ELSEVIER
ISPRS Journal of Photogrammetry & Remote Sensing 55 (2000) 164–175
PHOTOGRAMMETRY & REMOTE SENSING
www.elsevier.nl/locate/isprsjprs

Fuzzy spatial objects and their dynamics

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Received 12 February 1999; accepted 4 March 2000

Abstract

Many monitoring activities of environmental processes deal with phenomena that are fuzzy. Their spatial description cannot be based on the geometry of their boundaries, because these cannot be identified. In fact, such phenomena have an uncertain thematic description and that is often the reason why their spatial extent can only be determined with limited accuracy. We will therefore present in this paper a formalism for the description of objects without fixed boundaries, i.e., objects with a fuzzy spatial extent. A method based on this formalism will be developed for the extraction of fuzzy spatial object data from digital images or from field representations in a raster format.

This method can then be used for monitoring purposes when the spatial extent of fuzzy objects are identified at different epochs; the dynamics of such objects can then be obtained from these time series. Several parameters will be proposed to estimate the overlap of objects at successive epochs; with these parameters the state transitions of objects can be evaluated. Several types of state transition can be distinguished with these parameters such as shift, merge, and split of objects. These state transitions can then be combined to form the processes through which the objects evolve over time. The proposed method is applied in a coastal geomorphologic study of a barrier island in The Netherlands.

Keywords: spatial modelling; fuzzy objects; convex fuzzy spatial extent; object dynamics

1. Introduction

The monitoring of environmental features is often hampered by the fact that their definition is vague. This problem occurs in many applications like the monitoring of natural vegetation and forest areas, the development of land use, coastal development, etc. Such monitoring processes are often based on the use of remote sensing data from which the information about the relevant features is to be extracted. The vagueness of concepts and definitions in such applications has two effects:

1. The features can only be identified with a limited level of certainty so that there is a substantial fuzziness in their spatial description.
2. If the processes that are to be monitored are expressed through changes in the states of these features, then this monitoring can also be done with limited certainty.

These two aspects of monitoring will be discussed in this article. Firstly the conceptual aspects of the identification of fuzzy spatial objects will be dis-

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0924-2716/00/$ - see front matter
PII: S0924-2716(00)00017-4

cussed, then follows an explanation of how the uncertainty of object states affects the analysis of state transitions.

1.1. The identification of fuzzy spatial objects

The spatial extent of geo-objects is generally determined through the boundaries, or more precisely, through the position of the boundary points. The analysis of the geometric uncertainty of the objects is therefore often based on accuracy models for the co-ordinates of these points. The epsilon-band method is well known in this context (Dunn et al., 1990). The solutions for this problem that are found in the literature are not satisfactory though. The reason is that the geometric uncertainty of geo-objects is not only a matter of co-ordinate accuracy; it is rather a problem of object definition and thematic vagueness (see also the discussions on this topic in for example, Chrisman, 1991; Burrough, 1986; Burrough and Frank, 1995; Burrough and McDonnell, 1998; Goodchild et al., 1992). This latter aspect cannot be handled by a geometric approach alone.

This becomes apparent when mapping is based on feature extraction from digital images with a raster structure rather than with a vector structure commonly used in land surveying and photogrammetry. The uncertainty of remote sensing image classification is primarily considered to be thematic, and the certainty that a pixel belongs to a thematic class might be expressed through a likelihood function, which is evaluated in the classification process (Lillesand and Kiefer, 1994; Buiten and Clevers, 1993). Image segments can then be formed of contiguous sets of pixels falling under the same class. If these segments represent the spatial extent of objects then the uncertainty of the geometry of these objects is due to the fact that the value of the likelihood function varies per pixel (Canters, 1997; Fisher, 1996; Wickham et al., 1997; Usery, 1996; Brown, 1998; Gahegan and Ehlers, 1997). We will use the approach of Molenaar (1994, 1996, 1998) to explain how thematic uncertainty propagates to the geometry of objects when objects are extracted from image or raster data.

1.2. Object dynamics

For monitoring environmental phenomena we should be able to compare the states of spatial objects at different epochs. The literature on this topic is limited, and few publications discuss the dynamics of objects, particularly spatial change, in a generic way (Yuan, 1996; Hornsby and Egenhofer, 1997). There is even less literature about the dynamic behaviour of fuzzy objects. The detection of the dynamics of fuzzy objects is the second point to be addressed in this paper. We will follow the approach of Cheng (1998) and Cheng and Molenaar (1997) based on the evaluation of overlap of the spatial extents of objects in successive years. Several parameters will be explained for the identification of the type of process that has taken place between two epochs. The approach will be illustrated by means of an example dealing with the dynamics of sediments along the Dutch coast.

