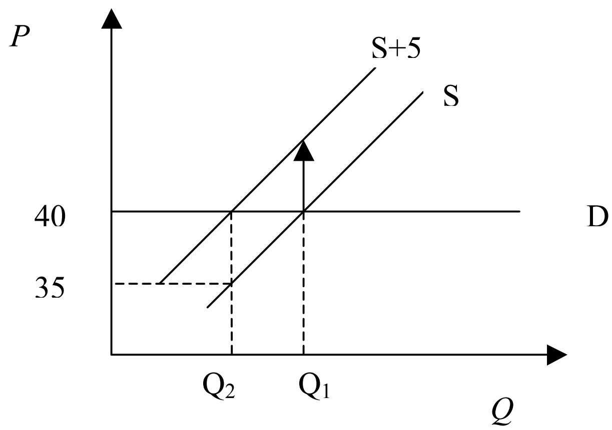
   The market will not clear. A price ceiling set below the equilibrium price will create excess demand.

   e. **The government sets a production quota, allowing only 5,000 units be produced.**

   The market will not clear. A quota limiting output below the equilibrium level will create excess supply since the price will be driven above the equilibrium price.

2. In a competitive market, there is currently no tax, and the equilibrium price is $40. The market has an upward-sloping curve. The government is about to impose an excise tax of $5 per unit. In the new equilibrium with the tax, what price will producers receive and consumers pay if the demand curve is:

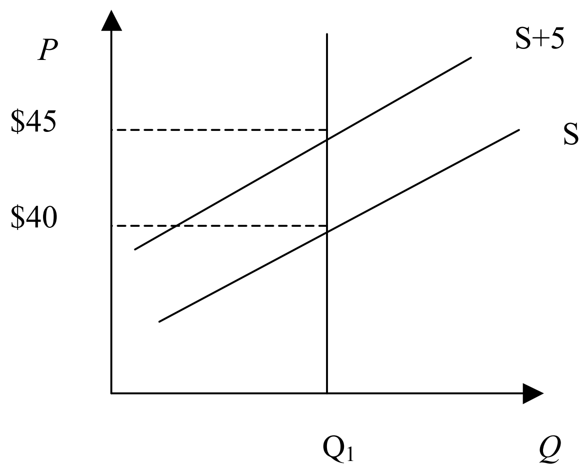
a) Perfectly elastic



With perfectly elastic demand, a $5 excise tax shifts the supply curve up by 5. The consumer price does not change. The producer price reduces to $35 (there has to be a $5 difference between consumer and producer price), they take on the whole burden on the tax. The equilibrium quantity falls from Q_1 to Q_2.

b. **Perfectly inelastic**.

This time the consumer bears the burden of the tax. Equilibrium quantity does not change, the price the consumer pays goes up to \$45 and the price the producer receives stays at \$40.



3. **The current equilibrium price in a competitive market is \$100. The price elasticity of demand is – 4 , and the price elasticity of supply is + 2 . If a tax of \$3 per unit is imposed, how much would you expect the equilibrium price paid by consumers to change? How much would you expect the equilibrium price received by producers to change?**

Using the formula

$$\frac{\Delta P^C}{\Delta P^S} = \frac{\in_{Q^S,P}}{\in_{Q^D,P}},$$

we can substitute – 4 for $\in_{Q^D,P}$ and +2 for $\in_{Q^S,P}$ to obtain

$$\frac{\Delta P^C}{\Delta P^S} = -0.5 \text{ or } \frac{\Delta P^S}{\Delta P^C} = -2 .$$

Both of these ratios tell us that if the consumer price goes up by one unit, the producer price goes down by two units. In total we have three units. Dividing the tax, \$3, by 3 units means each unit is worth \$1.

So, the price received by producers decreased by \$2 and the price consumers have to pay increased by \$1.

4. Suppose the market for cigarettes in a particular town has the following supply and demand curves: $Q^s = P$ and $Q^d = 50 - P$, where the quantities are measured in thousands of units. Suppose that the town council needs to raise $300,000 in revenue and decides to do this by taxing the cigarette market. What should the tax be in order to raise the required amount of money?

Suppose that the required tax is $\$t$ per unit. Let's start by writing down what we know In equilibrium with the excise tax:

1. $P^c = P^s + t$, where P^c is the consumer price, and P^s is the supplier price
2. $Q^s = Q^d$
3. Government revenue= $300,000 = Q^* * t$, where Q^* is the new equilibrium quantity.

