CT Scans: techniques and inversions

Greg Bensen

Geophysical Inverse Theory

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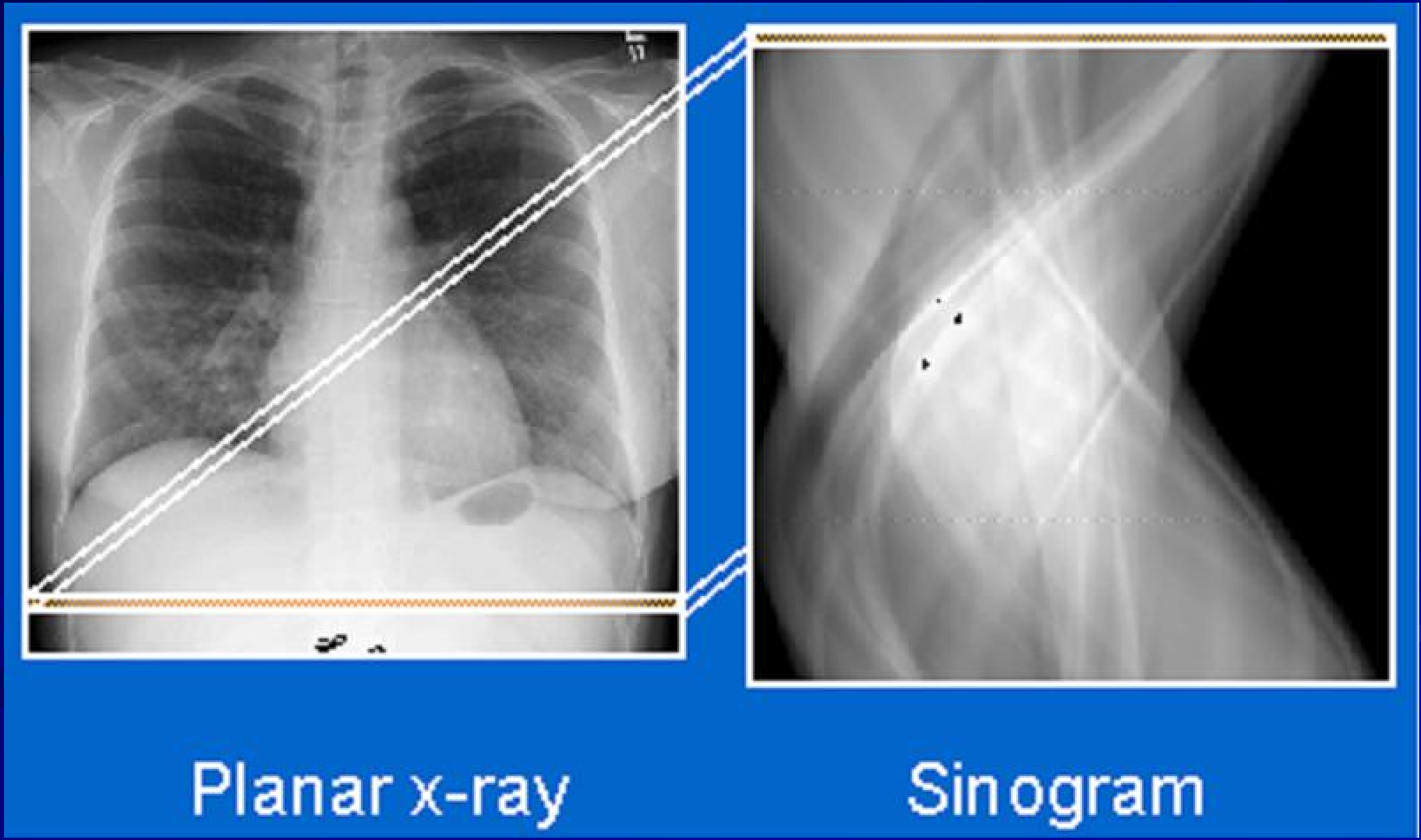
Outline

- Brief history
- Relevance to inverse theory
- Equipment (experiment setup)
- Mathematical underpinnings
- Examples
- Tradeoffs & limitations

Technique History

- Started with normal 2D x-rays (shadow-graphs)
- Base theory developed with 1917 “Radon transform” and work in the 1960’s facilitated by early computers
- 1972: US patent issued, Nobel prize awarded to same scientists in 1979
- Since then, most advances have been in instrument design (4 major implementations)
- Also improvements in computing power and minor algorithm improvements