1.3. The structure of this article

Section 2 explains some basic concepts for modelling fuzzy spatial objects and discusses their existential and extensional uncertainty. Then the concept of “spatially disjoint objects” will be elaborated for fuzzy objects, and objects with a convex fuzzy spatial extent will be defined as a generalisation of elementary crisp objects. Section 3 uses these concepts to formulate a method for the identification of fuzzy objects, and this will be demonstrated by an example where geomorphologic units are to be identified from a height raster in a coastal zone. Section 4 develops parameters for the evaluation of the overlap of the fuzzy spatial extents of objects determined at two successive epochs. These parameters will then be used to formulate a procedure to analyse the type of state transitions object went through between two epochs. Section 4 applies this procedure to the coastal zone of our example. Section 5 summarises the major findings and directions for further.

2. Uncertainty aspects of fuzzy spatial objects

Let $U_M = \{\ldots, O_i, \ldots\}$ be the universe of a map M, where the term “map” refers to a spatial database containing a terrain description. U_M is the collection of all terrain objects represented in this database. The syntactic approach for handling spatial object information developed in Molenaar (1994, 1996, 1998) makes it possible to distinguish three types of statement with respect to the existence of these spatial objects:

1. An existential statement asserting that there are spatial and thematic conditions that imply the existence of an object *O*.
2. An extensional statement identifying the geometric elements that describe the spatial extent of the object.
3. A geometric statement identifying the actual shape, size and position of the object in a metric sense.

These three types of statements are intimately related. The extensional and geometric statements imply the existential statement — if an object does not exist it cannot have a spatial extent and a geometry. The geometric statement also implies the extensional statement. All three types of statement may have a degree of uncertainty and although these statements are related, they give us different perspectives emphasising different aspects of uncertainty in relation to the description of spatial objects.

2.1. Existential uncertainty

The uncertainty whether an object O exists can be expressed by a function: $\text{Exist}(O) \in [0,1]$. If this function has a value of 1 we are sure that the object exists, if it has a value of 0 we are sure it does not exist. This latter case leads to a philosophical problem, because how can we make existential statements about objects that do not exist, or rather how can we identify non-existing objects and refer to them as an argument of this function? This problem will not be elaborated here, but we will follow a pragmatic approach by restricting the range of the function to $\text{Exist}(O) \in (0,1)$, meaning that the function can take any value larger than 0 and less or equal to 1. The uncertainty of the existential statement is due to the fact that observational procedures, such as photo-interpretation or satellite image analysis, can identify observational conditions suggesting that an object might exist at some location without giving definite certainty that it really exists as an independent object. The 'observed object' then gets an object identifier but it might, in fact, be a part of another object. The 'exist' function expresses in this case the uncertainty of the actual real-world state of the observed object. The problem arises due to the fact that in many GIS applications it is only possible to refer indirectly to real world objects through descriptions provided by observational systems. This problem is strongly related to the referential problem identified in philosophy (Evans, 1982; Neale, 1990; Quine, 1960). If the existence of objects is uncertain then the universe of a map becomes a fuzzy universe:

$$U_M = \{\ldots,\{O_i,\ \text{Exist}(O_i)\},\ldots\}$$

The members of this fuzzy universe are the objects with the function expressing the uncertainty of their existence. This situation is fundamentally different from the situation dealt with by the theory of fuzzy (sub)sets (Kaufman, 1975; Klir and Folger, 1988; Klir and Yuan, 1995; Zimmermann, 1985). There the existence of the members of the universal set is not uncertain, only the subsets of the universal set are fuzzy. Therefore, the concepts of the theory of fuzzy subsets and of fuzzy reasoning should be applied with care in our situation. If the universe of a map is not certain then what is?

We will try to formulate an answer to this question by considering the complete set of faces of a vector map (or the set of raster elements) as a universal set from which subsets can be generated by assigning faces to the spatial extent of objects. This means that the case where U_M consists of fuzzy objects is interpreted in the sense that the spatial extent of these objects are the fuzzy subsets of a universal set of faces. The formalism developed in Molenaar (1994, 1996, 1998) will help to elaborate this approach in the following sections of this paper.

2.2. Extensional uncertainty

Suppose that the geometry of the objects of some U_M is represented in a vector format, i.e., the geometry is described in nodes, edges and faces (or 0-, 1- and 2-cells). Let $\text{Geom}(M)$ be the geometric component of the map, i.e., it is the collection of all geometric elements describing the geometry of all objects of the universe. Let $\text{Face}(M)$ be the collection of all faces in $\text{Geom}(M)$, and let the function $\text{Part}_{22}[f,O]$ express the relation between a face $f \in$

Face(M) and an object $O \in U_M$. If this function has a value of 1 then the face belongs to the spatial extent of the object, if it has a value of 0 then that is not the case. We define the set:

$$\text{Face}(O) = \{ f \mid \text{Part}_{22}[f,O] = 1 \}$$

This is then the extensional statement of the object in the sense that Face(O) determines the spatial extent of O. In this notation the geometric description of the objects is organised per object. This formulation is valid for crisp objects, i.e., objects with identifiable boundaries (see the end of Section 2.4).

