FUZZY SETS APPROACH TO SPATIAL ANALYSIS AND PLANNING — A NONTECHNICAL EVALUATION

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ABSTRACT. Uncertainty in spatial analysis and planning has conventionally been treated as a consequence of random occurrence of exact spatial events. The present paper, however, argues that fuzziness is actually a major contributory factor to most of the uncertain spatial behavior. Exactitude in representing, analyzing, and predicting human behaviors over space and time is difficult if not impossible to accomplish in a fuzzy environment characterized by ambiguous or incomplete information and inexact cognitive and decision-making processes. Fuzzy sets theory is proposed as an appropriate framework for a formal representation and analysis of inexact spatial concepts, structures, and processes. In place of a technical presentation, the versatility of fuzzy sets theory in spatial analysis and planning is discussed from a pedagogical point of view. Applications of the theory to the conceptualization of imprecise spatial concepts, analysis of uncertain spatial behavior, and spatial decision analysis and planning are examined. Directions for further research in these areas are proposed. Compared to conventional methods, fuzzy sets approach appears to be more natural and powerful in analyzing imprecision of human spatial behaviors. Policy analysis and planning with inexact information and value-based standards can also be effectively handled. Though the development and applications of fuzzy sets theory to spatial analysis and planning has been limited and fragmentary, its potential contributions to various areas of research such as behavioral geography, cartography, remote sensing, and soft spatial data analysis are perceivable. It may prove to be a promising direction for the reconstruction of existing theories and the formulation of new theories and models of uncertainty in spatial analysis and planning.

Introduction

Imprecision appears to be a general characteristic of human behavior over space and time. Over the years, considerable difficulties have been encountered in attempting to analyze and predict with exactitude the spatial preference, choice, and movement of individuals. Traditionally, imprecision is equated with randomness. We are uncertain about a specific behavior because its occurrence is random. Based on this assumption, numerous probability models of spatial behavior have been developed.

Scrutinies of human behavior, however, reveal that randomness only accounts for a portion of uncertainty. Vagueness in human systems, on the other hand, seems to be a major attributing factor. Apparently, inexact concepts are rampant in human cognitive and decision-making processes. For example, when choosing the location of our home, we may employ criteria such as *easy* accessibility, *inexpensive, good* public services, and *low* crime rate. In deciding on a mode of transportation, *economical, relatively comfortable, good* service, and *low* accident rate may serve as standards. In determining a place to shop, the basic requirements may be *not too far away, relatively high* quality products, and *reasonable* prices. The italiazed terms are inexact concepts whose meanings are fuzzy. For example, there are no exact boundaries demarcating *economical* in terms of fare, or *not too far away* in terms of physical distance. Thus, uncertainty arising from the interpretation and processing of these inexact concepts has nothing to do with randomness but is directly related to fuzziness.

The basic difference between randomness and fuzziness is that randomness concerns uncertainty about the membership or nonmembership of an object in a nonfuzzy class, while fuzziness is associated with the gradual transition from nonmembership to membership of an object to a fuzzy class (Bellman and Zadeh, 1970). For instance, "The probability that John will go to store X is 0.8", is a probabilistic statement about the uncertainty of the occurrence of the nonfuzzy event "go to store X". "The grade of membership of 50 kilometers in the class, a *long* distance, is 0.6", however, is a statement concerning the membership of "50 kilometers" in the fuzzy class "long". Therefore, when fuzziness instead of randomness prevails, a theoretical framework, other than probability, which is capable of representing and analyzing inexact concepts and logics is required. Among existing analytical methods, fuzzy sets theory (Zadeh, 1965) is, perhaps, the most appropriate foundation for the analysis of inexactness in human systems.

