

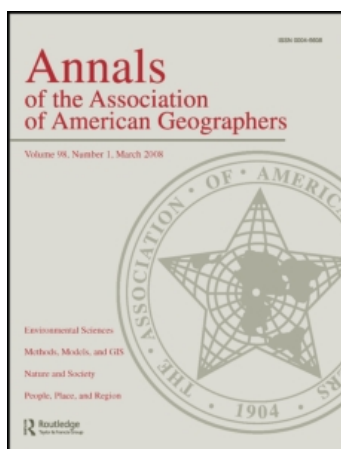
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FUZZY SETS AND SPATIAL CHOICE*

JOHN S. PIPKIN

ABSTRACT. Vagueness and ambiguities in individual perception pose conceptual and technical problems in the study of spatial choice. These problems may be attributed partially to deficiencies in the set-theoretic structures underlying our models of spatial cognition and evaluation. L. Zadeh's theory of fuzzy sets provides an explicit account of such ambiguities. The potential of this account includes applications in the areas of algebraic and probabilistic choice theory, distance perception, and in temporal and population aggregation.

INTENSIVE study of spatial patterns and behavior in terms of individual decision-making has been under way for more than a decade. Recent preoccupations and results are epitomized in the volume by Golledge and Rushton.¹ This, and most other recent work in behavioral geography, leans heavily on several crucial assumptions regarding 1) the existence or at least the constructability of action-spaces, life-spaces, mental maps, cognitive opportunity sets and the like; and 2) a value-based epistemology integrating these constructs with postulates on preference, utility, and choice. The power of these assumptions is evident: for example they underpin the scaling methodologies which have dominated recent work. The Golledge and Rushton volume, however, juxtaposes such technical analyses with other contributions, such as those of Burnett and Gould, which reveal continuing insecurity in the ontology and epistemology on which these assumptions are based, especially as they pertain to our most conspicuous single theme—spatial preference and choice.²

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* The partial support of the National Science Foundation, Grant SOC76-15525, is gratefully acknowledged.

¹ R. G. Golledge and G. Rushton, *Spatial Choice and Spatial Behavior* (Columbus: Ohio State University Press, 1976).

² K. P. Burnett, "Behavioral Geography and the Philosophy of Mind," in Golledge and Rushton, op. cit., footnote 1, pp. 23-48; and P. Gould, "Cultivating the Garden: A Commentary and Critique on Some

In view of the continuing scrutiny of the foundations of behavioral geography, it is surprising that little attention has been paid to the implications and limitations imposed by the set-theoretic bases of the assumptions mentioned above. The categories of our descriptive terminology are Boolean: the principle of the excluded middle holds. This principle governs our descriptive partitions of geographic space (into opportunity sets, for example), and the partitions we impute to cognitive space (into choice sets, for example). It also governs the descriptive predicates we assign to sites (for example, the attribute dimensions of shopping alternatives). Our value-based explanatory frameworks, such as choice functions and binary preference algebras, obey a truth-functional language based on the same principle. The salient assumption of traditional set theory is that set assignments are unambiguous, that is, set characteristic functions assume the values 0 or 1. Using sets and their complements this assumption implies that all universes of entities can be partitioned into exhaustive and mutually exclusive sets.

Set-assignment problems in the analysis of spatial choice and repetitive travel are well-known. Examples are the difficulty of establishing classes of destinations or of their attributes, and of determining the membership or nonmembership of pairs of sites in binary preference or indifference relations. The preference-utility-choice structures of consumer theory are purely Boolean, while probabilistic models of site choice entail a multinomial dis-

Multidimensional Speculations," in Golledge and Rushton, op. cit., footnote 1, pp. 83-94.

tribution whose parameter n counts the number of sites in a Boolean choice set. There are formidable difficulties in establishing such sets objectively for, say, grocery, drug, or newspaper purchases. It is also very difficult to credit consumers with the customary, unambiguous set assignments in perceived choice sets and subjective preference relations. These problems are viewed frequently as matters of operational definition or of measurement. At least part of their intractability, however, results from our commitment to traditional set theory.