For fuzzy objects the relation between face and object cannot be established with certainty so that we have $\text{Part}_{22}[f,O] \in [0,1]$. The spatial extent of a fuzzy object is therefore uncertain and is given by:

$$\text{Face}(O) = \{ f \mid \text{Part}_{22}[f,O] > 0 \}$$

Fig. 1 presents an example of this case.

The syntactic approach developed by Molenaar (1994, 1996, 1998) shows that the vector and the raster geometry have a similar expressive power. This implies that the handling of spatial uncertainty should in principle also be the same for both geometric structures. Hence, it must be possible to combine or even unify the vector and raster oriented approaches found in the literature. In case the geometry of the map is represented in a raster format the function $\text{Part}_{22}[\ ]$ is evaluated for each cell of the raster.

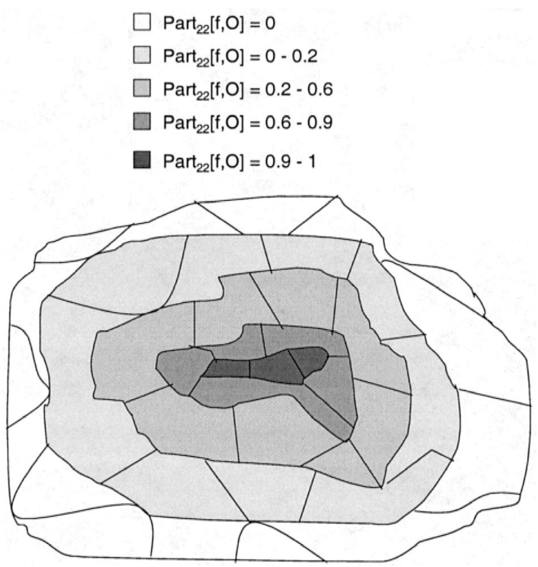

Fig. 1. Object with a fuzzy spatial extent.

Let the set of faces related to object O_a with certainty level c be:

$$\text{Face}(O_a \mid c) = \{ f_j \mid \text{Part}_{22}[f_j, O_a] \geq c \}$$

This is a conditional spatial extent of the object, comparable to the (strong) α-cut in Klir and Yuan (1995). With this set, we can define the conditional functions:

$$\text{Part}_{22}[f_j, O_a \mid c] = 1 \Rightarrow f_j \in \text{Face}(O_a \mid c)$$

$$\text{Part}_{22}[f_j, O_a \mid c] = 0 \Rightarrow f_j \notin \text{Face}(O_a \mid c)$$

The relationships between faces and the conditional spatial extents of an object are crisp.

Molenaar (1998) defined a notation where the geometric description of the area objects was organised per face. That notation can also be modified to handle uncertainty, and the set of area objects that have a fuzzy relationship with a face is then:

$$\text{AO}(f) = \{ O \mid \text{Part}_{22}[f,O] > 0 \}$$

With this expression it is possible to interpret a fuzzy area object as a fuzzy field. For each face f the function $\text{Part}_{22}[f,O]$ is then evaluated for each object of the map. If the map has a raster structure then the set AO( ) will be evaluated for cells.

Objects can then be combined into one layer by overlaying their fuzzy fields. Suppose that the fuzzy fields of n objects have been represented in a raster format and that these n raster layers are to be combined by an overlay operation. The attributes A_1 to A_n of the raster will represent the functions $\text{Part}_{22}[\text{cell},O]$ for the n objects, i.e., for each cell the attribute value a_k represents the value of the function $\text{Part}_{22}[\text{cell},O_k]$.

The field representation of fuzzy objects seems to be quite natural. This fact could explain why many mapping disciplines have such a strong affinity towards the field approach rather than the object approach. This approach is often used because of apparent syntactic simplicity. The consequence might then be that the object structure of certain terrain descriptions remains hidden and with it much of the semantic content of these descriptions.

2.3. Spatially disjoint objects

Suppose that objects are defined so that they form a spatial partition of the mapped area, the terrain description has then the structure of a single valued vector map (Molenaar, 1989, 1998). This means that the objects form a complete coverage of the mapped area and that they do not overlap, i.e., they are spatially exclusive. These requirements can be interpreted as follows for objects with a fuzzy spatial extent:

