Magnetic Resonance Imaging

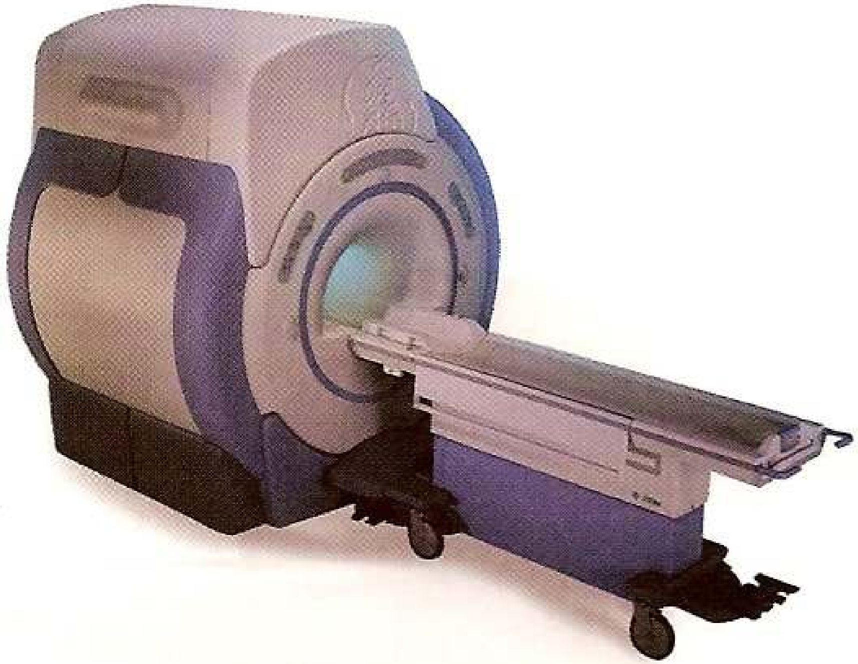
Bryan Killett

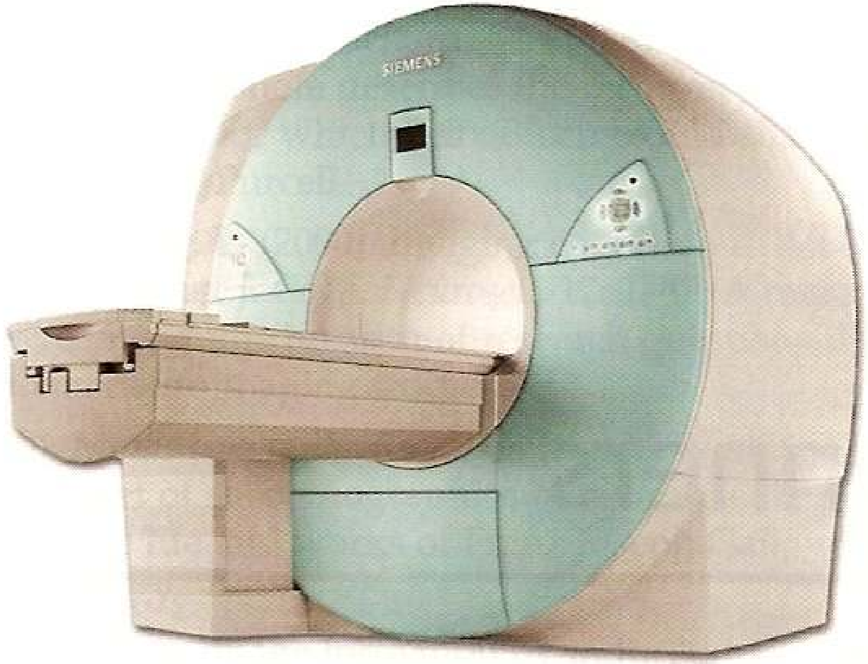
April 28, 2006

MRI Overview

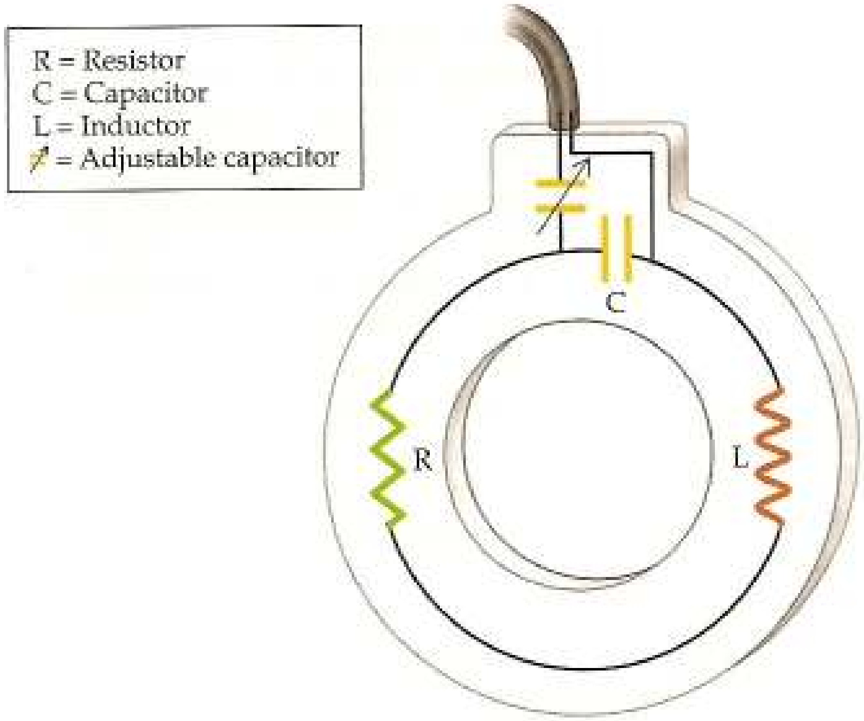
- A strong magnetic field forces nuclear spins into one of two states: a high energy state and a low energy state.
- Radio frequency pulses excite some nuclei into high energy states. These nuclei emit radio waves at specific frequencies as they fall back down to lower energy states.
- Non-uniform 'gradient' magnetic fields are added to the primary field. As a result, nuclei at different locations emit radio waves at different frequencies.
- This 'frequency space' data is inverted using an inverse FFT to obtain a 'coordinate space' image of the object.