It would be absurd to claim that reasonably adequate descriptive and explanatory accounts of spatial choice are inconsistent with the principle of the excluded middle or with Boolean truth-functional logics. The classical choice theories of economics and psychology have been successfully phrased in these terms. However, it is becoming increasingly doubtful that these classical tools are capable of providing a fully adequate account of the classes of vagueness and ambiguity uncovered by research on spatial cognition. Evidence on the intransitivity and temporal inconsistencies of preference relations may be accounted for in terms of ambiguous (vacillating) assignments in binary preference relations. I feel that simple introspection (or more sophisticated arguments from cognitive psychology) are adequate to convince us of occasional vagueness and ambiguity in the constitution of spatial choice sets.

The principle of the excluded middle in set-assignments and in truth values has been central to most logical calculi from the time of Aristotle to the present. Dissatisfaction with the "tyranny of either-or" is also of some antiquity. It is clearly revealed in the philosophy of Hegel; it has found recent expression in Georgescu-Roegen's analysis of nonarithmomorphic sets and in the development of three-, many-, and infinitely-many valued logics.³ A highly signifi-

³ N. Georgescu-Roegen, *The Entropy Law and the Economic Process* (Cambridge: Harvard University Press, 1972); S. Gale, "Conjectures on Many-Valued Logic, Regions, and Criteria for Conflict Resolution," Working Paper No. 10, *Research on Metropolitan Change and Conflict Resolution*, (Peace Science Department, University of Pennsylvania, 1975); S. C. Kleene, *Introduction to Metamathematics* (Amsterdam: North Holland Publishing Co., 1952); and J. Lucasiewicz, "Philosophical Remarks on Many-Values Systems of Propositional Logic," in S. McCall, ed., *Polish Logic, 1920-1939* (Oxford: Clarendon Press, 1967).

cant extension of the concept of set has been developed by Zadeh.⁴ He provides a uniquely specific and practical account of the vagueness and ambiguity in predicates which forms our main intuitive objection to the principle of the excluded middle. A fuzzy set A in a space X is defined by a characteristic or membership function μ_A whose values are not restricted to zero and one; in practice continuous functions are often used. This dispenses with the traditional Boolean condition of unambiguous membership or nonmembership. The value $\mu_A(x)$ is taken to measure the degree of membership of x in A .

In the past ten years the potential of fuzzy sets has been appraised in very diverse areas including biology, decision theory, cognitive psychology, pattern recognition, natural and machine languages, communications theory, logic, and engineering.⁵ A recent symposium addressed the relevance of fuzzy sets in cognitive and decision processes.⁶ Gale was among the first geographers to evaluate the utility of non-Boolean set theory in explanations of spatial behavior.⁷ He has appraised Zadeh's work in a general context of nonbinary logics, and in the specific context of the analysis of contingency tables. His primary concerns have been applications in alternative logics of explanation and approaches to regionalization and conflict resolution.

A glance at the literature on fuzzy sets suggests a very wide range of potential applications in the study of spatial choice and recurrent urban travel. For example, Tanaka and Mizu-

⁴ L. A. Zadeh, "Fuzzy Sets," *Information and Control*, Vol. 8 (1965), pp. 338-53; and idem, "Fuzzy Algorithms," *Information and Control*, Vol. 12 (1968), pp. 94-102.

⁵ For example: L. A. Zadeh, "Biological Application of the Theory of Fuzzy Sets and Systems," in L. D. Proctor, ed., *Biocybernetics of the Central Nervous System* (Boston: Little, Brown and Co., 1969), pp. 199-206; L. A. Zadeh, K. Fu, K. Tanaka, and M. Shimura, *Fuzzy Sets and Their Applications to Cognitive and Decision Processes* (New York: Academic Press, 1975); T. Kitagawa, "Fuzziness in Informative Logics," in Zadeh, Fu, et al., op. cit., pp. 97-124; and L. Zadeh, "Quantitative Fuzzy Semantics," *Information Sciences*, Vol. 3 (1971), pp. 159-76.

⁶ Zadeh, Fu, et al., op. cit., footnote 5.

⁷ Gale, op. cit., footnote 3; and idem, "Inexactness, Fuzzy Sets and the Foundations of Behavioral Geography," *Geographical Analysis*, Vol. 4 (1972), pp. 337-49.

moto give what is essentially an explicit fuzzy formulation of an urban auto-trip from the driver's viewpoint.⁸ The objective of my paper is to evaluate fuzzy-set theory in the context of the traditional value-based account of spatial choice. As indicated below, this approach is quite conservative. The next section outlines the interpretation of the inexactness in fuzzy sets. The following sections are structured according to Luce and Suppes' dichotomy between algebraic and probabilistic accounts of individual acts of choice.⁹