Based on the theory, the concept *long* in the measurement of distance can be characterized as

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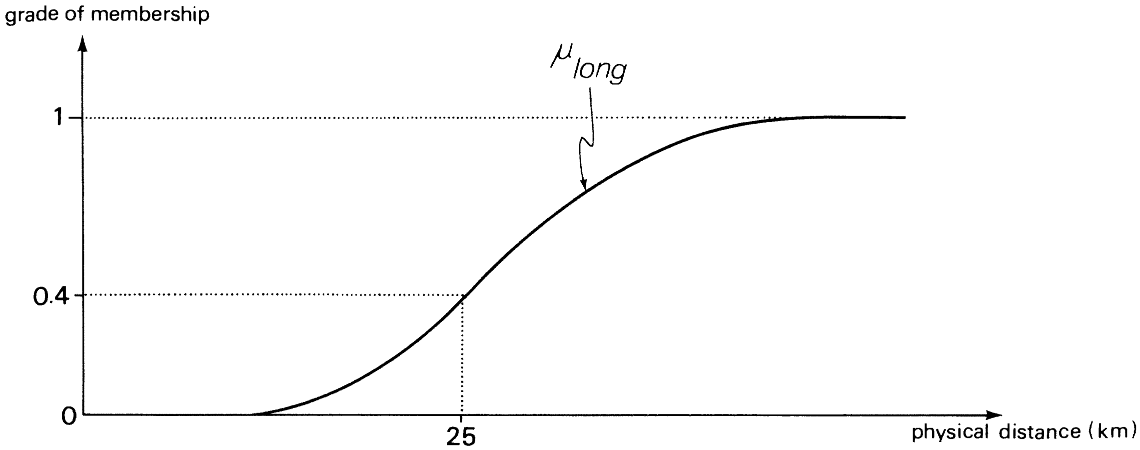

Figure. Membership function of the fuzzy subset *long*.

a fuzzy subset defined by a membership function, μ_{long}, which maps the universe of discourse, e.g. physical distance in kilometers, to a membership set M, e.g. [0, 1]. (See Figure).

The grade of membership μ_{long} (x) represents the degree of belonging of an element x in the universe of discourse U to the concept *long*, with μ_{long} (x) = 0 representing complete nonmembership and μ_{long} (x) = 1 indicating complete membership. The degree of belonging increases from zero to one as the value of x increases. The basic assumption is that a small variation in x should have at most a small effect on the degree of belonging of x to *long*. Thus, the concept *long* can be viewed as a continuous function which imposes a fuzzy restriction on the value of the base variable, physical distance. Obviously, such a characterization of a fuzzy concept is more natural than the Boolean logic which does not allow statements of intermediate truth.

Since its inception, the theory has been applied to a variety of problems such as pattern recognition, control, linguistics, logics, medical diagnosis, and decision analysis. Though a number of analysts (Gale, 1972; Pipkin, 1978; Ponsard, 1980a; Leung, 1982a) have argued that fuzzy sets theory can serve as an appropriate foundation for spatial analysis, the theory has developed in a slow and fragmentary way. This is true, of course, of the application of many other mathematical methods. To stimulate more research in this direction, I think it is timely to discuss, from a pedagogical point of view, the appropriateness of applying fuzzy sets theory to spatial analysis and planning.

In each of the following sections, applications of the theory to specific areas of spatial analysis and planning are first examined. Directions for further research are then proposed. Our discussion begins with the characterization of some fundamental spatial notions constituting a basis for spatial analysis in an inexact environment. In section III, the focus is centered on the analysis of uncertain spatial behavior in general and spatial economic behavior in particular. Section IV deals with issues of ambiguity in policy analysis and planning. Section V consists of plausible directions of research in areas which have not been analyzed in the previous sections but deserve serious attention. The paper then concludes with a remark of spatial analysis and planning through fuzzy sets theory. Though the present paper is not intended to serve as a technical review of the theory and its applications, an appendix summarizing the basic concepts is prepared as reference.

Conceptualization of imprecise spatial concepts

Spatial classification is perhaps one of the most fundamental issues in spatial analysis. Except for some situations where regional concepts are immaterial, most spatial analyses are directly or indirectly related to the concept of a region. Classification of climatic, agricultural, or economic regions is strictly a regional classification problem. Interegional flow, regional disparities of growth, and regional economic development all employ region as a basis of analysis.

Conventionally, a region is treated as an entity or a theoretical construct which can be exactly defined and delimited. Boolean logic, a two-valued system, has been employed to subdivide space into a set of mutually exclusive and exhaustive regions with clear-cut boundaries.