Each face belongs to an object of the universe of the map:

$$(\forall f \in \mathrm{Face}(M)) \Rightarrow \mathrm{Part}_{22}[f, U_M] = 1$$

The function $\mathrm{NPart}_{22}[f,O] = 1 - \mathrm{Part}_{22}[f,O]$ expresses then the certainty that face f does not belong to object O. The fact that the objects of the universe are spatially exclusive can be expressed by the fact the complement of the spatial extent of an object O_a is the combined extent of all other objects of the universe $U_M - O_a$:

$$(\forall f \in F_M)(\forall O_a \in U_M) \Rightarrow \mathrm{NPart}_{22}[f, O_a] = \mathrm{Part}_{22}[f, U_M - O_a]$$

so that in a fuzzy single valued vector map we have:

$$\mathrm{Part}_{22}[f, O_a] = 1 - \mathrm{Part}_{22}[f, U_M - O_a]$$

Let $\mathrm{XPart}_{22}[f,O]$ expresses the certainty that a face f belongs exclusively to object O and not to any other object. For a face f_k and an object O_i the value of this function is:

$$\mathrm{XPart}_{22}[f_k, O_i] = \mathrm{MIN}\left(\mathrm{Part}_{22}[f_k, O_i], \mathrm{MIN}_{j \neq i}\left(\mathrm{NPart}_{22}[f_k, O_j]\right)\right)$$

The minimum operators have been applied because we require that the face belongs to O_i *and not* to O_j for any $j \neq i$, this implies an *and* condition.

For single valued vector maps we should require that the extension of:

$$\mathrm{AO}(f_j) = \left\{O_a \mid \mathrm{MAXPart}_{22}[f_j, O_a] = \mathrm{MAX}_{O_i}\left(\mathrm{XPart}_{22}[f_j, O_i]\right)\right\}$$

contains only one element, i.e., there is only one object O_a for which the function XPart_{22} has a maximum value for face f_j. If there are more objects with the same maximum value then additional evidence is required for selecting a unique object.

2.4. Objects with a convex fuzzy spatial extent

An adjacency graph can be defined for the spatial extent of each object. The adjacency graph of an object O consists of:

1. Nodes representing the faces belonging to the spatial extent of O.
2. Edges so that each edge expresses the adjacency of the faces represented by the nodes it connects.

Elementary area objects can then be defined as objects that have a spatial extent consisting of a contiguous set of faces so that the adjacency graph of these faces is connected. This definition allows for elementary area objects (or even faces) with holes. This definition is valid for crisp objects.

This definition of elementary objects needs modification before it can be applied to fuzzy objects, but the original intention can be maintained. The concept of convex fuzzy sets as explained in Klir and Yuan (1995) can be used here, but it will be formulated differently according to Molenaar (1998). First, some supporting definitions will be formulated before a definition of elementary fuzzy area objects can be given.

Definition 1. *A fuzzy object O has nested conditional spatial extents if:*
$(\forall c_i, c_j \mid c_i > c_j) \Rightarrow (\mathrm{Face}(O \mid c_i) \subset \mathrm{Face}(O \mid c_j))$.

This means that the face set representing the spatial extent of an object for a large certainty level should be contained in the face set for a lower certainty level.

The second definition requires that for each certainty level the spatial extent of the object is connected, i.e., the face set has a connected adjacency graph.

Definition 2. *A fuzzy object is connected if:* $(\forall c > 0) \Rightarrow \mathrm{Face}(O \mid c)$ *is connected.*

When an object complies with this definition then each conditional spatial extent complies with the definition of crisp elementary area objects. With these two definitions objects with convex fuzzy spatial extents can be defined:

Definition 3. *A fuzzy object has a convex fuzzy spatial extent if it is connected and if its conditional extents are nested.*

Now we are ready to define elementary fuzzy objects:

Definition 4. *A fuzzy area object is elementary if it has a convex fuzzy spatial extent.*

This definition is indeed similar to the definition of elementary crisp objects, which is now a special case for the situation that the zone between $c = 0$ and $c = 1$ collapses to a width equal to 0, i.e., we get $\forall c_i, c_j \in (0,1] \Rightarrow \mathrm{Face}(O \mid c_i) = \mathrm{Face}(O \mid c_j)$.

This means that all conditional extents of crisp objects are identical.

3. The identification of fuzzy objects

The Survey Department of the Netherlands Ministry of Public Works makes yearly observations of height profiles along the Dutch coast to monitor changes of the beaches and dune areas. Geomorphologic units are to be extracted from the height data by means of a three-step procedure.

1. A full height raster is derived from the profiles through interpolation.
2. The cells of the raster are assigned to height classes related to the foreshore, beach and fore dune areas.
3. Contiguous regions will be formed consisting of cells belonging to these three classes.

The problem is that this can only be done with limited certainty because there is no fixed height value where the foreshore stops and the beach begins, nor is that the case for the transition form beach to fore dunes (Cheng et al., 1997). This means that the height classes related to the three types of regions can only be determined approximately because they are fuzzy.

Here we will discuss how the uncertainty propagates from the classification of the raster cells to the formation of segments of the raster. The example of coastal monitoring will be used to explain the more general situation where in the process of object identification the uncertainty of thematic data effects the determination of the spatial extents of the objects.