SOURCES OF INEXACTNESS IN FUZZY SETS

Various accounts have been given of the inexactness that distinguishes fuzzy from Boolean sets. Black, for example, distinguishes three types of inexactness in the assignment of words (concepts) to objects.¹⁰ *Generality* implies a one-to-many relationship between a word and concepts; *ambiguity* implies a many-to-one relationship; while *vagueness* implies imprecise concept boundaries. Each, according to Goguen, is encompassed in Zadeh's theory.¹¹ Gale proposes three different sources: 1) inexactness due to insufficient information (we admit the existence of a Boolean assignment but are unable to make it); 2) inexactness due to neutrality of predicates with respect to the object in question (e.g., noninclusive categories); and 3) inexactness due to the effects of secondary qualities.¹²

The most obvious existing framework for analysis of uncertainty is probability theory. In cases where the fuzzy measure is normalized on [0,1] there is a close formal similarity between the fuzzy measure and a probability measure. Most authors on fuzzy sets have been at pains to emphasize the difference. Bellman and Zadeh argue a specific distinction between ran-

domness and fuzziness.¹³ Randomness they take to entail uncertainty about the membership or nonmembership of an object in a Boolean set. Fuzziness pertains to membership in non-Boolean sets. They go on to argue that the mathematical tools of fuzzy sets are not only more appropriate for many problems than those of probability theory, but also simpler. Goguen writes:¹⁴

... a number of theories resemble our logic of inexact concepts. Perhaps probability is closest in spirit, since a probability distribution might be thought of as representing an inexact concept. But the manipulations allowed in probability theory are different from those our example suggests for fuzzy sets. In fact, the theory is a calculus of vagueness, ambiguity, and ambivalence rather than likelihood.

The fuzzy set concept is capable of capturing several types of vagueness in modelling spatial choice. At the individual level it provides an appealing description of at least two distinct classes of ambiguity:

1) Vagueness in the class membership of sites either in general opportunity (choice) sets, or possession of specific predicates describing a variety of site and distance attributes (e.g., price, service quality, and travel time). Existing empirical studies and introspection indicate that such attributes are indeed perceived in inexact ways.

2) Inexactness in the subjective values or preferences imputed to sites or individual attributes of sites. The fuzzy measure associated with a given binary preference may be interpreted as the subjective conviction with which it is held.

MODELLING INDIVIDUAL CHOICE

Fuzzified Algebraic Choice

The principal logical components of the algebraic account of choice as embodied in consumer, game, and statistical decision theories are as follows: a space X of alternatives (commodity bundles in consumer theory, "lotteries" of alternatives in game theory); a binary preference relation P defined on $X \otimes X$ on which certain partial or complete ordering assumptions are imposed; a utility or loss function

⁸ K. Tanaka and M. Mizumoto, "Fuzzy Programs and Their Execution," in Zadeh, Fu, et al., op. cit., footnote 5, pp. 41-76.

⁹ R. D. Luce and P. Suppes, "Preference, Utility and Subjective Probability," in R. D. Luce, R. R. Bush, and E. Galanter, eds., *Handbook of Mathematical Psychology* (New York: John Wiley & Sons, 1965), Vol. III, p. 256.

¹⁰ M. Black, "Reasoning with Loose Concepts," *Dialogue*, Vol. 2 (1963), pp. 1-12.

¹¹ J. A. Goguen, "The Logic of Inexact Concepts," *Synthese*, Vol. 19 (1968-69), pp. 325-73.

¹² Gale, op. cit., footnote 7, p. 340.

¹³ R. Bellman and L. A. Zadeh, "Decision-Making in a Fuzzy Environment," *NASA Contractor Report*, NASA CR-1594 (1975).

¹⁴ Goguen, op. cit., footnote 11, p. 327.

$U(x)$ in agreement with P ; and a choice function $C(x)$ usually defined by maximization or minimization of U over subsets of X . This framework figures quite explicitly in some work in behavioral geography and is implicit in much more.¹⁵ A fundamental decomposition of the properties of location into site and distance attributes is frequently made.