Such a system ignores the fact that two spatial units are only similar, with respect to specified attributes, to a certain degree, or their interaction exists in varying degrees of intensity. Realistically, gradual variations of most spatial phenomena often make it difficult if not impossible to establish a non-fuzzy boundary so that a dichotomous regionalization makes sense. Vagueness of the regional names, usually words or sentences in our everyday language, further compounds the difficulty of deriving well-separated regions. A region is obviously a fuzzy theoretical construct with an ambiguous boundary. A spatial unit may belong to a region only to a certain degree. In place of the Boolean logic, a new theoretical foundation is required to construct a more natural characterization of the regional concepts.

Based on its ambiguity, Gale (1972, 1975, 1976), Ponsard (1980a), and Leung (1983) have proposed that a region can be conceptualized as a fuzzy subset having imprecise spatial connotations. In climatic classification, for example, a "hot region" may be characterized as a fuzzy subset *hot* defined by a membership function μ_{hot} which maps the value of a base variable x, e.g. temperature (°C), into a membership set [0, 1], with μ_{hot} (x) indicating the degree of belonging of an area having x°C to the *hot* region. Thus, instead of imposing an arbitrary cut-off point between *hot* and *not hot* regions, gradual transition from membership to non-membership of a *hot* region is employed. In general, such a characterization is consistent with how we feel about hotness and the more or less continuous variation of temperature over space. As a result, regions may be separated only to a certain degree. Overlapping of regions, ordinarily observed in reality, is preserved.

Since a region is only a class name which comes from everyday language, then regional classification is technically linguistically-based. Employing a concept of a linguistic variable (Zadeh, 1975a, b, c) and a theory of possibility (Zadeh, 1978), Leung (1983) has proposed a spatial classification framework so that linguistically-based regional concepts can be formally represented and regional assignment can be performed. With respect to a set of attributes, a region is characterized by a linguistic proposition which is translated into a possibility distribution defining the fuzzy connotation of the region name.

For example, a climatic region may be characterized by a linguistic proposition, "Region X is *hot* and *wet*", which is defined by a possibility distribution function

$$\text{Poss (temperature} = \text{u, precipitation} = \text{v)} = \min (\mu_{hot}\,(\text{u}), \mu_{wet}\,(\text{v})). \quad (1)$$

It states that if the temperature is u and precipitation is v, then the possibility of a region being considered as *hot* and *wet* equals the minimum of its grade of membership of being *hot* and that of being *wet*.

If the temperature of a spatial unit Y is *around* α and its precipitation is *around* β, then the possibility of assigning Y to a *hot* and *wet* region X is

$$\begin{aligned}&\text{Poss (temperature (Y) is } \textit{around } \alpha \text{ and}\\&\text{precipitation (Y) is } \textit{around } \beta |\\&\text{temperature (X) is } \textit{hot} \text{ and}\\&\text{precipitation (X) is } \textit{wet})\\&= \min (\sup \min (\mu_{around\ \alpha}(\text{u}), \mu_{hot}(\text{u})),\\&\sup \min (\mu_{around\ \beta}(\text{v}), \text{u}_{wet}(\text{v})). \quad (2)\end{aligned}$$

The assignment equation states that the possibility of assigning spatial unit Y to region X is equal to the minimum value of the height of intersection of fuzzy subsets *around* α and *hot* with respect to temperature and that of *around* β and *wet* with respect to precipitation. A simpler interpretation is that the possibility of assigning spatial unit Y to region X depends on how well the pattern (fuzzy subsets) characterizing Y matches that of X in each characteristic (in this example, they are represented by the height of intersection of *around* α and *hot* in temperature, and that of *around* β and *wet* in precipitation). The possibility of Y being considered as a *hot* and *wet* region is equal to the smallest height of all the intersections. This smallest height is the minimum threshold applied to all characteristics.

Thus, a region is a linguistically characterized concept with inexact meaning. Due to the imprecise information about a spatial unit, it may only be assigned to a region to a certain degree. Complete membership, however, is possible if its characterizing patterns are identical to those of the region.

Sometimes, in regional classification, the number of regions and their names are not predetermined. Conventionally, techniques such as cluster analysis, discriminant analysis, and other grouping algorithms are employed to partition a data space into a set of mutually exclusive and exhaustive regions. Regional names are then subjectively assigned. With fuzzy information, fuzzy clustering techniques (Bezdek, 1981) can be applied. The method identifies regions as fuzzy clusters which constitute a fuzzy partition of a data space. That is, a spatial unit may belong to several regions with varying degrees of intensity. The partition fuzziness depends on how exact the data points are.