Planar X-ray or “shadowgraph” and sinogram



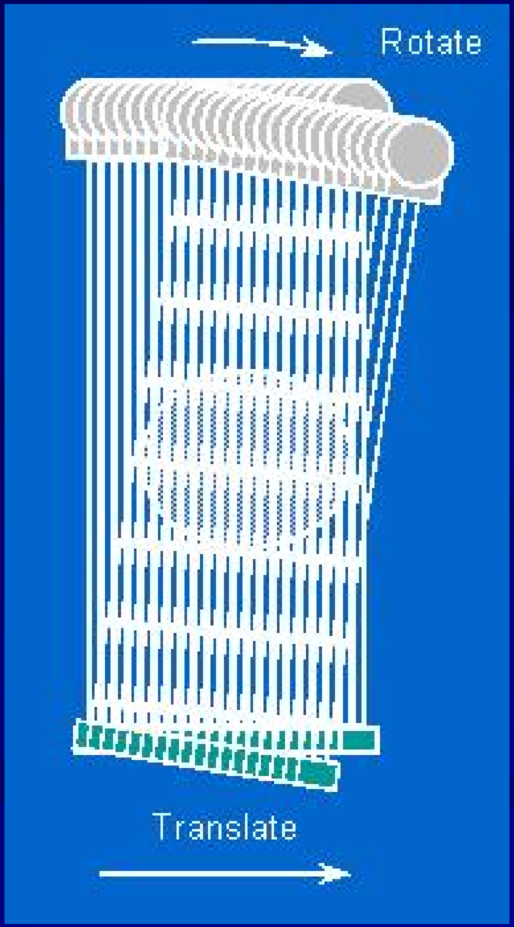
Relevance to Inverse Theory

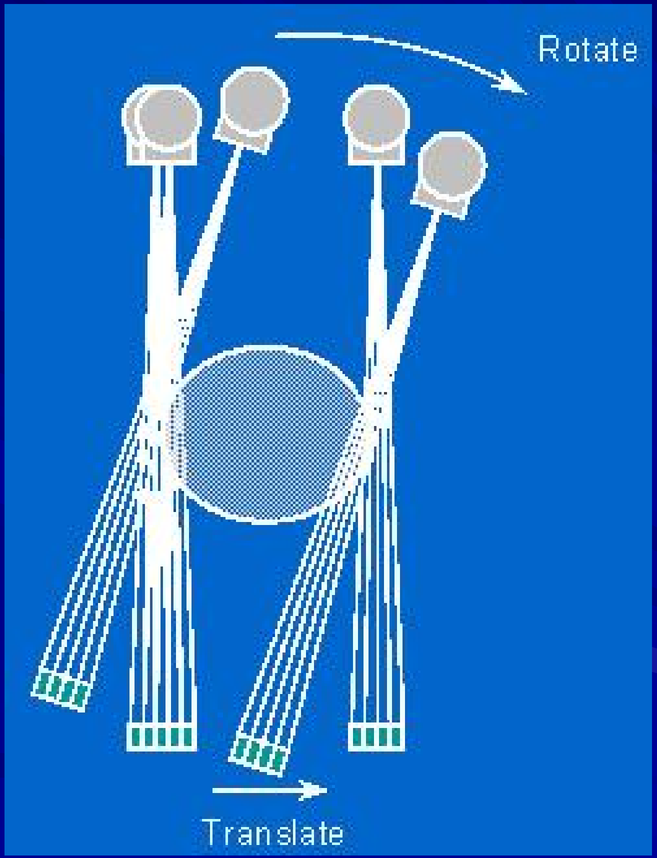
- What's measured?
  - Measures linear attenuation coefficient at a given angle which is a measure of x-ray absorption
- What to we want to know?
  - 2D or 3D structure, distinguishing different materials with high precision
- How do we do it?
  - Take these x-ray strips, apply an application specific filter or two and take the sum over 180˚

The equipment

- Has gone through 4 generations.
  - Progressively faster (4 minutes to 0.5 sec)
  - More accurate
  - More complicated processing