Fuzzy counterparts have been constructed for each of the mathematical entities involved in this paradigm. For example, Zadeh provides a fuzzy extension of binary relational algebra including analogs of ordering (preference) and equivalence (indifference) relations.¹⁶ Shimura provides techniques for the reconstruction of fuzzy preference relations from questionnaire data which are equivalent to the usual (non-fuzzy) scaling algorithms.¹⁷

In the nonfuzzy algebra of preference relations, the most general binary relation is specified as a subset of $X \otimes X$. Additional assumptions such as the weak-order postulates imply a linear or partial ordering which may in turn be decomposed into an asymmetric, nonreflexive preference relation and an equivalence relation of indifference. In the literature on destination choice these relations are applied to pairs of destinations with respect to a given trip purpose. A complete linear ordering of all sites (which is recognized to be a very strong cognitive assumption) imputes to consumers a perfect ranking, with no two sites sharing an indifference class of the relation. Weaker postulates on partial orderings permit indifference groupings and breaks in the chain of preference. In each of these cases, including the most general weak ordering, membership of pairs of destinations in the preference or indifference relation is Boolean.

It is clear that the fuzzy set concept offers potential for a significant generalization of the

concept of a spatial preference ordering. In the fuzzy version, the pairs of sites are imputed a degree of membership in the relation. Just as techniques exist for compounding preferences over the components of choice objects into a single, aggregate preference relation, so fuzzy relations may be compounded. The obvious decomposition, again, would distinguish site and distance attributes. Especially appealing is the fact that a variety of sources of exactness may be subsumed in the measure function. For example, it might describe (value-free) subjective vagueness about membership of the sites in an ill-defined class of destinations, the subjective "degree of conviction" with which a particular preference rating (evaluation) is held, as well as the evaluation itself. In repetitive travel for familiar goods the latter might dominate, whereas in shopping for an unfamiliar product or an unpremeditated gift-item at Christmas the first source of vagueness would be the more important.

A simpler approach to a fuzzy algebraic account of spatial choice is to develop a choice function directly, rather than inducing one from an underlying preference relation. The classical Boolean model frames choice as a set function determined by maximization of a utility function taking values at points in a space S of sites (vectors of site and distance attributes). A very simple generalization of destination choice might be constructed as follows. Define choice as a fuzzy set $C \subset S$ with membership function μ_c . Each single attribute (parking space, service quality, travel time) may be taken as another fuzzy subset $C_i \subset S$. We may construct C in several different ways from the sets C_i . We might simply stipulate $\mu_c = \sum_i \mu_{c_i}$. Or C might be defined as the fuzzy union or intersection of the C_i , thus

$$C = \bigcap_i C_i \text{ which implies } \mu_c(x) = \min_i \mu_{c_i}(x);$$

$$x \in X$$

$$\text{or } C = \bigcup_i C_i \text{ which implies } \mu_c(x) = \max_i \mu_{c_i}(x); x \in X.$$

Each of these alternatives (the attribute-compounding assumptions) possesses different behavioral implications. For example, the set-intersection definition of aggregate choice implies that site choice is determined by a minimum threshold simultaneously applied to all site and link attributes. Practical difficulties in implementing this account of choice will be

¹⁵ For example, G. Rushton, "Decomposition of Space-Preference Functions," in Golledge and Rushton, op. cit., footnote 1, pp. 119-33; and D. Demko and R. Briggs, "A Model of Spatial Choice and Related Operational Considerations," *Proceedings, Association of American Geographers*, Vol. 3 (1971), pp. 49-51.

¹⁶ L. A. Zadeh, "Similarity Relations and Fuzzy Orderings," *Information Sciences*, Vol. 3 (1971), pp. 177-200.

¹⁷ M. Shimura, "Fuzzy Sets Concept in Rank Ordering Objects," *Mathematical Analysis and Applications*, Vol. 43 (1973), pp. 717-35.

considerable; but I feel that it provides a far more attractive account of preference formation than the classic algebraic model. It permits specific incorporation of several sources of inexactness, including fuzziness in the class (functional type) of destination with respect to a given trip purpose, and cognitive inexactness in the predicate "preferred." Of course, several accounts of actual choice are consistent with the formulation sketched here. Deterministic choice might entail maximizing μ_c with respect to x , or we might specify a choice probability for each x proportional to $\mu_c(x)$.