Given that we can put quantity supplied and demanded in terms of consumer prices (see additional things we know below) we have three equations and three unknowns (t, P^c, P^s)

We also know

4. $Q^s = P^s$
5. $Q^d = 50 - P^c$

Using equation 2, and substituting in equations 4 and 1 we get:

$$Q^s = Q^d$$
$$P^S = 50 - P^C$$
$$P^S = 50 - \left(P^S + t\right)$$
$$2P^S = 50 - t$$
$$P^S = 25 - \tfrac{1}{2}t$$

Since the equilibrium quantity $Q^* = Q^s = P^S$, we have

$$Q^* = 25 - \tfrac{1}{2}t.$$

Now we can use equation 3 to determine how much the tax is. The council needs to raise $300,000, we must have $tQ^* = 300,000$. Substituting for Q^*, we obtain

$$300,000/t = 25 - \tfrac{1}{2}t$$
$$300,000 = 25t - \tfrac{1}{2}t^2$$
$$t^2 - 50t + 600 = 0$$
$$(t - 20)(t - 30) = 0$$

So, we have two possible values for the tax: $t = 20$ and $t = 30$. Either one would generate the $300,000 in tax revenues, but a $20 tax would do so with a smaller deadweight loss than a $30 tax would.

5. **Suppose the market for corn in Pulmonia is competitive. No imports and exports are possible. The demand curve is**

$$Q^d = 10 - P^d,$$

where Q^d is the quantity demanded (in millions of units) when the price consumers pay is P^d. The supply curve is

$$Q^s = \begin{cases} -4 + P^s & \text{if } P^s \geq 4 \\ 0 & \text{if } P^s < 4 \end{cases}$$

where Q^s is the quantity supplied (in millions of units) when the price producers receive is P^s.

   a. **What are the equilibrium price and quantity?**

   Setting $Q^d = Q^s$, we obtain

$$10 - P = -4 + P$$

$$P^* = 7$$

   Substituting this result into the demand equation gives us

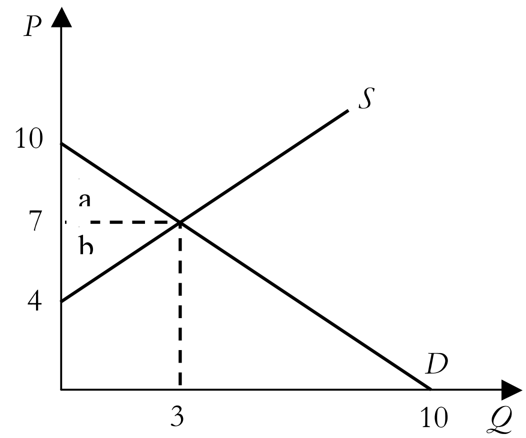
$$Q^* = 10 - P^*$$

$$Q^* = 3$$

   Therefore, the equilibrium price is $7 per unit, and the equilibrium quantity is 3 million units.

   b. **At the equilibrium in part a, what is consumer surplus? Producer surplus? Deadweight loss? Show all of these graphically.**

   The perfectly competitive equilibrium is depicted in the graph below:



The consumer surplus is represented by the area a and is equal to $\frac{1}{2}(3)(10-7) = \$4.5 \text{ million}$. The producer surplus is represented by area b and is equal to $\frac{1}{2}(3)(7-4) = \$4.5 \text{ million}$. There is no deadweight loss when the equilibrium is perfectly competitive.

c. Suppose the government imposes a tax of $\$2$ per unit to raise government revenues. What will the new equilibrium quantity be? What price will buyers pay? What price will sellers receive?

If the government imposes a tax of \$2 per unit, $P^d = P^s + 2$. Setting $Q^d = Q^s$ and substituting for P^d, we obtain

$$10 - (P^s + 2) = -4 + P^s$$

$$12 = 2P^s$$

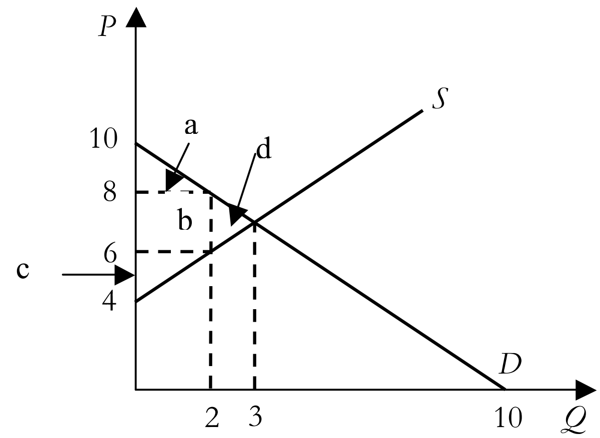
$$P^s = 6$$

Substituting back into $P^d = P^s + 2$ yields $P^d = 8$, and substituting $P^d = 8$ into the demand function $Q^d = 10 - P^d$ yields $Q^d = 2$.