In addition to the concept of affinity, regional classification may be based on interactions among spatial units. Hierarchical systems of regions may be derived accordingly. Since interaction exists with varying degrees of intensity, conventional binary representation is usually inadequate. To have a better approximation to reality, degree of interaction should be employed as a basic concept in deriving hierarchical regions. Leung (1980a) has applied concepts of a fuzzy binary relation to model sociometric structures in which vagueness of liking or disliking between individuals prevails. Based on the degree of liking or disliking, small groups with fuzzy boundaries are identified and a hierarchical group structure is established. If we treat spatial units as individuals and interaction between spatial units as liking or disliking, regions can likewise be determined and hiearchical systems be constructed accordingly (Ponsard, 1980a).

Therefore, the major contribution of fuzzy sets theory to spatial classification is that it provides a more natural definition of a region and a more appropriate procedure for regionalization. Instead of avoiding ambiguity while it exists, fuzzy sets approach gears to the formal representation and analysis of uncertainty in spatial classification problems.

Vagueness in regional classification is not the unique feature in spatial analysis. Spatial cognition and perception are also inexact in nature. Basic concepts such as distance, connection, and direction are ambiguous in our cognitive environment. In place of numbers, subjective distance, connection, and direction may take on as their values the words or sentences in our everyday language (Leung, 1982a). For example, the value of distance may be *long, short, very long,* or *somewhat short* which are terms with fuzzy spatial connotations. The connection between two places may be cognized as *close* or *not too close*. A specific direction may be identified as *a bit west or somewhat east and somewhat north*. These spatial concepts, in fact, can be characterized as linguistic variables whose values are linguistic terms generated by a syntactic rule. A semantic rule then gives each term its meaning, a fuzzy subset. For instance, the membership function μ_{north} is a characterizing function which associates a physical measurement of direction with the linguistic term *north*. Once the formal representation of the linguistic spatial variables is constructed, operations in fuzzy sets theory can be employed to analyze or execute uncertain spatial decision rules.

Thus, imprecise spatial concepts can be effectively characterized via fuzzy sets theory and a theory of possibility. Though research in this area has been limited, the potentiality for future development appears to be promising. In addition to a more extensive analysis of various ambiguous spatial concepts, further research should aim at the conceptualization of time, especially subjective time. Ideally, a space-time framework can be constructed to serve as a basis for the analysis of inexact spatial structures and processes. The possibility of building a spatial language should also be explored.

Analysis of uncertain spatial behaviour

As previously discussed, complexity of human systems, insufficient or inexact information, and ambiguous cognitive and decision-making processes are major factors contributing to the uncertainty of spatial behavior. However, conventional spatial theories or models assume perfect information and precise cognitive and decision-making processes. Such an assumption is especially prominent in spatial economic analysis. Consumers are assumed to be capable of making a clear-cut discrimination with regard to different goods or places. Their preferences take on a Boolean structure. A producer, likewise, is assumed to possess perfect information about the market and complete control over inputs and outputs. His utility of profit is again Boolean. Thus, a consumer's or a producer's decision-making process is exact and their spatial behavioral patterns are certain.

Such a two-valued system of preference-utility-choice structure, nevertheless, often fails in its ability to explain uncertain spatial behavior in an inexact environment. Pipkin (1978), Leung (1979a, 1980b), Nijkamp (1979), and Ponsard (1979, 1980a, b, 1982) all argue that with prevailing fuzziness in our information system, preference and utility are generally vague. Individuals may not be able to discriminate perfectly between different goods or places. Their preference and utility are thus fuzzy. This section attempts to examine the basic idea along this line of reasoning through several spatial economic problems.

Market area analysis is an issue which has been extensively studied in spatial economic analysis. Given the location of a number of firms in a market, the typical problem is to determine how the market should be divided among the firms. By common practice, a market is subdivided into a set of mutually exclusive and exhaustive market areas. A basic assumption of this approach is that consumers can discriminate between different goods or places with exactitude. Their preference for a firm is exact so that no consumers choose more than one firm in their travel-making decisions.