To provide a more concrete example, consider fuzzy modelling of the specific attribute, distance. Inexactness in the perception of distance is well studied. For example, subjective estimates of physical distance or travel time frequently are rounded to simple cognitive units of miles or minutes. We might formalize this imprecision by defining fuzzy sets (distance predicates) in a space of points, X . The predicate "reasonably close to" might be made precise as a fuzzy set $A \subset X \otimes X$. We might specify

$$\mu_A(x,y) = \alpha e^{-\beta d(x,y)} \quad \alpha, \beta > 0,$$

where d is actual Pythagorean distance (or travel time). Fuzzy distance minimization from a fixed point such as the home may be formulated as a fuzzy set $B \subset X$ interpreted as "approximately the shortest." Finally, a fuzzy account of absolute time and distance perception may be specified by fuzzy sets in a space of distances. As a trivial example, the set "roughly half a mile" might have membership function $\alpha \exp\{-\beta|d-.5|\}$.

Some experimental results on the fuzzy perception of distance are reported by Kochen who indicated that about one-half of his sample population interpreted "far distances" as a fuzzy set with a membership function which was monotone and continuous in real distance.¹⁸

Fuzzified Probabilistic Choice

The distinction between the algebraic and the probabilistic models lies in their account of a single act of choice with respect to a single alternative. In the algebraic model this choice is determinate (determined, for exam-

ple, by utility maximization). In the probabilistic account it is prescribed by the outcome of a Bernoulli experiment which leads, under the simplest aggregation assumptions, to a multinomial model of repetitive destination choice.

The extension of probability theory to fuzzy sets entails an extension of a probability measure to fuzzy subsets of a sample space S . The probability of an arbitrary fuzzy event E is defined as the integral of μ_E over S with respect to the probability measure.¹⁹

In the discussion of fuzzified algebraic choice above, it was indicated that fuzzy postulates may be imposed either on a binary preference algebra (inducing a fuzzy choice function), or directly upon the choice function itself. The same logical possibilities exist in the probabilistic case, though in fact no literature exists on stochastic preference relations which are also non-Boolean. Nonfuzzy stochastic preference relations have been described in the psychological literature, together with such ancillary concepts as weak and strong stochastic transitivity.

The superficial (formal) similarity between the stochastic binary relations described by Marschak and fuzzy binary relations described by Zadeh is so strong that the substantive difference requires reemphasis.²⁰ Stochastic transitivity traditionally has measured uncertainty or unpredictability in an either-or response. A fuzzy relation addresses such intrinsically irresolvable or vague binary propositions as Shimura's "roses are more beautiful than cherry blossoms."²¹

For brevity, only fuzzy-stochastic choice functions will be considered here. It is quite straightforward to phrase probabilistic destination choice in these terms. Consider the predicate C (the fuzzy set "preferred") defined above. There are many ways in which a probability measure P might be defined relative to μ_c . As a concrete example, suppose $P(x_i)$ is pro-

¹⁹ L. A. Zadeh, "Probability Measures and Fuzzy Events," *Journal of Mathematical Analysis and Applications*, Vol. 23 (1968), pp. 421-27.

²⁰ J. Marschak, "Binary-Choice Constraints and Random Utility Indicators," in K. J. Arrow, S. Karlin, and P. Suppes, eds., *Mathematical Methods in the Social Sciences* (Stanford: Stanford University Press, 1960), pp. 312-29.

²¹ M. Shimura, "Fuzzy Sets Concept in Rank Ordering Objects," *Information Sciences*, Vol. 3 (1971), p. 718.

¹⁸ M. Kochen, "Applications of Fuzzy Sets in Psychology," in Zadeh, Fu, et al., op. cit., footnote 5, pp. 395-408.

portional to $\mu_c(x_i)$, which is the measure of the intersection of fuzzy preference sets describing a range of site and trip-link attributes. The result would express $P(x_i)$ as a function of site and link attributes in a form analogous to the traditional Luce-Huff and multinomial logit models.

A more general class of probabilistic destination choice models has been described.²² They derive from Thurstone's classic theory of discriminial distributions and incorporate a cognitive choice mechanism involving maximization of stochastic utilities. Randomness and normality in individual discriminial distributions are traditionally derived by applying the Central Limit Theorem to many independent affective factors which condition the evaluative process. The existing literature on fuzzy sets applied to cognitive processes contains no reference to the potential reinterpretation of at least some of the sources of variability as fuzzy rather than probabilistic. In formalistic and conceptual terms, this is an easy task.

Bellman-Zadeh Fuzzy Decision Theory

Bellman and Zadeh have formulated a general theory of fuzzy decisions which logically encompasses both algebraic and probabilistic forms.²³ The distinctive feature of their approach is the formal identity of constraints, goals, and decisions, all of which figure as fuzzy subsets of a choice space. Consider the following goals and constraints for destination choice, defined over a set X of sites (x_i is a vector of site and link attributes):

- G_1 "prices at x_i are quite reasonable"
- G_2 "service quality at x_i is fairly good"
- .
- .
- .
- C_1 "the range of goods at x_i is adequate"
- C_2 "it doesn't take too long to get to x_i "
- .
- .
- .