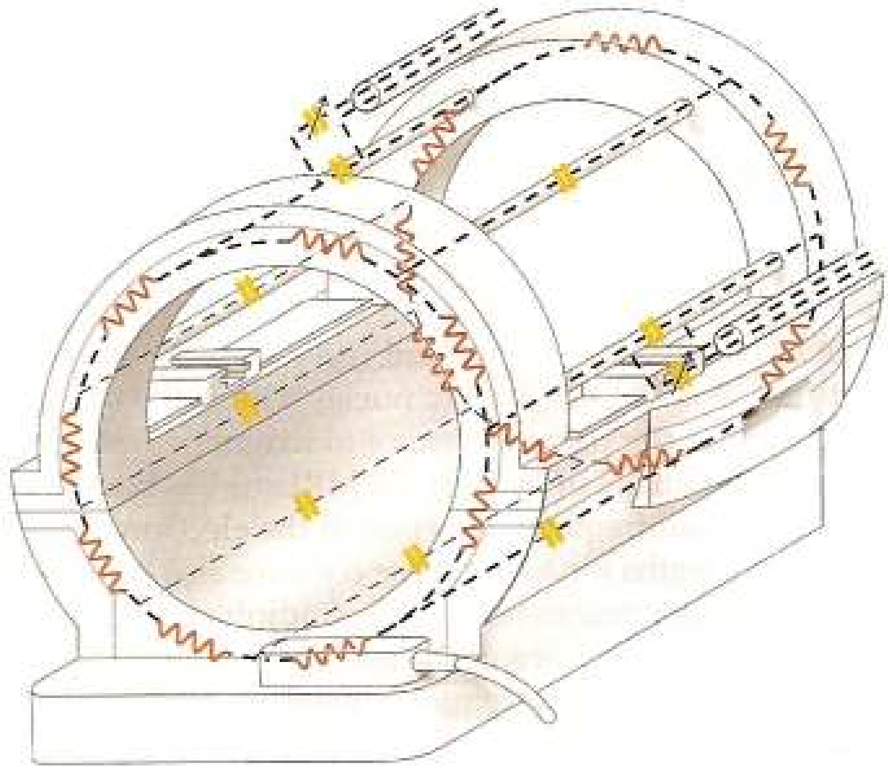
MRI Overview



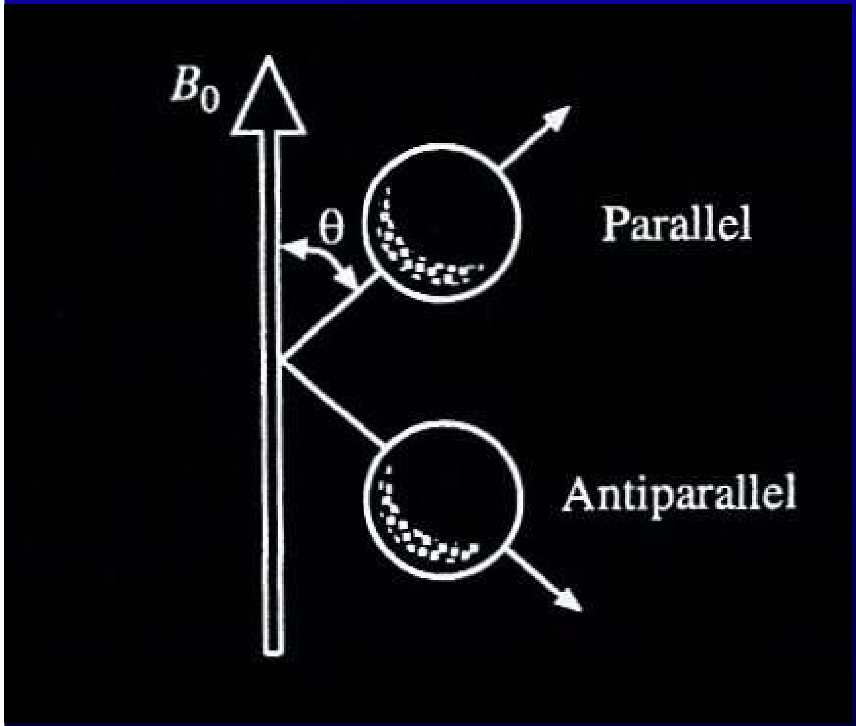


MRI Overview





Nuclear Spins in a Magnetic Field

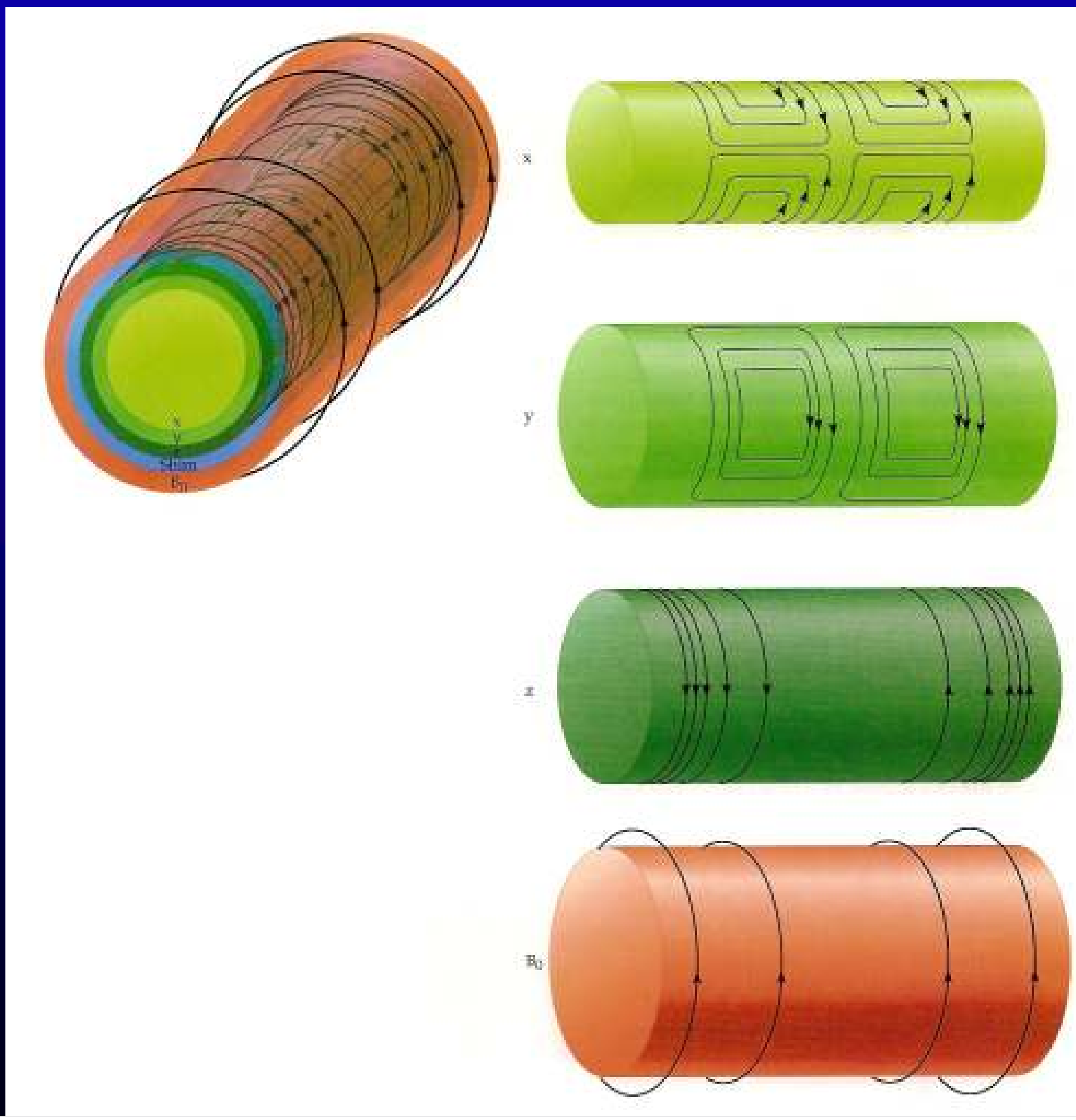


- Due to quantum mechanics, nuclear spins are not free to align directly along the field.
- Hydrogen nuclei, for example, can only align in two directions: ‘parallel’ or ‘antiparallel’.

