Problem Set 5 Answers

1. Ch 7, Problem 7.2

A grocery shop is owned by Mr. Moore and has the following statement of revenues and costs:

<table>
<thead>
<tr>
<th>Description</th>
<th>Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>Revenues</td>
<td>$250,000</td>
</tr>
<tr>
<td>Supplies</td>
<td>$25,000</td>
</tr>
<tr>
<td>Electricity</td>
<td>$6,000</td>
</tr>
<tr>
<td>Employee salaries</td>
<td>$75,000</td>
</tr>
<tr>
<td>Mr. Moore’s salary</td>
<td>$80,000</td>
</tr>
</tbody>
</table>

Mr. Moore always has the option of closing down his shop and renting out the land for $100,000. Also, Mr. Moore himself has job offers at a local supermarket at a salary of $95,000 and at a nearby restaurant at $65,000. He can only work one job, though. What are the shop’s accounting costs? What are Mr. Moore’s economic costs? Should Mr. Moore shut down his shop?

The accounting costs are simply the sum: $25,000 + $6,000 + $75,000 + $80,000 = $186,000.

The economic costs also include the opportunity cost of the land rental ($100,000) and the extra salary Mr. Moore would earn if he selected his next best alternative ($95,000 - $80,000 = $15,000). So, the economic costs are $186,000 + $100,000 + $15,000 = $301,000.

Should Mr. Moore shut down his shop? His economic costs exceed his revenues by $301,000 - $250,000 = $51,000. Since his economic costs are greater than his revenues, he should shut down his shop.
2. Ch 7, Problem 7.5

A firm uses two inputs, capital and labor, to produce output. Its production function exhibits a diminishing marginal rate of technical substitution.

a. If the price of capital and labor services both increase by the same percentage amount (e.g., 20 percent), what will happen to the cost-minimizing input quantities for a given output level?

If the price of both inputs change by the same percentage amount, the slope of the isocost line \((- \frac{w}{r})\) will not change. Since we are holding the level of output fixed, the point at which the isocost line is tangent to the (fixed) isoquant does not change. Therefore, the cost-minimizing quantities of the inputs will not change.

b. If the price of capital increases by 20 percent while the price of labor increases by 10 percent, what will happen to the cost-minimizing input quantities for a given output level?

If the price of capital increases by a larger percentage than the price of labor, the isocost lines become flatter (since \(\frac{w}{r}\) decreases). This means that the point of tangency will move to the southeast as shown in the diagram below:

Another way to think about this is to realize that when the price of capital increases by a larger percentage than the price of labor, labor has become cheaper relative to capital. The firm responds by substituting away from capital in favor of labor.

In sum, the cost-minimizing quantity of labor should increase, and the cost-minimizing quantity of capital should decrease.
3. Suppose the production of digital cameras is characterized by the production function \( Q = LK \), where \( Q \) represents the number of digital cameras produced. Suppose that the price of labor is $10 per unit and the price of capital is $1 per unit.

a. Graph the isoquant for \( Q = 121,000 \).

![Diagram showing the isoquant for Q = 121,000]

b. On the graph you drew for part a, draw several isocost lines including one that is tangent to the isoquant you drew. What is the slope of the isocost lines?

![Diagram showing isocost lines]

The slope of the isocost lines is \(-w/r = -10/1 = -10\).

c. Find the cost-minimizing combination of labor and capital for a manufacturer that wants to produce 121,000 digital cameras. Mark this point on the graph you drew for part a.

We recognize that the production function \( Q = LK \) is Cobb-Douglas; therefore, we will have an interior solution to this problem.

We can begin by setting up the minimization problem:
\[ \min_{L,K} \quad TC = wL + rK \]
\[ \text{s.t.} \quad Q_0 = LK \]

Using the quantity constraint we sub. in L or K into the objective function.

\[
\min_L \quad TC = wL + r \frac{Q_0}{L} \\
\frac{\partial TC}{\partial L} = w - \frac{rQ_0}{L^2} = 0 \\
\text{solving} \\
L = \sqrt{\frac{rQ_0}{w}} \text{ This is the labor demand function} \\
\]

We can sub in for r, w and Q to find \( L^* = 110 \)

Subbing the labor demand into quantity constraint we can obtain the capital demand function.

\( K^* = 121,000 / 110 = 1100 \)

Therefore, the cost-minimizing combination of labor and capital is \( (L^*, K^*) = (110, 1100) \). This point is marked in the graph below:

\[ d. \text{See part c for labor demand function} \]
$L = \sqrt{\frac{rQ_0}{w}}$  
This is the labor demand function

Subbing into constraint: $K = \frac{Q_0}{L} = \frac{Q_0}{rQ_0/w} = \sqrt{\frac{wQ_0}{r}}$

e. Suppose, instead, that the technology for producing digital cameras is described by $Q = (L^{1/2} + K^{1/2})^2$. Now what is the cost-minimizing combination of labor and capital for a manufacturer that wants to produce 121,000 digital camera?