²² J. S. Pipkin, "Two Probabilistic Revealed Preference Models of Choice Process in Recurrent Urban Travel," unpublished doctoral dissertation, Northwestern University, 1974.

²³ Bellman and Zadeh, op. cit., footnote 13; and K. Asai, H. Tanaka, and T. Okuda, "Decision-Making and its Goal in a Fuzzy Environment," in Zadeh, Fu, et al., op. cit., footnote 5, pp. 257-78.

Goal and constraint sets are concatenated, for example, by fuzzy set intersection $G = \cap G_i$, $C = \cap C_i$, and the decision set D is defined as $G \cap C$. The induced fuzzy measure μ_D may be used to prescribe a decision according to deterministic, nondeterministic, or probabilistic decision rules.

LONGITUDINAL AND CROSS-SECTION AGGREGATION

Other structures in the fuzzy set literature appear useful in temporal and population aggregation, including conditioned fuzzy sets and higher order (type- n) fuzzy sets.

Temporal Aggregation

Conditioned or parameterized fuzzy sets are defined as families of fuzzy sets A_t on a space X , when $\mu_A(x, t)$ depends on a parameter, t .²⁴ If C is defined as the predicate "preferred" over a space of sites, parameterization of μ_C with a time variable t provides a framework for learning (or other) theories on the temporal behavior of fuzzy choice. For example, in the context of a learning process on travel time in a metric space of times Y , the predicate "shortest travel time" may be defined as a fuzzy set $S \subset Y$, with characteristic function $\mu_S(T, t)$. A learning process involving the sharpening of time perception in discrete or continuous time might have a form similar to

$$\mu_S(T, t) = \exp\{-\alpha(t)|T - T_0|\};$$

$$T \in S, \alpha(t) > 0$$

where T_0 is the actual shortest travel time and α is an increasing function of trial index, t .²⁵

²⁴ L. A. Zadeh, "Towards a Theory of Fuzzy Systems," in R. E. Kalman and N. DeClaric, *Aspects of Network and Systems Theory* (New York: Holt, Rinehart and Winston, 1971), p. 474.

²⁵ It seems worthwhile to clarify in some detail the logical status of the components in this example. Two points in urban space are fixed—say, a respondent's home and his new work place. We assume that the individual's perception of the "true" shortest time T_0 under given traffic conditions is fuzzy. I feel that this assumption is very reasonable. Thus in a space of times Y the respondent will be more or less inclined to agree that a specified time $T \in Y$ is indeed the shortest travel time. The degree of his agreement is represented by the measure function of a fuzzy set $S \subset Y$, where S defines the predicate "shortest travel time" over Y . This measure is dependent on a time parameter, t . The secular change in t is governed by a function, α . For the illustrative measure function given

Population Aggregation

A parameterized family of fuzzy preference functions might be used to aggregate over a population of individuals differing both in the perceived content of the spatial choice set and the preference associated with sites in it. As a simple example, consider a large population in which the i^{th} individual's preference over a set of sites S is expressed as a fuzzy set $P(i) \subset S$, with characteristic function $\mu_P(s,i)$, $s \in S$. With a deterministic individual decision rule defined by maximization of μ , the number of individuals choosing a destination S is prescribed by the set of extrema of the functions $\mu_P(s,i)$. A probabilistic decision rule with choice probabilities proportional to the fuzzy measures implies a multinomial distribution of visit frequencies analogous to the mixed multinomial distributions traditionally used to describe choice in heterogeneous populations.

An alternative mechanism for population aggregation is provided by second or higher-order fuzzy sets. A first-order fuzzy set has a measure function taking values in a real interval. A second-order fuzzy set has a measure function whose range space is a set of first-order fuzzy sets.²⁶ Let P represent a "population" space of individuals and S a space of sites. A sample of travellers or questionnaire respondents may be taken as either a Boolean or fuzzy set $A \subset P$. For a given individual $\mu_C(p)$ defines a fuzzy preference predicate $C \subset S$, describing inexact site evaluations by individual p . One merit of this approach is that the sample of respondents may itself be characterized as a fuzzy subset of a population. The measure associated with an individual might be construed as his degree of membership in a class such as "blue-collar workers," "persons living close to a food supermarket," and so on. This is a more natural approach to heterogeneity in a population than the usual attempts to constitute homogeneous (and highly restrictive) samples. The measure function of a population might be specifically defined in terms of distance to trip destinations: this would provide an alternative to the usual dichotomy of localized and dis-

persed samples. Gradations in both sample membership and preference might then be handled simultaneously.