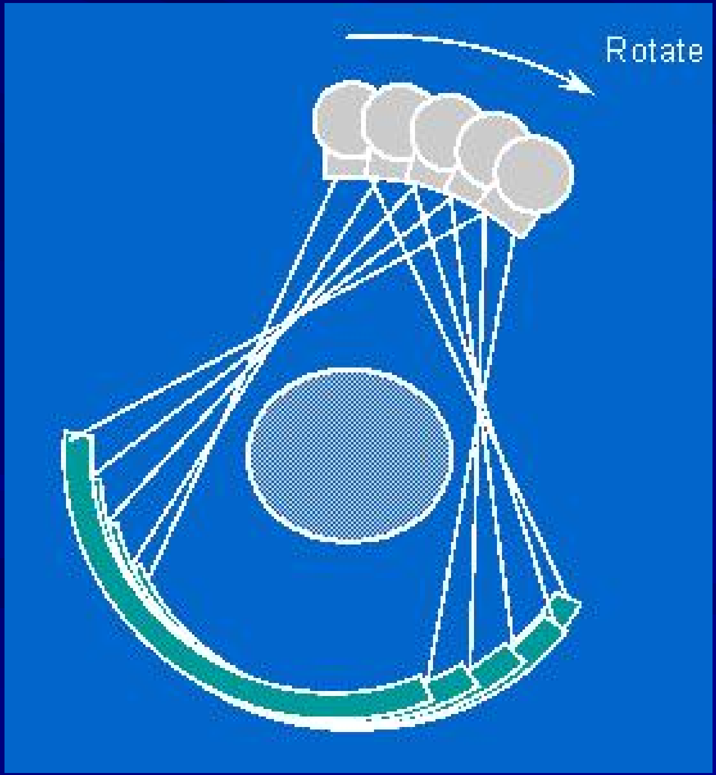
Generation 1: point source and detector
generation 2: narrow fan beam, array of detectors

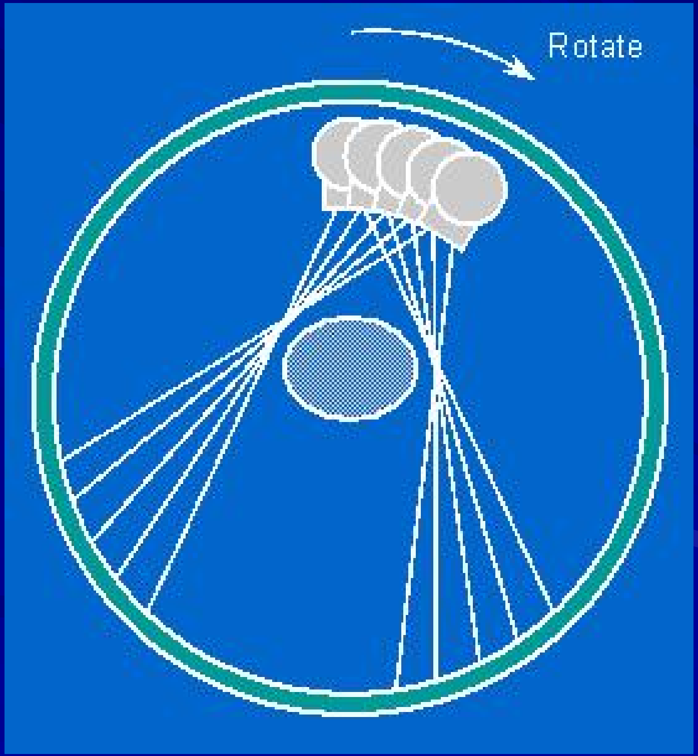




Third and fourth generation arrived at the same time. Third has rotating source and detector arrays. Most common equipment.

Fourth generation:
Array of sources and detectors. Nearly instantaneous, less moving parts and fewer artifacts.