This production function is unfamiliar, so we should first determine whether or not it meets the criteria for an interior solution:

1. Are the isoquants downward sloping? Since $MP_L = \left(\frac{L^{1/2}}{L^{1/2} + K^{1/2}}\right)L^{-1/2} > 0$ and $MP_K = \left(\frac{K^{1/2}}{L^{1/2} + K^{1/2}}\right)K^{-1/2} > 0$, “more is better” of both inputs. Therefore, the isoquants are downward sloping.

2. Do the isoquants exhibit diminishing $MRTS_{L,K}$? Note that $MRTS_{L,K} = MP_L / MP_K = \left(\frac{K}{L}\right)^{1/2}$. We already know that the isoquants are downward sloping, so moving down along an isoquant means increasing $L$ and reducing $K$. If we do that, then the denominator of $MRTS_{L,K}$ increases and the numerator of $MRTS_{L,K}$ decreases; so overall, $MRTS_{L,K}$ decreases as we move down along an isoquant. Therefore, the isoquants do exhibit diminishing $MRTS_{L,K}$.

3. Are the isoquants smooth? Yes, since there is no weird break in the equation for the production function.

4. Do the isoquants touch the axes? Yes. The equation for the isoquant in question is $121,000 = \left(\frac{L^{1/2}}{L^{1/2} + K^{1/2}}\right)^2$, and the input combinations $(121000, 0)$ and $(0, 121000)$ are both points on that isoquant.

In sum, we can go ahead with trying to find an interior solution because criteria 1 through 3 have been met; however, since the isoquants touch the axes, we could get a corner solution.
\[ \text{Min } TC = wL + rL \]
\[ \text{s.t. } Q = (L^\frac{1}{2} + K^\frac{1}{2})^2 \]

Using the quantity constraint we sub. in L or K into the objective function.

Rewriting the quantity constraint: \( K = (Q^\frac{1}{2} - L^\frac{1}{2})^2 \)

\[ \text{Min } TC = wL + r(Q^\frac{1}{2} - L^\frac{1}{2})^2 \]
\[ \frac{\partial TC}{\partial L} = w + 2r(Q^\frac{1}{2} - L^\frac{1}{2})(-\frac{1}{2} L^{-\frac{1}{2}}) = 0 \]
\[ L^\frac{1}{2} = \frac{rQ^{\frac{1}{2}}}{(w + r)} \]
\[ L^* = \frac{r^2Q}{(w + r)^2} = \frac{121,000}{11^2} = 1,000 \]

Subing \( L^* \) back into the quantity constraint: \( K^* = (121,000^\frac{1}{2} - 1000^\frac{1}{2})^2 = 100,000 \).

Therefore, the cost-minimizing combination of labor and capital is \((L^*, K^*) = (1000, 100000)\).

4. Ch 7, Problem 7.14

Suppose a production function is given by \( Q = \min\{L, K\} \). Draw a graph of the demand curve for labor when the firm wants to produce 10 units of output \((Q = 10)\).

The production function \( Q = \min\{L, K\} \) indicates that the inputs are perfect complements. The cost-minimizing combination of labor and capital for a given level of output \( \overline{Q} \) is \((\overline{L}, \overline{K}) = (\overline{Q} / a, \overline{Q} / b)\). In this case, the coefficients, \( a \) and \( b \), are both 1; so, \((L, K) = (\overline{Q}, \overline{Q})\). This indicates that the input demand curve for labor is \( L = Q \) and that the input demand curve for capital is \( K = Q \).

Note that the demand for labor does not depend on the price of labor \( w \). In particular, if the firm wants to produce 10 units of output, its demand for labor is simply \( L = 10 \). We sketch this demand curve below:
To reiterate, the demand curve is vertical because the demand for labor does not vary with the price of labor \( w \).

5. Ch 7 problem 7.19

A plant’s production function is \( Q=2KL+K \). The price of labor services \( w \) is $4 and of capital services \( r \) is $5 per unit.

a) In the short-run, the plant's capital is fixed at \( K = 9 \). Find the amount of labor it must employ to produce \( Q=45 \) units of output.

Since we know \( K \) and \( Q \) we can use the quantity constraint to find the short-run amount of labor needed to produce \( Q=45 \). \( 45 = 2(9)L + 9 \), so \( L^* = 2 \).

b) How much money is the firm sacrificing by not having the ability to choose its level of capital optimally?

The optimal solution is the long-run cost minimizing amounts of \( L \) and \( K \). In the long-run capital is not fixed.

\[
\begin{align*}
\text{Min } & \quad TC = wL + rK \\
\text{s.t.} & \quad Q_0 = 2KL + K
\end{align*}
\]

Rewriting the budget constraint and subbing it into the objective function we get:

From the quantity constraint: \( k = \frac{Q_0}{2(L + 1/2)} \)
\[
\begin{align*}
\text{Min } TC &= wL + r \left( \frac{Q_o}{2(L + \frac{1}{2})} \right) \\
\frac{\partial TC}{\partial L} &= w - \frac{rQ_o}{2(L + \frac{1}{2})^2} = 0 \\
L^* &= \sqrt{\frac{rQ_o}{w}} \frac{1}{2} = \sqrt{\frac{5\times 45}{8}} \frac{1}{2} = 4.8 \\
K^* &= \frac{45}{2(4.8 + \frac{1}{2})} = 4.24
\end{align*}
\]

The firm’s costs in the long-run are: 
\[ TC = 4\times 4.8 + 5\times 4.24 = 40.4 \]
The firm’s costs in the short-run are: 
\[ TC = 4\times 2 + 5\times 9 = 53 \]

Therefore, the firm loses about $12.6 because of the constraint on capital.