Carlucci and Donati (1977) have demonstrated that when criteria governing a decision are fuzzily evaluated, preference for a firm becomes vague. Market areas appear to be fuzzy clusters. Overlapping instead of clear-cut demarcation of market areas is observed. The situation is in many ways similar to the regionalization problem discussed in section II.

When criteria such as accessibility, quality, service, and price are evaluated as fuzzy subsets such as *easy, good, good,* low respectively, Leung (1980b) proposes to construct consumer's preference for a firm as a convex fuzzy subset. Employing a separation theorem, market area between firms is determined. Again, overlapping of market areas turns out to be a general phenomenon rather than an exception. Applying the concept of a separation threshold, the degree of separability of the whole market is derived. Except for special circumstances, the market can only be subdivided to a certain degree. The conclusion then is that existing ambiguity in consumer's preference structure prevents a mutually exclusive subdivision of a market.

Imprecision not only exists in travel-making decisions, but also plays an important role in locational choice. With imprecise and incomplete information, evaluations of the relative importance of locational factors and attractiveness of sites may not be exact. Binary characterization of locational factors, *important* and *not important,* and sites, *attractive* and *not attractive,* is usually difficult. *Important* and *attractive* are in fact fuzzy subsets. A locational factor is important to a certain degree, so is the attractiveness of a site. Consequently, binary locational choice structure is inappropriate.

Based on this rationale, a fuzzy-set locational choice rule with hierarchical objectives is formulated (Leung, 1979a). The decision rule is a max-min composition of a series of fuzzy binary relations which relays, through the hierarchy, the relative importances of individual sites in satisfying various level objectives. The result of the analysis shows that a site may only be preferred over another to a certain degree.

Similar in spirit but different in approach, Bona, Inaudi, and Mauro (1980) reason that criteria for a residential choice are usually vague. For example, *not too far away* and *good enough* can be judgements on the locational factors "distance to work" and "quality of housing" respectively. Based on the fuzzy evaluations, a satisfaction function is constructed. Residential choice is derived through the maximization of the satisfaction function. The problem turns out to be a nonlinear programming problem.

Employing fuzzy preference and utility, Ponsard (1979, 1980b) investigates a key issue of spatial economic analysis, namely a consumer's and a producer's spatial equilibrium, in a fuzzy environment. Conventionally, a consumer's spatial equilibrium is obtained through the maximization of an exact utility function subject to an exact budget constraint. When inexactness prevails, nevertheless, a utility function and a budget constraint may only be fuzzily specified. For example, a constraint may be expressed as a fuzzy statement, "The total budget should be smaller than a or *not much greater than* a". Under this circumstance, a consumer's spatial equilibrium can only be derived through the maximization of the fuzzy utility subject to a fuzzy budget constraint. The problem boils down to a fuzzy mathematical programming problem. By the same token, a producer's spatial equilibrium is derived via the maximization of a fuzzy utility of profit subject to a fuzzy technological constraint (Ponsard, 1980b). It is demonstrated that the equilibrium does not necessarily have to be achieved at a point or a set of points on the technological frontier since the production set is fuzzy in nature. The advantage then is that the decision-maker can exhaust all possible situations in which equilibrium can be attained.

When a consumer's utility function is exact but the budget constraint is fuzzy, or a producer's utility of profit is exact but the technological constraint is fuzzy, spatial equilibrium is derived from maximizing an exact objective function subject to a fuzzy constraint. The result reduces to the solution of a fuzzy integral (Ponsard, 1982).

Thus, the non-Boolean preference-utility-choice structure is mainly a result of inexact information and fuzzy cognitive and decision-making processes. Though research on fuzzy spatial behvaior is fragmentary, interesting and more realistic results, as reviewed, in market area analysis, locational choice, and spatial equilibrium analysis have been derived. To have a more thorough examination of the power of the fuzzy sets approach and to enable reconstructions of spatial models, spatial behavior in various socio-economic systems needs to be analyzed. Further research should also focus on the analysis of dynamic spatial systems in a fuzzy environment, albeit mathematically more complicated. Hopefully, general theories of spatial behavior can be constructed, and human behavior over space and time can be better approximated.