There are three height classes in this example so that for each raster cell $\boldsymbol{P}$ a vector will be evaluated:

$$[L(\boldsymbol{P},C_1), L(\boldsymbol{P},C_2), L(\boldsymbol{P},C_3)]^T \, (0 \le L(\boldsymbol{P},C_k) \le 1)$$

where $L(\boldsymbol{P},C)$ represents the membership function value of grid cell $\boldsymbol{P}$ belonging to class C. For each class C_k regions can be identified consisting of cells with $L(\boldsymbol{P},C_k) > \mathrm{Threshold}_k$. Each region can then be interpreted as the fuzzy extent of a spatial object belonging to a class C_k. In many applications, fuzzy spatial overlaps among objects are permitted, so that the objects have fuzzy transition zones (Burrough, 1996; Usery, 1996). In the transition zones, the pixels might belong to multiple objects.

The geomorphologic units of this example have been defined so that they should form a partition of the mapped area, and that can only be the case if the classes are spatially exclusive. Each grid cell should therefore belong to only one height class, and consequently to only one object. Therefore, let $\mathrm{N}L[\boldsymbol{P},C_k] = 1 - L[\boldsymbol{P},C_k]$ represent not-membership, i.e., the certainty that $\boldsymbol{P}$ does not belong to class C_k, and let $\mathrm{X}L[p_{ij},C_k]$ express the certainty that the grid cell belongs to class C_k and not to any other class C_l with $l \ne k$. The latter can be expressed by means of a minimum operator

$$\mathrm{X}L[\boldsymbol{P},C_k] = \mathrm{MIN}\left(L[\boldsymbol{P},C_k], \mathrm{MIN}_{l \ne k}(\mathrm{N}L[\boldsymbol{P},C_l])\right) \quad (1)$$

$\boldsymbol{P}$ should belong to only one class, requiring only that there is one class for which the function $\mathrm{X}L[\ ]$ has a maximum value for $\boldsymbol{P}$. If there are more classes with the same maximum values then additional evidences are required in order to select a unique class. It can be represented as

$$\text{If} \quad \mathrm{X}L[\boldsymbol{P},C_k] = \mathrm{MAX}_{c_l}(\mathrm{X}L[\boldsymbol{P},C_l])$$

$$(l=1,\ldots,N) \quad \text{then} \quad D[\boldsymbol{P},C_k]=1, \tag{2}$$

$$\text{otherwise} \quad D[\boldsymbol{P},C_k]=0.$$

After assigning the cells to classes, an area S_a of class type C_k will be formed by the following two rules, Molenaar (1996, 1998),

$$\text{for all grid cells } P_{ij} \in S_a, D[P_{ij},C_k] = 1$$

and

$$\text{if } P_{kl} \in S_a \quad \text{and} \quad \mathrm{ADJACENT}[P_{kl},P_{ij}] = 1$$

and

$$D[P_{ij},C_k]=1 \quad \text{then} \quad p_{ij} \in S_a \tag{3}$$

ADJACENT$[P_{kl},P_{ij}]$ expresses the adjacency relationship between grid cells P_{kl} and P_{ij}, and its value is either 0 or 1. P_{ij} will only be assigned to S_a if $D[P_{ij},C_k]=1$. The certainty that this assignment is correct depends on the certainty that the cell has been assigned correctly to C_k. Therefore, the relationship between P_{ij} and S_a can be written as

$$\mathrm{Part}[P_{ij},S_a] = \mathrm{MIN}(D[P_{ij},C_k],\mathrm{X}L[P_{ij},C_k]) \tag{4}$$

Eq. (4) expresses how the thematic uncertainty (uncertain classification of grid cells) propagates into the extensional uncertainty (i.e., fuzzy spatial extent) of an object. For example, let a grid cell have membership values for three classes:

$$L[\boldsymbol{P},C]_{t_1} = \begin{Bmatrix} 0.2 \\ 0.7 \\ 0.0 \end{Bmatrix} \quad \text{so that}$$

$$\mathrm{N}L[\boldsymbol{P},C]_{t_1} = \begin{Bmatrix} 1-0.2 \\ 1-0.7 \\ 1-0.0 \end{Bmatrix} = \begin{Bmatrix} 0.8 \\ 0.3 \\ 1.0 \end{Bmatrix}$$

and according to Eq. (1),

$$\mathrm{X}L[\boldsymbol{P},C]_{t_1} = \mathrm{MIN}\begin{Bmatrix} 0.2,0.3,1.0 \\ 0.7,0.8,1.0 \\ 0.0,8.0,0.2 \end{Bmatrix} = \begin{Bmatrix} 0.2 \\ 0.7 \\ 0.0 \end{Bmatrix}$$

With these results we find that

$$\mathrm{X}L[\boldsymbol{P},C_2] = \mathrm{MAX}_{C_i}(\mathrm{X}L[p,C_i]) \quad (i=1,2,3)$$

$$\text{so that} \quad D[\boldsymbol{P},C_2]=1.$$

This means that this cell is assigned to class C_2 with certainty 0.7.