Nuclear Spins in a Magnetic Field

- Each nucleus precesses (like a top) around the primary field at its 'Larmor' frequency: γB
- γ is known as the gyromagnetic ratio, which is unique for each type of nucleus.
- For example, γ for hydrogen is 42.58 MHz/T
- An RF pulse at the Larmor frequency will excite a nuclei from its lower (parallel) state to its upper (antiparallel) state. The nuclei will then radiate at its Larmor frequency as it falls back down to its lower energy state.

Localization of Spins in 3D



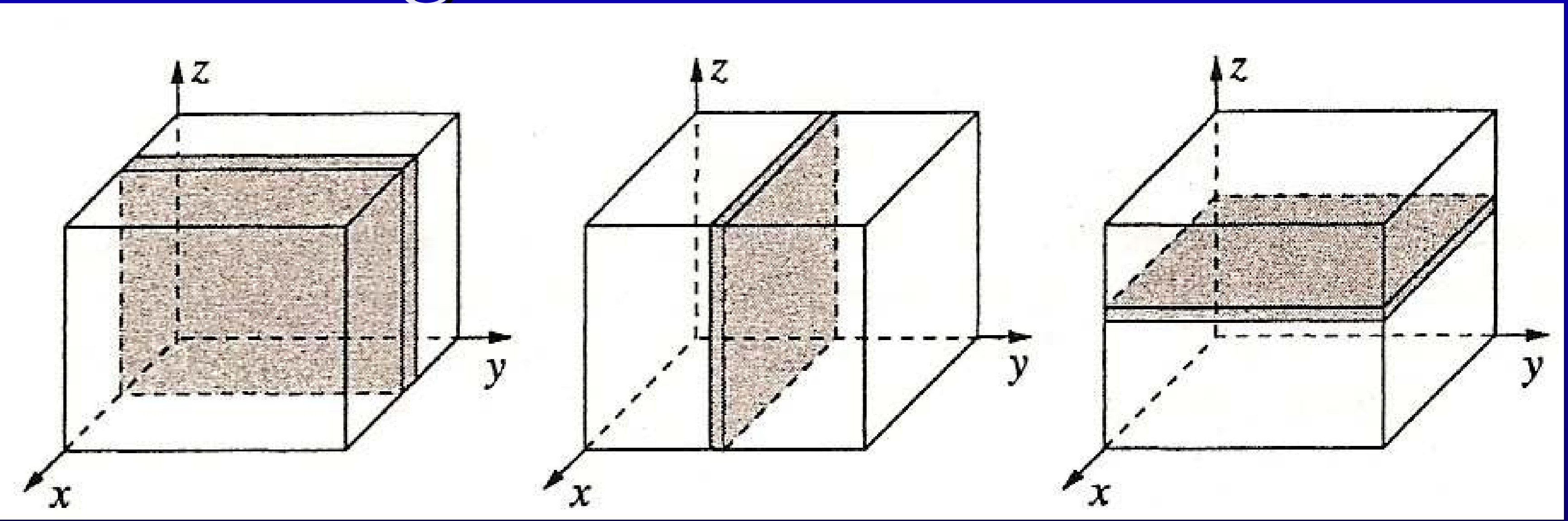
-> X Gradient Field Coil

-> Y Gradient Field Coil

-> Z Gradient Field Coil

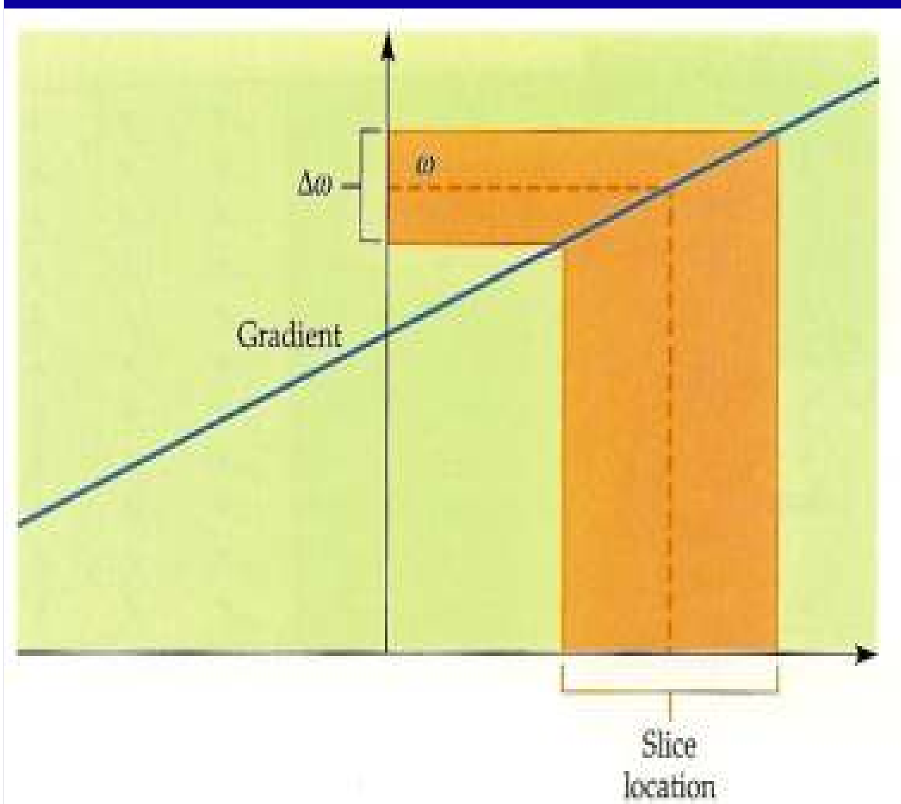
-> Primary Field Coil

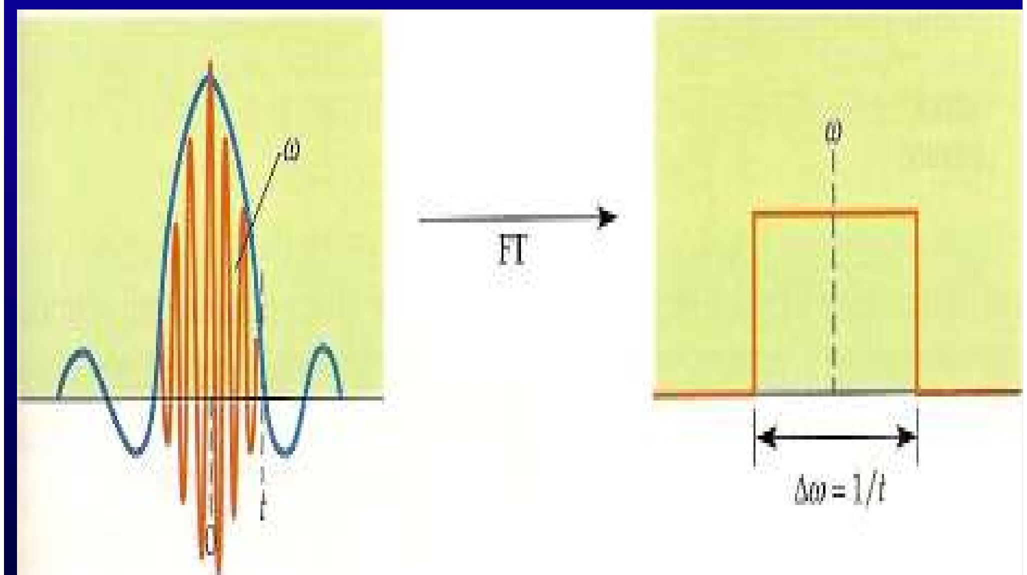
First Dimension: Selecting a Slice via RF Excitation



- Idea: excite nuclei only if they lie in a thin 2D slice of the object.
- To accomplish this, turn on the gradient field in the desired direction only while the RF excitation pulse is activated.