Multicriteria decision-making and planning

Spatial decision-making and planning often involve multiple objectives, usually conflicting, and multiple decision-making units, usually with conflicting interests. In conventional analysis, decision-making units are well-separated clusters, alternatives are well-defined, and evaluations of the effectiveness of alternatives in achieving prescribed criteria are exact. Precise discrimination of policies or alternatives is thus imposed. In reality, as emphasized throughout, such a state of certainty seldom exists. Vagueness in our decision-making processes may prevent exact evaluations of alternatives. In place of precise numerical scores, judgement with linguistic terms may have to be employed.

In formulating a fuzzy choice problem, Nijkamp (1979) employs fuzzy subsets such as fairly irrelevant, relevant, or very relevant in the priority ranking of criteria. The efficiency of an alternative is characterized by terms like *very high value, high value, normal value,* or *low value*. The dominance between alternatives is described as *great difference, normal difference,* and *negligible difference*. Based on this fuzzy information about the criteria and alternatives, pair-wise comparison of alternatives in terms of their performances is then constructed, and a dominance structure is derived for all alternatives. Subsequently, alternatives can be rank-ordered and selection can be made accordingly. The advantage of this approach is that imprecise concepts can be incorporated into a multicriteria decision-making framework.

In an analysis of project selection, Leung (1979b) employs a criterion function by which the relative merits of projects with respect to their *worth, cost,* and *risk* are evaluated. Linguistic judgements such as *high* worth, *low* cost, and *slight* risk are employed to characterize alternatives. The criterion function is constructed through the intersection of the linguistic variables *worth, cost,* and *risk*. Alternatives are then selected on the basis of their degrees of compatibility in the criterion function. Such a procedure is demonstrated to be versatile to solve project selection problems interwoven with imprecise information and judgemental standards.

A basic concept in multicriteria decision analysis is the notion of a comprimise solution. A compromise solution is sought whenever conflicting objectives are involved. To enable a dynamic multigroup, multicriteria conflict resolution with inexact information, Leung (1981e) proposes an extended model of a displaced ideal in which value-based rules characterized by linguistic variables are embedded. The process consists of two interactive and iterative stages. The first stage is conflict resolution within a decision-making unit. The notion of a local ideal, the best alternative within a decision-making unit, is formulated as a basis for deriving a local compromise solution. A linguistic variable, proximity, with values such as *close to* and *very close to* is employed to define the proximity of an alternative to the ideal solution with respect to a criterion. The local compromise solution is derived from minimizing a distance function between any alternative and the local ideal. A similar procedure is also applied to obtain the global ideal solution, the best alternative for all parties concerned. The global compromise solution, the alternative which has the least deviation from the global ideal, is again derived from a distance minimization procedure.

To enhance the generality of the ideal solution method and to better approximate to real-life conflict resolution, Leung (1981f) further proposes a concept of a fuzzy ideal for multicritera conflict resolution. Conventionally, the ideal and the compromise solutions are point-valued with exactitude. Since the evaluation rules are linguistically-based and are inexact in nature, the ideal derived from them is demonstrated to be fuzzy. By employing a concept of a linguistic variable and a theory of possibility, the fuzzy ideal is defined by a m-cell whose element is a composite linguistic proposition about a criterion. For example, with respect to net return, the performance of the ideal alternative, taking all alternatives into consideration, may be specified as "net return should be *between 5 and 11 million dollars* and *around 9 million dollars* and *a little more than 6 million dollars*". Its numerical counterpart, a fuzzy region, is delimited by fuzzy intervals imposed by the associated possibility distributions translated from the composite linguistic propositions.

The advantage of the concept of a fuzzy ideal is that it allows for qualitative as well as quantitative evaluations of criteria and alternatives. Without confining the ideal solution to a point, the concept provides a set of ideal values for deriving compromise solutions. Thus, it enables us to exhaust all possible ways in which a compromise solution may be achieved. The conflict resolution process becomes more flexible and powerful. To allow for exact solution, zero-in procedures, defuzzification mechanisms, are also constructed to obtain point-valued solution.