Fig. 2(a) (b) and (c) show the spatial extents of foreshore, beach, and fore dune in a coastal geomorphology study (Cheng et al., 1997). Each of these figures shows the fuzzy spatial extent of one of the objects. They have been combined into one layer in Fig. 2(d). This figure shows that no crisp boundaries can be identified but that there are transition zones where grey-values express the uncertainty of the assignment of raster cells to the objects.

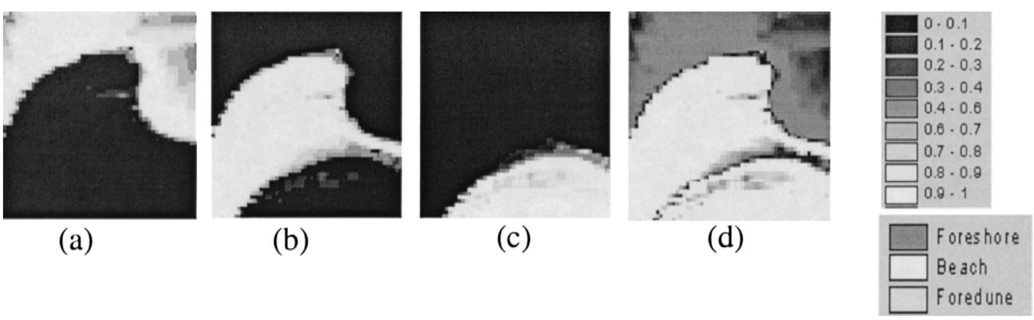

Fig. 2. Fuzzy classification and fuzzy objects in 1989.

4. The identification of state transitions of fuzzy objects

The procedure in the previous section identifies the regions that represent the spatial extents of objects in one epoch. The regions at different epochs should be linked to form lifelines of the objects. This can be realised under two assumptions:

1. Each object has a convex fuzzy spatial extent, i.e., we are dealing with elementary fuzzy objects as defined in Section 2.4.
2. The natural phenomena are changing gradually, so that the changes of the objects from year to year are limited.

This implies that the spatial extents of one and the same object at different epochs should have a larger overlap than the spatial extents of different objects. Under this assumption we can find the successor of a spatial extent at epoch t_n by calculating its spatial overlaps with all the spatial extents at epoch t_{n+1}.

The overlap of two regions S_a and S_b can be found through the intersection of their two cell sets, which is a simple raster-based operation.

$$\mathrm{OVERLAP}(S_a, S_b) = \mathrm{Cells}(S_a) \cap \mathrm{Cells}(S_b) \qquad (5)$$

$\mathrm{Cells}(S_a)$ and $\mathrm{Cells}(S_b)$ represent the sets of raster cells belonging to region S_a and S_b, respectively.

The spatial extents of the objects are fuzzy, and the evaluation of their overlap should take care of

Table 1
Identification and presentation of state transition

Regions at T_1	Regions at T_2	Overlay	Indicators: $\mathrm{Roverl}(S_b\|S_a)$ / $\mathrm{Roverl}(S_c\|S_a)$	Indicators: $\mathrm{Roverl}(S_a\|S_b)$ / $\mathrm{Roverl}(S_a\|S_c)$	Indicators: Similarity	State Transition	Symbol
A	B		Large	Large	High	shift(S_a;S_b)	
A	B C		Small	Large	Low	split(S_a;S_b,S_c)	
			Small	Large	Low		
B C	A		Small	Large	Low	merge(S_b,S_c; S_a)	
			Small	Large	Low		
A	B		Large	Small	Low	expand(S_a;S_b)	
A	B		Small	Large	Low	shrink(S_a;S_b)	
	B	0	/	0	/	appear(S_b)	
A		0	0	/	/	disappear(S_a)	

that. The possibility of a grid cell to be part of the overlap of two fuzzy regions can be defined according to Dijkmeijer and De Hoop (1996),

$$\mathrm{Over}\left[S_a,S_b \mid P_{ij}\right] = \mathrm{MIN}\left\{\mathrm{Part}\left[P_{ij},S_a\right],\mathrm{Part}\left[P_{ij},S_b\right]\right\} \quad (6)$$

Part$[P_{ij}, S_a]$ and Part$[P_{ij}, S_b]$ can be evaluated according to Eq. (4). The size of P_{ij} is considered to be 1 here, so that the size of a fuzzy region S is defined as

$$\mathrm{Size}(S) = \sum_{P_{ij}} \mathrm{Part}\left[P_{ij},S\right] \quad \text{where} \quad P_{ij} \in \mathrm{Cells}(S) \quad (7)$$

The size of the overlap of two fuzzy regions is then

$$\mathrm{SOVERLAP}(S_a,S_b) = \sum_{P_{ij}} \mathrm{Overl}\left[S_a,S_b \mid P_{ij}\right] \quad (8)$$

where $P_{ij} \in \mathrm{Cells}(S_a) \cap \mathrm{Cells}(S_b)$.