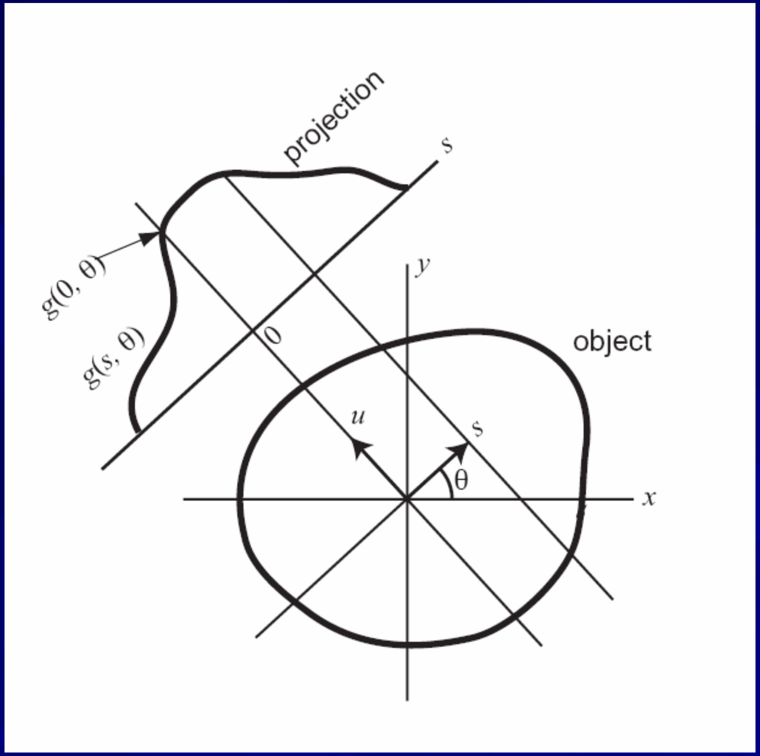




The math

- Radon transformation (Radon, 1917) is the base theory stating that a path through 2D space can be brought to an integral over 1 variable.
- Followed by Radio Telescope fan beam measurement inversion (Bracewell and Riddle 1967). Good for homogeneous and inhomogeneous scan distribution.
- Finally brought to medical technology for radiographs and electron micrographs; used for reconstruction of 2D maps to 3D (Ramanchandran and Lakshminarayanan, 1971).

Radon Transformation



$$g(0,\theta) = \iint_{-\infty}^{\infty} f(x,y)\,\delta(x\cos\theta + y\sin\theta)\,dx\,dy$$

Observation at (0,θ) is an integral through the body f(x,y) times a delta function defined by the path.

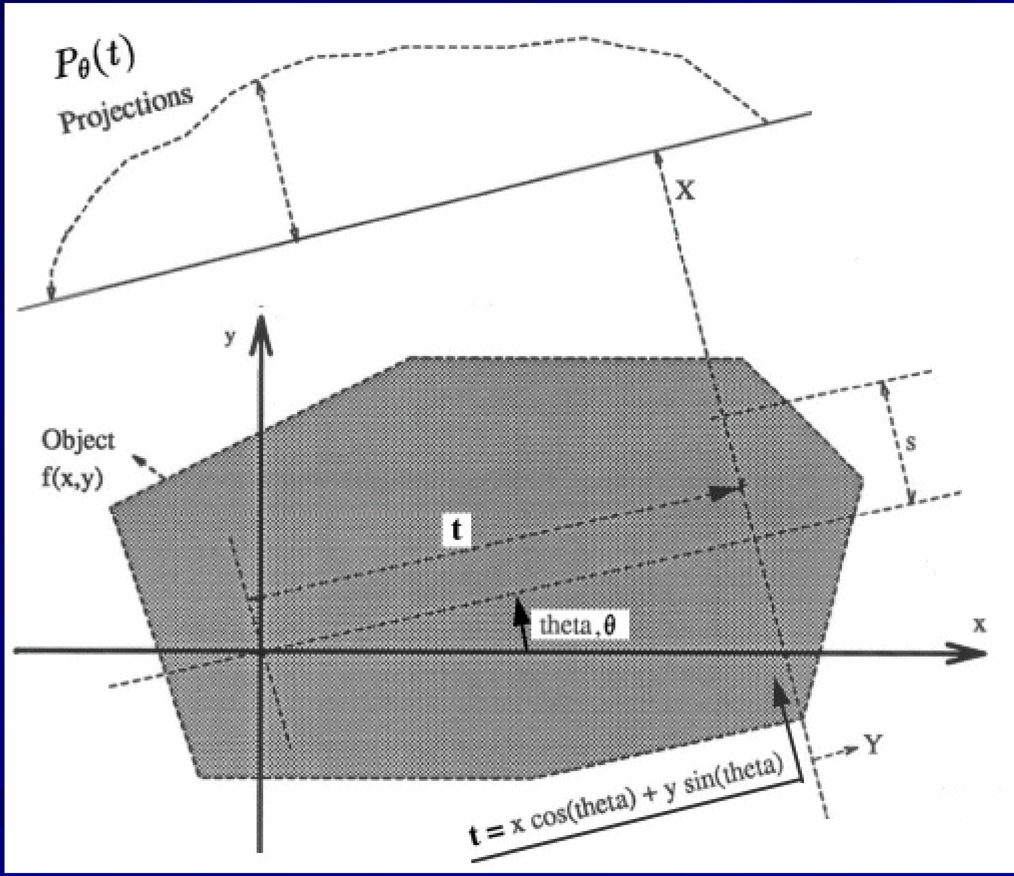
or for all s

$$g(s,\theta) = \iint_{-\infty}^{\infty} f(x,y)\,\delta(x\cos\theta + y\sin\theta - s)\,dx\,dy$$