Thus, the new equilibrium quantity is 2 million units, the price buyers will pay is \$8 per unit, and the price sellers will receive is \$6 per unit.

d. At the equilibrium in part c, what is consumer surplus? Producer surplus? The impact on government revenue? Deadweight loss? Show all of these graphically.

The new equilibrium is depicted in the graph below:



The consumer surplus is represented by area a and is equal to $\frac{1}{2}(2)(10-8) = \$2 \text{ million}$. The producer surplus is represented by area c and is equal to $\frac{1}{2}(2)(6-4) = \$2 \text{ million}$. The impact on government revenue is represented by area b and is equal to $(2)(8-6) = \$4 \text{ million}$. The deadweight loss is represented by area d and is equal to $\frac{1}{2}(3-2)(8-6) = \$1 \text{ million}$.

e. Suppose the government has a change of heart about the importance of kumquat revenues to the happiness of the Boornian farmers. The tax is removed, and a subsidy of $1 per unit is granted to kumquat producers. What will the equilibrium quantity be? What price will the buyer pay? What price (including the subsidy) will kumquat farmers receive?

If the government repeals the tax and implements a subsidy of $1 per unit, $P^s = P^d + 1$. Setting $Q^d = Q^s$ and substituting for P^s, we obtain

$$10 - P^d = -4 + (P^d + 1)$$

$$13 = 2P^d$$

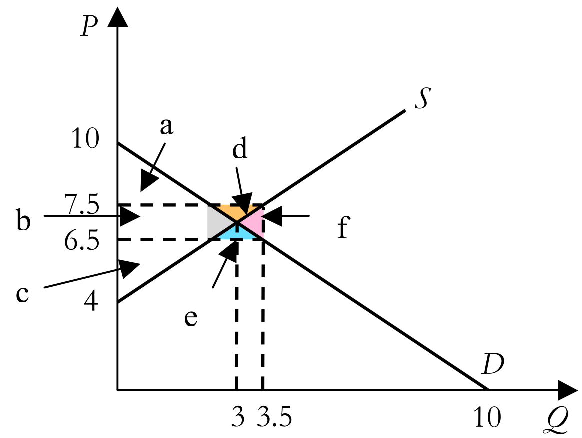
$$P^d = 6.5$$

Substituting back into $P^s = P^d + 1$ yields $P^s = 7.5$, and substituting $P^d = 6.5$ into the demand function $Q^d = 10 - P^d$ yields $Q^d = 3.5$.

Thus, the new equilibrium quantity is 3.5 million units, the price buyers will pay is $6.50 per unit, and the price sellers will receive is $7.50 per unit.

f. At the equilibrium in part e, what is consumer surplus? Producer surplus? What will be the total cost to the government? Deadweight loss? Show all of these graphically.

The new equilibrium is depicted in the graph below:



The consumer surplus is represented by the sum of the areas A, B, and E:

$$CS = \tfrac{1}{2}(3.5)(10 - 6.5) = \$6.125 \text{ million}.$$

The producer surplus is represented by areas C, B, and D :

$$PS = \tfrac{1}{2}(3.5)(7.5 - 4) = \$6.125 \text{ million}.$$

The total cost to the government is represented by areas B, E, D, and F:

$$\text{Cost to Government} = (3.5)(7.5 - 6.5) = \$3.5 \text{ million}.$$

The deadweight loss is represented by area F:

$$DWL = \tfrac{1}{2}(3.5 - 3)(7.5 - 6.5) = \$0.25 \text{ million}.$$

g. Verify that for your answers to parts b, d, and f the following sum is always the same: consumer surplus + producer surplus + impact on the government budget + deadweight loss. Why is the sum equal in all three cases?

The sum in all three cases is \$9 million. These sums are all the same because the deadweight loss measures the difference between total economic surplus under the competitive outcome (CS + PS) and total economic surplus under a form of government intervention (CS + PS + impact on government budget).