Let R_i be the set of regions at epoch T_i, and let $S_a \in R_1$ and $S_b \in R_2$. The following indicators can be used to evaluate the relationships between regions at the two epochs. First of all, the relative fuzzy overlap between two regions can be defined as

$$\mathrm{ROverl}(S_b \mid S_a) = \mathrm{SOVERLAP}(S_a,S_b)/\mathrm{Size}(S_a) \quad (9)$$

$$\mathrm{ROverl}(S_a \mid S_b) = \mathrm{SOVERLAP}(S_a,S_b)/\mathrm{Size}(S_b) \quad (10)$$

where ROverl$(S_b \mid S_a)$ represents the relative overlap with respect to S_a, and ROverl$(S_a \mid S_b)$ is the relative overlap with respect to S_b. The similarity of two fuzzy regions can be defined as

$$\mathrm{Similarity}(S_a,S_b) = \frac{\mathrm{SOVERLAP}(S_a,S_b)}{\sqrt{\mathrm{Size}(S_a)\mathrm{Size}(S_b)}} \quad (11)$$

Using these indicators, object state transitions can be identified between two epochs. Seven cases are shown in Table 1.

The combinations of indicator functions behave differently for these seven cases. State transitions can now be identified by the following process:

For all $S_b \in R_2$ compute Size(S_b)
For all $S_a \in R_1$ do
  > compute Size (S_a)
For all $S_b \in R_2$
  > compute SOVERLAP(S_a,S_b)
  > compute Roverl$(S_b \mid S_a)$, Roverl$(S_a \mid S_b)$, Similarity(S_a,S_b)
  > evaluate shift $(S_a;S_b)$, expand$(S_a;S_b)$, shrink$(S_a;S_b)$
  > evaluate split $(S_a;\ldots S_b,\ldots)$, appear(S_b)
  > evaluate merge $(\ldots,S_a,\ldots;S_b)$, disappear(S_a)

The split process implies that one region $S_a \in R_1$ splits into several regions $S_b \in R_2$, and the merge process implies that many regions $S_a \in R_1$ merge into one region $S_b \in R_2$.

5. Dynamics of fuzzy objects

The procedure of the previous section identifies possible dynamic relationships between regions at two different epochs. Regions thus related can be

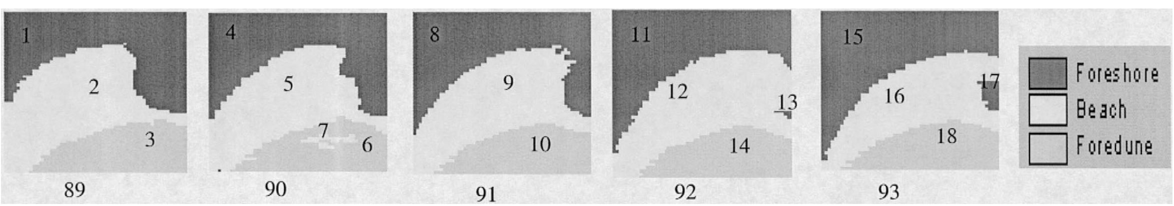

Fig. 3. Regions detected in the years 1989 to 1993.

Table 2
Fuzzy overlaps among fuzzy regions

Year	Region	Area	Overlap with regions in the next year				Class Type
1989	1	1108.1	937.5	81.8	0.0	0.0	Foreshore
	2	1246.8	106.3	1104.8	9.2	0.0	Beach
	3	644.3	0.0	12.7	572.5	27.5	Foredune
1990	4	1138.7	975.0	76.0	0.0		Foreshore
	5	1229.7	76.0	1129.5	2.6		Beach
	6	586.8	0.0	0.0	564.3		Foredune
	7	28.0	0.0	0.0	26.3		Beach
1991	8	1101.3	862.7	116.9	6.4	0.0	Foreshore
	9	1260.1	87.3	1146.6	0.0	0.5	Beach
	10	609.8	0.0	3.3	0.0	605.7	Foredune
1992	11	1004.9	751.5	6.8	0.0	0.0	Foreshore
	12	1288.7	119.3	1101.1	38.9	2.8	Beach
	13	6.4	0.0	1.6	4.6	0.0	Foreshore
	14	625.7	0.0	2.7	0.0	604.4	Foredune

linked to form lifelines of objects that may have “shifted”, “expanded” or “shrunk” between two successive epochs. The regions that appeared at a specific moment represent new objects, and regions that disappeared at some moment represent disappearing objects. Furthermore, “merging” and “splitting” objects can be identified. The procedure to identify dynamic objects will be explained by means of an example based on the regions detected in our coastal zone area in the years 1989 to 1993 (Fig. 3).