and after some tricks, get it as just a function of ‘u’

$$g(s,\theta) = \int_{-\infty}^{\infty} f(s\cos\theta - u\sin\theta,\, s\sin\theta + u\cos\theta)\,du$$

Scan inversion



Projection into Fourier space

$$P_\theta(t) = \int_{(\theta,t)line} f(x,y)ds \qquad (2.1)$$

$$P_\theta(t) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\ [\delta(x\cos(\theta) + y\sin(\theta) - t)\ dx\ dy] \qquad (2.2)$$

$$S_\theta(w) = \int_{-\infty}^{\infty} P_\theta(t)e^{-j2\pi wt}dt \qquad (2.4)$$

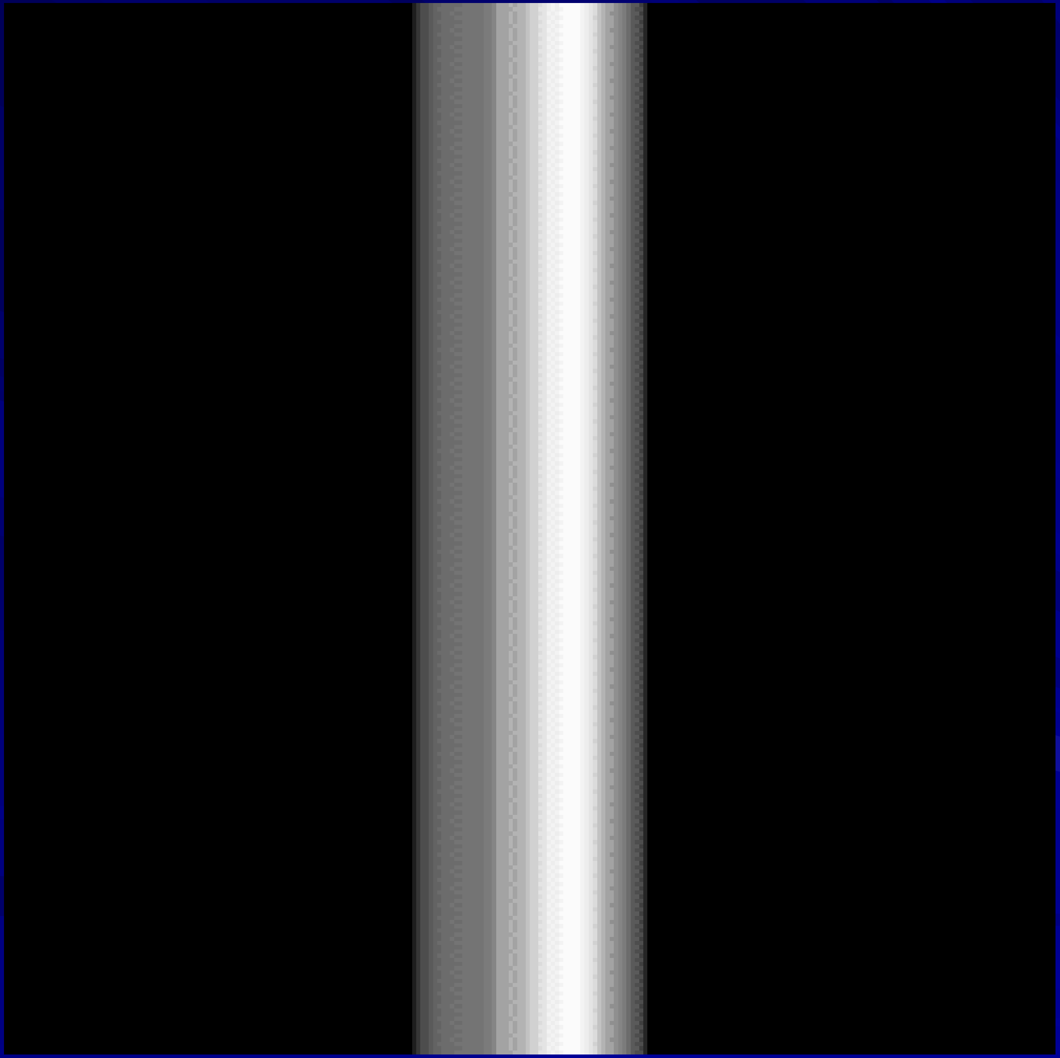
Fourier transform of a projection at a given (theta, t)