First Dimension: Selecting a Slice via RF Excitation

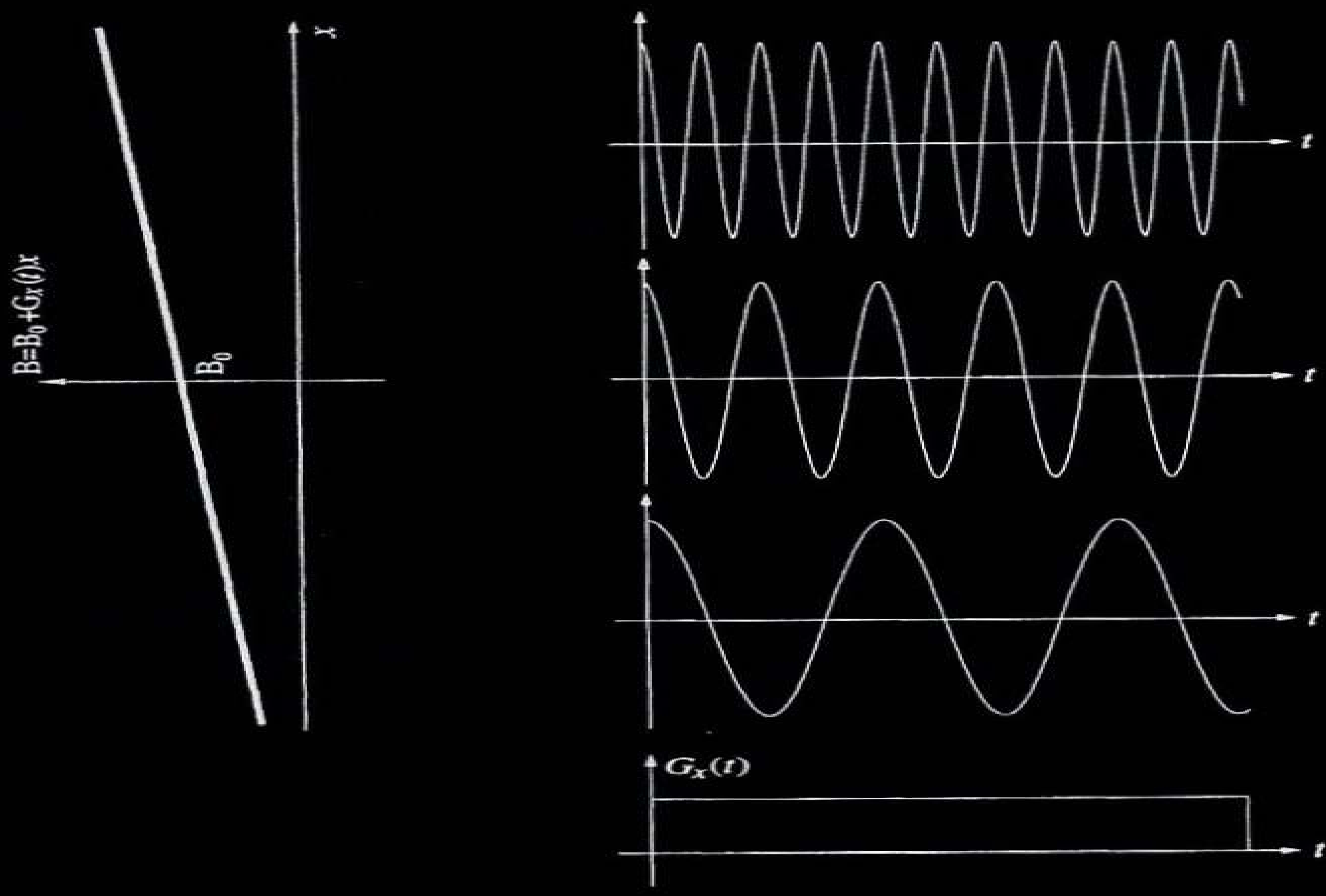




Second Dimension: Frequency Encoding

- Idea: the Larmor frequency (γB) is linearly dependant on the magnetic field strength, so a magnetic gradient will record a nucleus's position along that gradient by shifting its Larmor frequency.
- To accomplish this, turn on the appropriate gradient *after* the RF excitation pulse is activated, and keep it on until the signal has decayed away.