6. Suppose that demand and supply curves in the market for corn are

$$Q^d = 20{,}000 - 50P \quad \text{and}$$
$$Q^s = 30P$$

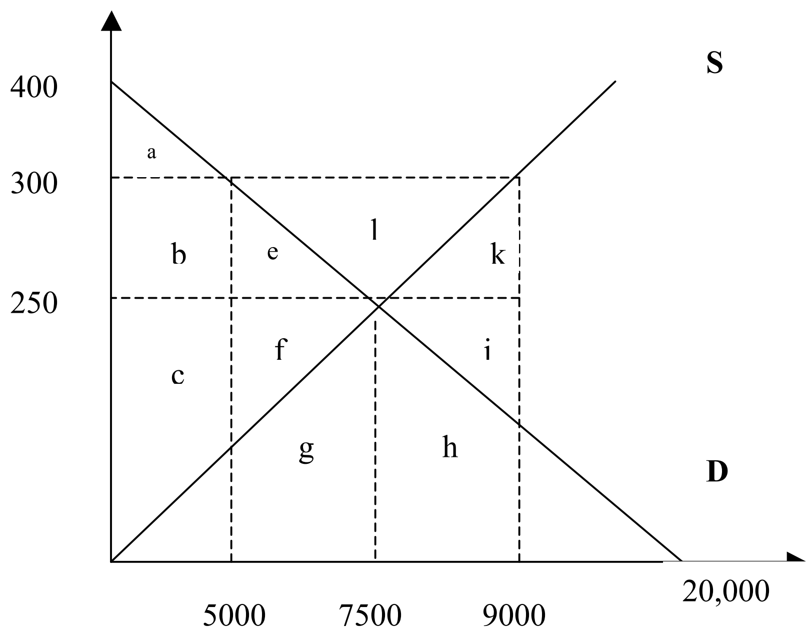
Suppose that the government would like to see the price at \$300 per unit and is prepared to artificially increase demand by initiating a government purchase program. How much would the government need to spend to achieve this? What is the total deadweight loss if the government is successful in its objective.

Without government intervention we can determine equilibrium price and quantity by setting quantity equal to demand.

$$Q^d = Q^s$$
$$20,000 - 50P = 30P$$
$$P^* = 20,000/80$$
$$P^* = 250$$

Plugging P* back into either the Q^d *or* Q^s

$$Q^* = 7500$$



If the price is increased to $300, suppliers would like to produce $9,000 (plug 30 into the supply curve 30*300). Consumer would only demand $5,000 (plug 300 into the demand curve 20,000-50*300). Thus, there is excess demand in the market. In order to keep the price at 300, the government must by the 4,000 units which is not demanded. This will cost the government 4,000*300 = $1.2 million.

To figure out deadweight lost lets calculate the net benefit before and after the government intervention. Remember net benefit is the sum of consumer and producer purchase, and government purchase.

Net Benefit
Before Intervention: CS=A+B+E PS=C+F
A+B+C+E+F

After the Intervention: CS = A PS = B+E+L+F+C GR= - (E +L+ K+F+J+G+H)
A+B+C-K-J-G-H

Change in net benefit is : A+B+C-K-J-G-H-A-B-C-E-F = -E-F-K-J-G-H

Deadweight loss is: E + F + K + J + G + H

Area G + H + J + K represents production costs that are incurred for units of corn t hat no one consumes.

Area E + F represents benefits no longer captured by anyone.