6. A firm’s long-run total cost curve is \[ TC(Q) = 40Q - 10Q^2 + Q^3. \] Over what range of output does the production function exhibit economies of scale? Over what range does it exhibit diseconomies of scale? At what quantity is minimum efficient scale?

Economies of scale occur when \( MC < AC \); diseconomies of scale occur when \( MC > AC \); and minimum efficient scale occurs where \( MC = AC \) (if \( MC \) is increasing at that point). Clearly, the first step is to calculate \( MC \) and \( AC \):

\[
\begin{align*}
MC(Q) &= 40 - 20Q + 3Q^2 \\
AC(Q) &= 40 - 10Q + Q^2
\end{align*}
\]

Economies of scale occur when \( MC < AC \):
\[
40 - 20Q + 3Q^2 < 40 - 10Q + Q^2 \]
\[
2Q^2 - 10Q < 0 \\
Q(2Q - 10) < 0
\]

Hence, we have economies of scale when \( Q < 5 \).

Similarly, diseconomies of scale occur when \( MC > AC \). Hence, we have diseconomies of scale when \( Q > 5 \).
And finally, since $AC$ is decreasing (economies of scale) for $Q < 5$ and increasing (diseconomies of scale) for $Q > 5$, it must be the case that minimum $AC$ occurs at $Q = 5$. Hence, the minimum efficient scale is $Q = 5$.

7. Ch 8 problem 8.10: For each of the total cost functions, write the expression for the total fixed cost, average variable cost, and marginal cost:

a. $TC(Q) = 10Q$
   
   $TFC = 0, AVC = 10, MC = 10$.

b. $TC(Q) = 160 + 10Q$
   
   $TFC = 160, AVC = 10, MC = 10$.

c. $TC(Q) = 10Q^2$
   
   $TFC = 0, AVC = 10Q, MC = 20Q$.

d. $TC(Q) = 160 + 10Q^{1/2}$
   
   $TFC = 160, AVC = 10Q, MC = 5/Q$.

8. Ch 8 problem 8.11: A firm produces a product with labor and capital as inputs. The production function is described by $Q = LK$. Let $w = 1$ and $r = 1$. Find the equation for the firm's long-run total cost curve as a function of quantity $Q$.

From Question 3 we know the input demand functions:

$L = \sqrt{\frac{rQ_0}{w}}$

$K = \sqrt{\frac{wQ_0}{r}}$

We sub these back into the total cost function:

$TC = w\sqrt{\frac{rQ_0}{w}} + r\sqrt{\frac{wQ_0}{r}}$

$TC = \sqrt{wrQ_0} + \sqrt{wrQ_0} = 2\sqrt{wrQ_0}$

when $r = 1$ and $w = 1$

$TC = 2\sqrt{Q_0}$
9. Ch 8 problem 8.13: A firm produces a product with labor and capital. Its production function is described by \( Q = L + K \). Let \( w = 1 \) and \( r = 1 \) be the prices of labor and capital respectively.

a. Find the equation for the firm’s long-run total cost curve as a function of quantity \( Q \) when the prices of labor and capital are \( w = 1 \) and \( r = 1 \).

With a linear production function, the firm operates at a corner point depending on whether \( w < r \) or \( w > r \). If \( w < r \), the firm uses only labor and thus sets \( L = Q \). In this case, the total cost (including the fixed cost) is \( wQ \). If \( w > r \), the firm uses only capital and thus sets \( K = Q \). In this case, the total cost is \( rQ \). When \( w = r = 1 \), the firm is indifferent among combinations of \( L \) and \( K \) that make \( L + K = 10 \). Thus, we have \( TC(Q) = Q \).

b. Find the solution to the firm’s short-run cost-minimization problem when capital is fixed at a quantity of 5 units (i.e. \( K = 5 \)), and \( w = 1 \) and \( r = 1 \). Derive the equation for the firm’s short-run total cost curve, average cost curve and marginal cost curve as a function of \( Q \).

When capital is fixed at 5 units, the firm’s output would be given by \( Q = 5 + L \). If the firm wants to produce \( Q < 5 \) units of output, it must produce 5 units and throw away \( 5 - Q \) of them. The total cost of producing fewer than 5 units is constant and equal to $5, the cost of the fixed capital. For \( Q > 5 \) units, the firm increases its output by increasing its use of labor. In particular, to produce \( Q \) units of output, the firm uses \( Q - 5 \) units of labor, for a cost of \( Q - 5 \), and 5 units of capital, for a cost of 5. Thus, \( STC(Q) = Q - 5 + 5 = Q \).