THE FORM OF FUZZY MEMBERSHIP FUNCTIONS

Successful implementation of ideas from the fuzzy set literature in the study of spatial choice awaits specification of appropriate mathematical forms for the imputed membership functions, and the development of appropriate estimation procedures. Ultimately, we require measurement techniques for recovery of fuzzy measure functions from reported and revealed preferences. Our knowledge of appropriate forms is slight, for we have no tradition of explicitly modelling cognitive vagueness and imprecision. Even the topology appropriate to specific choice problems is obscure. For the fuzzy set defined by the predicate "a place quite close to home," we obviously require a continuous treatment of space. For the fuzzy set "a place where I am quite likely to be able to buy the latest Newsweek," the fuzzy measure presumably should be defined on the discrete set of urban commercial establishments (or the set of newsagents and drugstores).

Intuition suggests straightforward and plausible ways in which fuzzy measure functions could be estimated. On a simple level, homogeneous or heterogeneous respondent samples might be canvassed as to the membership of sites in specified categories. The measure values might be defined in terms of the positive response proportions. More elaborate schemes of line-drawing and length-estimation could be used to calibrate fuzzy distance-predicates, while map-drawing experiments would permit estimation of the measures of spatial sets.

Although the general characteristics of fuzzy measures may be established empirically, it is clearly impossible to identify such functions precisely.²⁷ Instead, as in the usual parameter estimation contexts, a function must be fitted from a limited class of acceptable measures (such as linear or continuous forms). Bellman, Kalaba, and Zadeh deal with reconstruction of a fuzzy measure exemplified (sampled) on a

the individual's perception of the shortest travel time sharpens.

²⁶ L. Zadeh, "Calculus of Fuzzy Restrictions," in Zadeh, Fu, et al., op. cit., footnote 5, pp. 1-39.

²⁷ For example see the comments on empirical continuity of the measures in Kochen, op. cit., footnote 18.

finite set of points.²⁸ Bellman et al. write:²⁹

In most practical situations the a priori information about the characteristic function of a fuzzy set is not sufficient to construct an estimate of it which is "optimal" in a meaningful sense. Thus in most instances one is forced to resort to a heuristic rule . . . with the only means of judging the "goodness" of the estimate so obtained lying in experimentation.

A very noticeable feature of the fuzzy set literature in a variety of fields is the ad hoc and even apparently casual way in which the form of the fuzzy measures are specified. Nowhere are clear criteria given, beyond the obvious requirements of intuitive acceptability and mathematical simplicity. Although no standardization is apparent in the literature, Zadeh refers to the obvious advantages of a standardized family of measure functions, capable of being fitted for specified fuzzy sets.³⁰ He provides two standardized forms approximating the normal density and distribution functions in shape.

An alternative to the assumption of simple but arbitrary forms for fuzzy membership functions is demonstrated by Kochen.³¹ Here a series of behavioral assumptions on subjective "degree of agreement" is used to deduce a specific characteristic function. The set under consideration is the fuzzy set of real numbers corresponding to the predicate "much larger than x " where x is stipulated. Kochen assumes that subjective degree of belief changes in proportion to its current value, and in proportion to current disagreement. This yields a logistic characteristic function for the predicate. It is probable that these or analogous axioms can be found to describe our perception of distance and travel time, though I find them somewhat less plausible for predicates describing site preferences.

CONCLUDING REMARKS

Vagueness and ambiguity in cognition, and ambivalence, vacillation, and inconsistency in evaluation, invariably accompany the workings of the mind in as complex an arena as urban space. These problems may be viewed, at least in part, as assignment problems. In the case of

cognition, they arise with respect to the conceptual and linguistic categories we operationalize as choice sets. In the case of evaluation they arise with respect to the set structures in our algebraic and probabilistic models of choice. Typically these ambiguities have been attributed to human cognitive limitations, and handled essentially in the vein of statistical error. An alternative and potentially fruitful procedure is to account for these problems as shortcomings in the concept of set itself. If these arguments have any force, it is natural to attack the ambiguities directly, and to attempt to treat graduated set membership in explicit, analytical terms. Zadeh's abstract theory provides a language for this analysis, and I have attempted to demonstrate its applicability to some distinctively geographic problems.