Based on the concept of a fuzzy ideal, dynamic conflict resolution through a theory of a displaced fuzzy ideal is formulated (Leung, 1982b). Displacement of a fuzzy ideal is activated by changes in the evaluations of alternatives with respect to criteria, or the deletion or addition of criteria or alternatives. A concept of a displaced fuzzy local ideal is constructed to resolve conflict within a decision-making unit. A notion of a displaced fuzzy global ideal is formulated to resolve conflicts among decision-making units. Again, the whole process is interactive and recursive in nature.

In addition to the problems discussed above, planning with inexact objectives and constraints is a common multicriteria decision-making problem. When objectives and constraints are exactly specified, a wealth of mathematical programming techniques is available to find an optimal solution of an optimization problem. Unfortunately, exact information is difficult to ascertain. Imprecise information and inexact cognitive and decision-making processes tend to force planners to formulate vague objectives and constraints. For example, instead of requiring the total usage of a resource to be less than or equal to a specific value α, we may only be able to restrict it to a tolerance level "*around* α", a fuzzy subset. Likewise, in place of maximizing an objective function, we may only be capable of specifying a satisfaction level within which the value of the objective function is achieved. A fuzzy linear programming framework has been applied to solve urban and regional programming problems with inexact objective and exact and inexact constraints (Leung, 1982c). Kacprzyk and Straszak (1980, 1981) have also proposed a multistage fuzzy mathematical programming method to determine an optimal sequence of controls for integrated regional development with respect to a fuzzy notion of quality of life.

The major advantage of the fuzzy mathematical programming approach is its ability to handle both exact and inexact constraints or objectives. Planning with tolerance is thus made possible. It appears to be a profitable direction for the development of robust programming methods.

Though only a few aspects of multicriteria decision-making and planning have been examined in this section, the versatility of fuzzy sets approach is persuasive. Other problems such as locational externalities, spatial justice, location-allocation problems, and spatial welfare economics which involve value judgements should be investigated in further research. Fuzzification of conventional decision-making models or techniques, then lead to better approximation of reality and more general results. Future development, perhaps, should be geared to the construction of a completely different framework, e.g. a purely linguistic approach, so that spatial decision analysis and planning can be examined with new perspectives.

Some plausible areas of research

In the previous sections, fuzzy sets analysis of various spatial analytical and planning problems have been reviewed and directions for further research have also been proposed. Nevertheless, current fragmentary research efforts should only be viewed as an initial probing of the fuzzy sets theoretical approach in spatial analysis and planning. Hopefully, it can stimulate more comprehensive and in depth research in the field.

To explore other possible applications, I attempt, in this section, to suggest several areas to which fuzzy sets theory can be profitably applied. Ideally, researchers in these areas will pursue and expand on them.

Since fuzzy sets theory is developed for the representation and analysis of imprecise concepts, it can be employed as an analytical foundation for behavioral geography (Gale, 1972). Problems such as mental maps, way finding, spatial cognition, perception, and decision are mostly inexact in nature. Their characterizations and interpretations can be effectively handled by concepts of fuzzy sets theory. The analysis of subjective distance, connection, and direction is an obvious example (Leung, 1982a). With more research in the field, a more cohesive analytical framework of inexact spatial behaviors may be developed in the future.

In the mapping and interpretation of spatial configurations and images, geometry and topology are methods ordinarily employed. Conventional figures such as a square, a circle, or an equilateral triangle are exactly identified. However, geometrical figures sometimes cannot be exactly defined or identified, especially on a cognitive basis. A figure may be identified as *more or less* a square, *almost* a circle, or *somewhat like* an equilateral triangle. The representation and identification of these fuzzy figures can be handled by fuzzy sets theory. Lee (1976), for example, has suggested some procedures in identifying fuzzy versions of an equilateral triangle. So, to enrich our inventory of plausible spatial configurations, it may be worthwhile to investigate possible applications of fuzzy sets theory in cartographical research. Furthermore, the theory may also serve as a basis for the representation and interpretation of fuzzy images in remote sensing or air-photo interpretation.

In recent years, analysis of soft data has become a major concern in social sciences, especially in regional economics, sociology, psychology, geography, and transportation planning. The movement essentially stems from the realization that, contrary to conventional assumption, most of the concepts and information in social science research are in fact noncardinal. Soft data (categorical, ordinal, or fuzzy), as demonstrated throughout this paper, are obviously rampant in spatial analysis and planning. Fuzzy sets theory in general and possibility theory in particular can be employed as a foundation for the development of a new set of techniques for soft spatial data analysis.