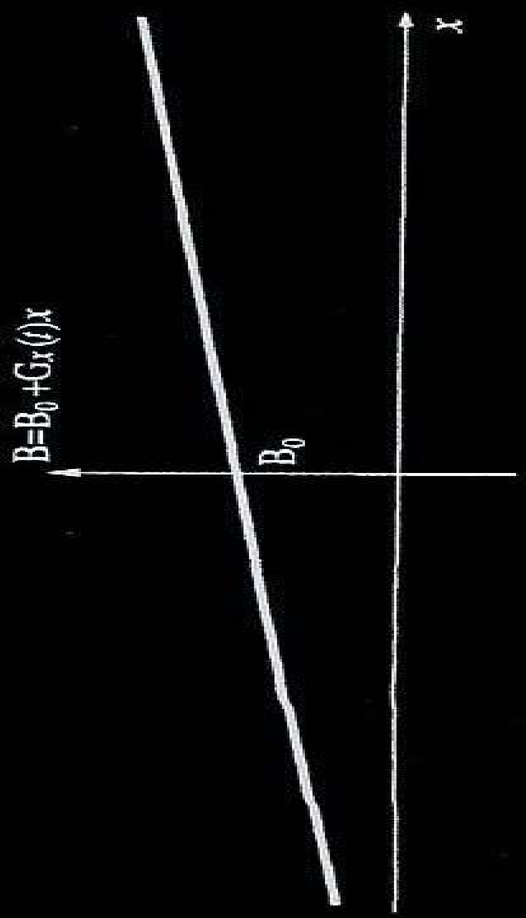
Second Dimension: Frequency Encoding

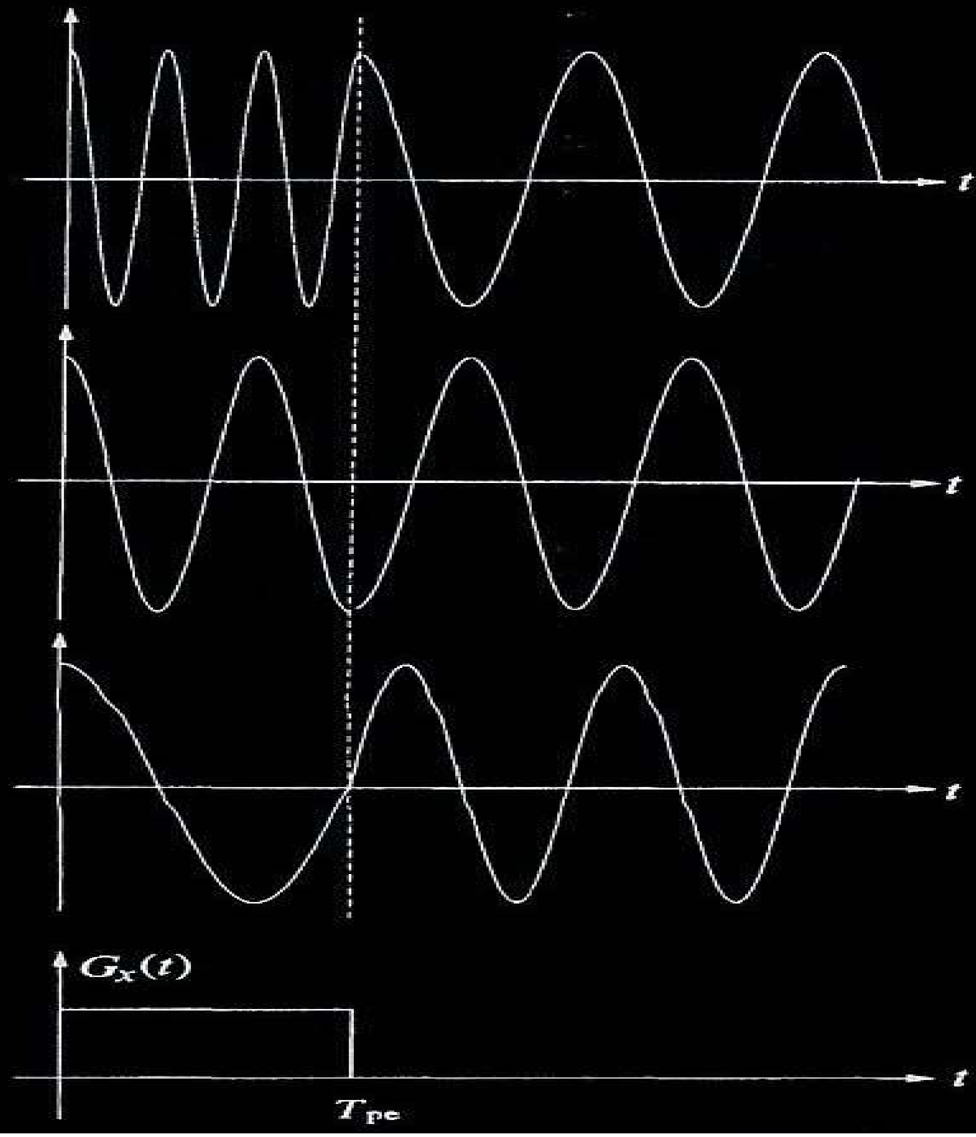


Third Dimension: Phase Encoding

- Idea: let the phase of a nucleus's signal be dependent on its position.
- To accomplish this, turn on the appropriate gradient *after* the RF excitation pulse is activated, but turn it off after a short amount of time.

Third Dimension: Phase Encoding





How to Image a 2D Slice Along the Z-axis

- Pick a slice position z_0 and a slice width Δz. Turn on the z-gradient coil.
- Turn on an RF excitation pulse that excites nuclei in the desired slice. Immediately afterwards, turn off the z-gradient coil.
- Simultaneously turn on the x and y gradient coils. After a short amount of time, turn off the y-gradient coil. This will result in an image with frequency encoding in the x-direction and phase encoding in the y-direction.

Image Reconstruction

Definitions:

- Spin density: ρ(x,y,z). (This is the image we want)
- MR signal: S(t). (This is the data we record)
- Primary field: $B_0(t)$. (The fields are the model)
- Gradient fields: $G_x(t)$, $G_y(t)$
- "Slice Magnetization" M(x,y):

$$M(x,y) \propto \int_{z_0-\frac{\Delta z}{2}}^{z_0+\frac{\Delta z}{2}} dz * \rho(x,y,z)$$

Image Reconstruction

1D Fourier Transform:

$$S(t) = \int dx * M(x) \exp(-i\omega t)$$

Insert the definition of the resonance frequency:

$$\omega t = 2\pi\gamma B t = 2\pi\gamma\left(B_0 + G_x x + G_y y\right)t$$

To account for time-dependent gradient fields, integrate:

$$\omega t \rightarrow 2\pi\gamma \exp\left(-i\gamma\int_0^t d\tau * \left(B_0 + G_x(\tau)x + G_y(\tau)y\right)\right)$$

Image Reconstruction

Finally, generalize to 3D to obtain the forward problem:

$$S(t) = \iint dxdy * M(x,y)\exp\left(-2\pi i\gamma\int_0^t d\tau * \left(B_0 + G_x(\tau)x + G_y(\tau)y\right)\right)$$