Perhaps the greatest methodological advantage of the fuzzy set concept applied to spatial choice is the formal unity it endows on apparently disparate concepts such as choice set, value, goal, constraint, and decision. They share the status of fuzzy sets over an appropriately defined choice space. As indicated above, the inexact site-preference predicate may subsume inexact destination class membership and inexact preference-formation over a range of component attributes of sites and links. Concatenation of these sets (for example by fuzzy set intersection) yields a decision space in which either deterministic or probabilistic decision rules may be defined.

The concept of a compound and fuzzy spatial choice set propounded above also permits an attractive integration of the cognitive and affective aspects of spatial choice. Formulations of urban cognitive space by Wolpert, Horton and Reynolds, and others have emphasized the dichotomy of preference and information.³² Clearly, information is primordial to preference formation, but evidence has accumulated that informational judgments such as subjective estimates of the distance or travel time are not independent of site preferences.³³ Evaluations

²⁸ Zadeh, op. cit., footnote 4.

²⁹ R. Bellman, R. Kalaba, and L. A. Zadeh, "Abstraction and Pattern Classification," *Journal of Mathematical Analysis and Applications*, Vol. 13 (1966), p. 3.

³⁰ Zadeh, op. cit., footnote 26, p. 29.

³¹ Kochen, op. cit., footnote 18, p. 397.

³² J. Wolpert, "Behavioral Aspects of the Decision to Migrate," *Papers and Proceedings of the Regional Science Association*, Vol. 15 (1965), pp. 159-69; and F. E. Horton and D. R. Reynolds, "Effects of Urban Spatial Structure on Individual Behavior," *Economic Geography*, Vol. 47 (1971), pp. 36-48.

³³ For example, D. L. Thompson, "New Concept: Subjective Distance," *Journal of Retailing*, Vol. 39 (1965), pp. 1-6.

(preference predicates), subjective distance predicates, and predicates portraying subjective information on the class membership of sites in opportunity sets, may each be defined as fuzzy subsets of a choice space representing a discrete or continuous subset of urban space.

One of the most appealing specific applications lies in the treatment of subjective distance or travel time. Although considerable attention has been devoted to finding the transformation linking physical and perceived distance, the result is usually a simple, one-to-one "psychophysical" function, such as a power or logarithmic form. Because these functions are one-to-one and because the continuum of subjective distance has the same cardinality as physical distance, the imputed judgments of distance are still arbitrarily precise. The usual way to capture vagueness in individual distance judgments is by a statistical distribution around a specified mean. A more direct way is to assume that our perception of space is intrinsically fuzzy. One straightforward way to accomplish this, as shown above, is to define distance predicates (e.g., "the distance between x and y is one mile") as fuzzy subsets of an appropriate universal set of pairs of points in R^2 .

Despite the empirical and methodological appeal of this unified account of inexactness in spatial choice, substantial problems remain to be overcome in implementing it. Although the epistemological difference between fuzzy set and probability measure is evident and has been emphasized in the literature, the practical implications of the distinction are less clear. It is

logically possible that psychologically-based but purely probabilistic accounts of individual choice behavior will prove adequate for all our needs. It is also possible that a clear-cut empirical decision between fuzzy and (Boolean) probabilistic formulations may never be made; still, the fuzzy model may well provide a conceptually more natural account of subjective inexactness in choice set specification and in preference formation.

Little is known of cognitively appropriate forms for the fuzzy characteristic functions. The existing literature is pervaded by ad hoc assumptions guided by mathematical simplicity. Few estimation frameworks have been offered to reconstitute the measures. The discussion above implies two questions that must be resolved. Firstly, is it possible to deduce appropriate functional forms from postulates on individual cognition? Secondly, how may suitable forms (based on theory or convenience) be fitted to data?

Fuzzy measure functions are themselves phrased in the usual (Boolean) mathematics. This poses, as Goguen indicates, a question of whether we can gain exact (nonfuzzy) knowledge of concepts which themselves are inexact.³⁴ Preliminary results in the fuzzy set literature, such as those of Kochen, suggest that we can. At minimum, there is no problem of principle in approximating inexact set measures with exact characteristic functions.

³⁴ Goguen, *op. cit.*, footnote 11.