One more, hang in there

$$S_\theta(w) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)e^{-j2\pi wt}dxdy \qquad (2.8)$$

$$f(x,y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} S_\theta(w)e^{j2\pi(ux+vy)}dudv \qquad (2.10)$$

Back projection in action



Filtered back projection

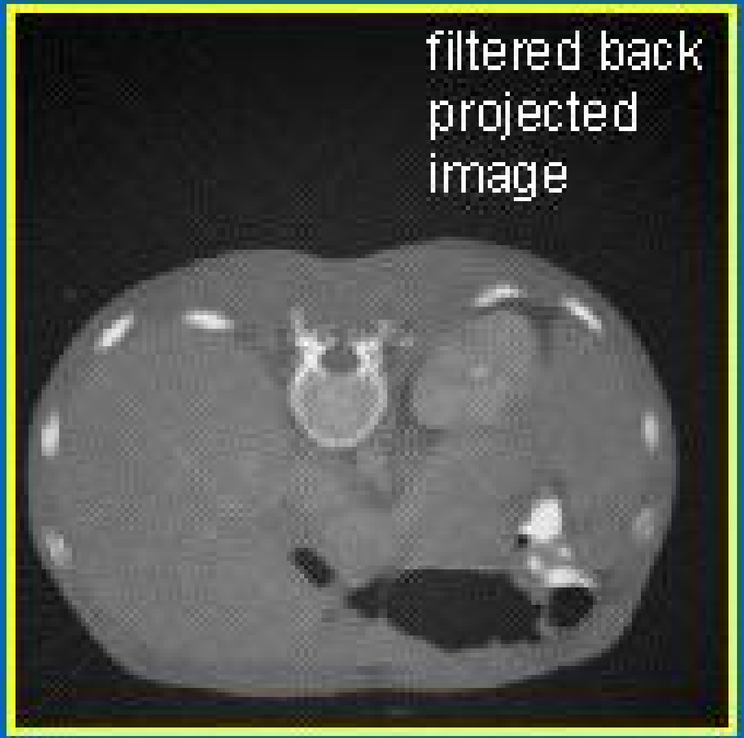
- Previous slides were continuous, this is a discrete 4 step process
  - Find the Fourier Transform of the 1D projections
  - Filter these projections through frequency domain multiplication

$$Q_\theta(t) = \int_{-\infty}^{\infty} S_\theta(w)\,|w|\,e^{i2\pi wt}dw \qquad (2.19)$$

  - Compute IFFT
  - Back project through summation, not integration

$$f(x,y) = \sum_{i=1}^{K} Q_{\theta_i}(x\cos(\theta_i) + y\sin(\theta_i)) \qquad (2.21)$$

Filtered and unfiltered back projection



Noise sources & limitations

- Noise Sources
  - Some random noise is always present as in most measurements
  - Metallic objects
  - Patient movement
  - Big folks
  - Instrument artifacts
- Limitations
  - Limit on permissible radiation dose
  - Requirement of near real time results

Tradeoffs

- Tradeoffs exist between
  - Slice thickness and z-accuracy
  - Resolution and dose size
  - High filter frequency and noise
  - To a lesser extent, speed (real time) and image quality

Last slide

- Back projection is a powerful and quick technique for 2D slice reconstruction in x-ray tomography
- Not ideal for seismic tomography because of
  - Strongly inhomogeneous path distribution
  - Off path propagation
  - Broad sensitivity regions
  - Relatively smaller amplitude variations

References

- Most figures from http://www.impactscan.org/slides/xrayct/index.htm

Animation from http://cryoem.berkeley.edu/~nieder/em_for_dummies/back_projection.html

Equations from http://www.sv.vt.edu/xray_ct/parallel/Parallel_CT.html