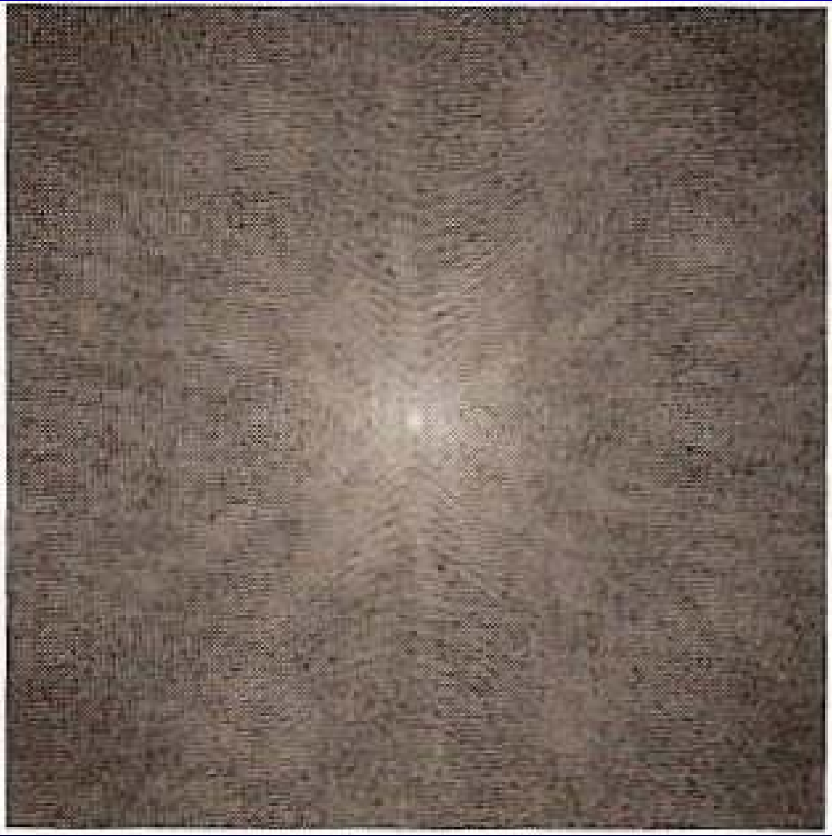
Most commonly, MR researchers use “k-space”:

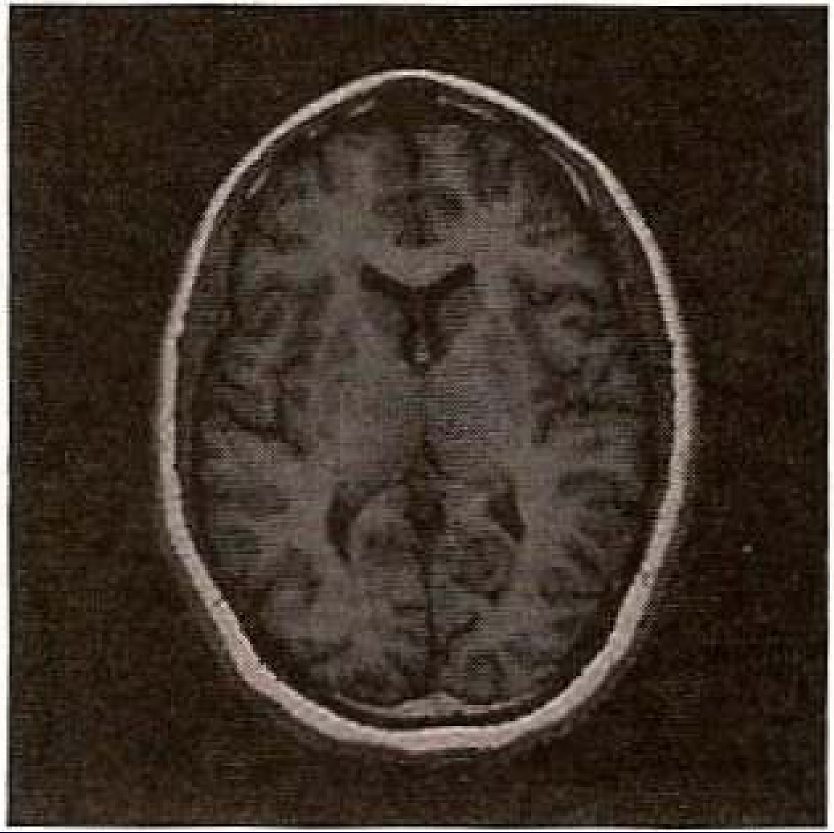
$$S(t) \rightarrow S(k_x, k_y)$$

Where $\mathrm{k_x}$ and $\mathrm{k_y}$ are defined as:

$$\mathrm{k_x} = \int_0^t d\tau * G_x(\tau) \text{ and } \mathrm{k_y} = \int_0^t d\tau * G_y(\tau)$$

From k-space to coordinate space





References

- Principles of Magnetic Resonance Imaging: A Signal Processing Perspective, by Zhi-Pei Liang, et. al.
- Functional Magnetic Resonance Imaging, by Scott A. Huettel, et. al.
- Magnetic Resonance Imaging, by E. Mark Haacke, et. al.