Table 2 presents the fuzzy sizes of regions and the fuzzy overlaps of regions in four successive years. With the indicators of Section 4 we can link the regions as shown in Fig. 4. These links indicate that the connected regions are most likely the spatial extents of an object in successive years. For exam-

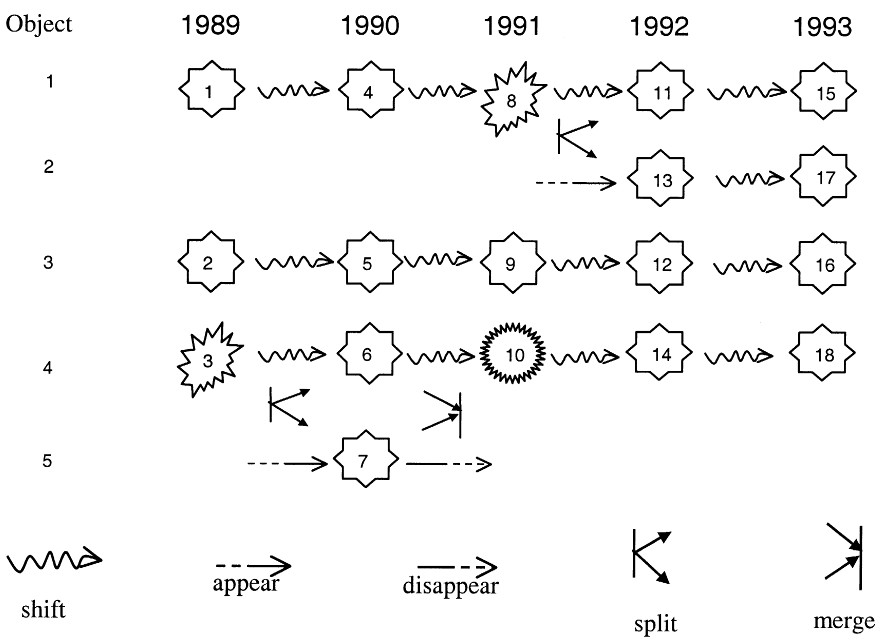

Fig. 4. Identified fuzzy objects and processes.

ple, region 1 has been linked with 4 and 4 with 8; region 3 has been linked with region 6 and 6 with 10. We also see a new region in 1990 (region 7). By checking the overlap of this region with the regions at 1989 and 1991, we find overlaps with regions 3 and 10; these regions are linked by a line also.

For example, the spatial overlap of region 3 in 1989 (S_a) and region 6 in 1990 (S_b) is 572.5 (Soverl(S_a,S_b)), and here $S_a = 644.3$, $S_b = 586.8$. So,

$$\text{ROvel}(S_b \mid S_a) = 572.5/644.3 = 0.819$$

$$\text{ROvel}(S_a \mid S_b) = 572.5/586.8 = 0.976$$

$$\text{Similarity}(S_a \mid S_b) = 0.894$$

These two regions are very similar to each other and can be considered as instances of the same object 3 at two epochs. As there are differences between the boundaries of these two regions, we conclude that object 3 has shifted from region 3 in 1989 to region 6 in 1990. We also calculated the similarities between region 3 (as S_a) and region 7 (as S_b),

$$\text{ROvel}(S_b \mid S_a) = 27.5/644.3 = 0.043$$

$$\text{ROvel}(S_a \mid S_b) = 27.5/28.0 = 0.982$$

$$\text{Similarity}(S_a \mid S_b) = 0.205$$

These two regions are not similar, but region 7 is more or less contained in region 3. This means that a new object appeared in 1990, resulting from a split from object 3. Analysis of the overlaps between regions of 1990 and 1991 shows that region 7 disappeared in 1991, it was merged into object 3 (region 10 in 1991).

With this approach, the objects and their transition processes can be identified, see Fig. 4. The icons represent the extents of the objects at different epochs. The symbols represent the types of state transition. For example, it can be seen that object 7 was split from object 3 between 1989 and 1990 and merged again into object 10 between 1990 and 1991.

6. Conclusion

This paper presents a method to identify fuzzy objects and their dynamics from field data sampled at different times. The method has been illustrated by means of an example based on a coastal geomorphologic study. It is expected that this method would be useful for applications where vague thematic concepts lead to uncertainty in the spatial descriptions of objects. This is often the case when environmental processes are monitored.

Our example showed how the uncertainties in the field observation data and in the definition of object classes propagate into the identification process of the spatial extents of objects at different epochs. Therefore, the spatial uncertainty of objects is due to the uncertainties of their thematic aspects. The formalism of Section 2 made this explicit. With this formalism a distinction could be made between the existential, extensional and geometric uncertainty of spatial objects. This formalism also shows that in this respect there is no difference between raster and vector representations of such objects.

The fuzzy spatial extents at different epochs represent the states of objects. The state transition processes of these objects can be determined through the relationships between these states. These relationships were evaluated by means of parameters that were derived from the formalism of the parameters in Section 2. With the state transition processes the evolution of the fuzzy objects could be traced.

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