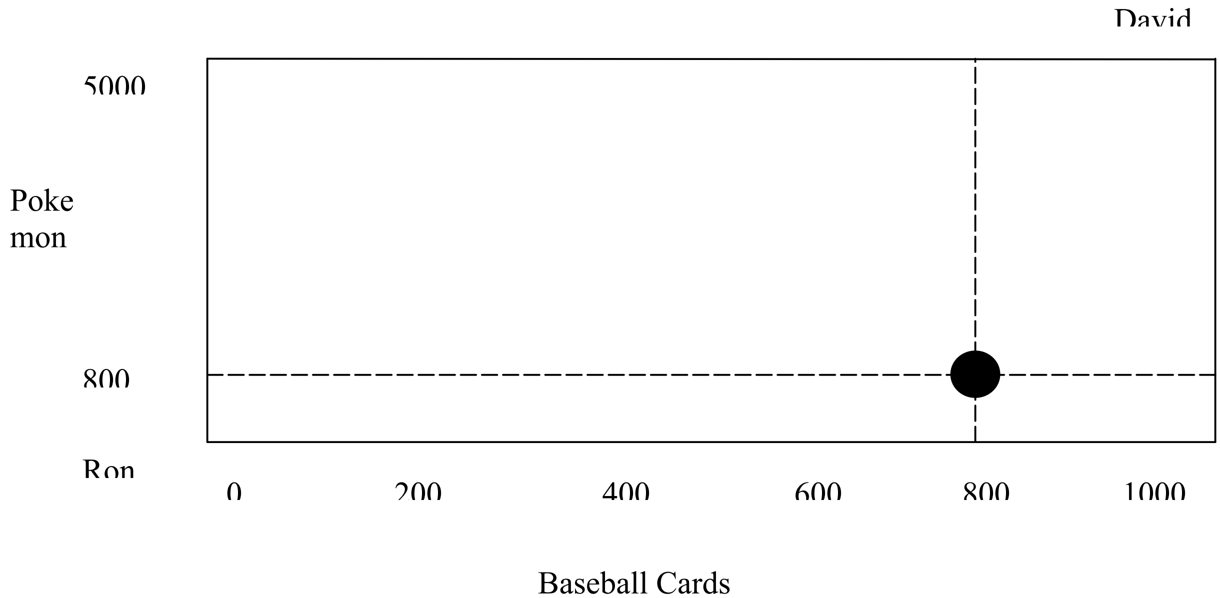
7. Two consumers, Ron and David, together own 1000 baseball cards and 5000 Pokemon cards. Let X_R denote the quantity of baseball cards owned by Ron and Y_R denote the quantity of Pokemon cards owned by Ron. Similarly, let X_D denote the quantity of baseball card owned by Don and Y_D the number of Pokemon cards owned by David. Suppose, further, that

For Ron $MRS^R_{x,y} = \dfrac{Y_R}{X_R}$

For David $MRS^D_{X,Y} = \dfrac{Y_D}{2X_D}$

Finally, suppose $X_R = 800 \quad Y_R = 800 \quad X_D = 200 \quad \mathrm{Y}_D = 4200$

(a) Draw an Edgeworth box that shows the set of feasible allocation in this simple economy.



(b) Show that the current allocation of cards is not economically efficient.

To be economically efficient, the MRS for the two consumers must be equal. At this allocation we have:

$$MRS_{X,Y}^{R} = \frac{Y_R}{X_R} = \frac{800}{800} = 1$$

$$MRS_{X,Y}^{D} = \frac{Y_D}{2X_D} = \frac{4200}{2(200)} = 10.5$$

Since $MRS^D > MRS^R$, the current allocation is not economically efficient.

(c) Identify a trade of cards between David and Ron that makes both better off.

$$MRS_{X,Y}^{R} = -\frac{\Delta Y}{\Delta X} = \frac{Pokemon}{Baseball} = 1$$

$$MRS_{X,Y}^{D} = -\frac{\Delta Y}{\Delta X} = \frac{Pokemon}{Baseball} = 10.5$$

Ron is willing to trade 1 pokemon card for 1 additional baseball card.
David is willing to trade 10.5 pokemon cards for 1 additional baseball card.

There are many allocations that could make them both better off. For instance David gives for instance if Ron give 9 pokemon cards in exchange for one baseball card both consumers will be better off.

8. Two firms together employ 100 units of labor and 100 units of capital. Firm 1 employs 20 units of labor and 80 units of capital. Firm 2 employs 80 units of labor and 20 units of capital. The marginal products of the firms are as follows:

$$MP_L^1 = 50 \quad MP_K^1 = 50 \quad \mathrm{MP}_\mathrm{L}^2 = 10 \quad MP_K^2 = \mathbf{20}$$

Is this allocation of inputs economically efficient?

To satisfy input efficiency, the marginal rates of technical substitution must be equal across firms. Here we have

$$MRTS_{l,k}^1 = \frac{MP_l^1}{MP_K^1} = \frac{50}{50} = 1$$

$$MRTS_{l,k}^2 = \frac{MP_l^2}{MP_k^2} = \frac{10}{20} = 0.5$$

So, the allocation of inputs is not economically efficient.

9. **The market demand curve for a monopolist is given by P=40-2Q.**

a) What is the marginal revenue function for the firm?

$$TR = P * Q = 40Q - 2Q^2$$

$$MR = \frac{\partial TR}{\partial Q} = 40 - 4Q$$

Alternatively you could have noted that the marginal revenue curve has two times the slope of the demand curve.

b) What is the maximum possible revenue that the firm can earn?

Maximize revenues where $\frac{\partial TR}{\partial Q} = MR = 0$

Therefore Q* will be 10

To determine price we plug Q=10 back into the market demand curve.

P* = 40-20 = 20

So total revenue will be TR = Q*P* = 10*20 = 200