Though uncertainty may be due to randomness or fuzziness, it may in fact be a result of both. For example, "What is the probability of having a *warm* day tomorrow?", is a question on the probability of the random occurrence of the fuzzy event *warm*. To analyze this type of phenomenon, confluence of probability and fuzzy sets theory may be necessary (Zadeh, 1968; Leung, 1981d). From combining the two methods, analysis of uncertain spatial structures and processes may be made more powerful.

Theoretically, fuzzy sets theory appears to be more efficient in analyzing inexact concepts. Several pragmatic problems, albeit important, have not received as much attention as they should have. First, the membership function defining a fuzzy subset is a priori in most fuzzy sets research. Bellman and Zadeh (1970) have argued that since a membership function is subjectively defined, it is then impossible to derive it empirically. A number of researchers, however, think otherwise. Kochen (1975), Dreyfuss, Kochen, Robinson, and Badre (1975), Hersh, Caramazza, and Brownell (1979), Paelinck (1979), and Leung (1981a) have performed empirical analyses on the determination of the membership function. Linguistic hedges modifying the meaning of a fuzzy subset have also been subjected to a few experiments (Hersch and Caramazza, 1976; Leung 1981b, c; MacVicar-Welan, 1977). Though the empirical results are in general encouraging, the basic problem of obtaining the most representative membership function has not been solved. To enhance the applicability of fuzzy sets theory to human systems research in general and spatial analysis and planning in particular, more empirical analysis is necessary. Gougen's concept of social truth (Goguen, 1979) may be a plausible research direction. Secondly, in combining fuzzy subsets, operations such as max-min-, algebraic product, or algebraic sum are employed. Sufficient experimental supports, however, have yet to be obtained (Rödder, 1975). Extensive research on how closely these operations approximate human manipulation of fuzzy concepts should be performed. Lastly, inferential procedures, such as max-min composition, involving inexact concepts has not been investigated empirically. Experimental analysis in this area may prove to be important in improving the theoretical development of fuzzy inferential procedures.

Conclusion

I have examined in this paper the development and applications of fuzzy sets theory to spatial analysis and planning. Directions for further research in various areas have also been proposed. Though the current review is intended to be as complete as possible, it is by no means to be an exhaustive account of the literature in the field. The author may not be aware of other completed or on going research, especially those which are not documented in English. Hopefully, this paper can act as a catalyst for the building of a more complete literature in this area of research. Though plausible directions for further research may be far from ideal in terms of breadth and depth, they are proposed to stimulate a comprehensive and thorough research effort in exploring the fuzzy sets approach to spatial analysis and planning.

One should not have the misconception that fuzzy sets researchers fuzzify exact systems unnecessarily, perhaps over-enthusiastic ones may have such a tendency. On the contrary, fuzzy sets approach is developed out of necessity. As demonstrated throughout this paper, a large number of spatial and planning problems simply cannot be analyzed or solved with exactitude when our systems are highly complex, and our information and decision-making processes are ambiguous. Fuzzy sets theory may not provide the best or the ultimate approximation to uncertain human behavior, it is, nevertheless, one of the most appropriate and promising frameworks for the analysis of inexact spatial and planning systems. The incorporation of fuzzy sets theory will certainly enrich our existing mathematical and quantitative methods in spatial analysis and planning.

Appendix

This appendix is only a summary of the basic concepts of fuzzy sets theory which are relevant to the discussion in this paper. A more thorough exposure may be found in Zadeh (1965), Kaufmann (1975), Negoita and Ralescu (1975), and Dubois and Prade (1979).

1. *Fuzzy subset.* Let U be a universe of discourse, let x be an element of U. Then a fuzzy subset A in U is a set of ordered pairs

   $\{(x, \mu_A(x))\}$, for all $x \varepsilon U$,

   where, μ_A: U→M is a membership function which takes its values in a totally ordered set M, the membership set, and $\mu_A(x)$ indicates the grade of membership of x in A.
2. *Height.* The height of a fuzzy subset A is the least upper bound of A, defined as height (A) $= \sup \mu_A(x)$.
3. *Convex fuzzy subset.* A fuzzy subset A is convex if and only if $\mu_A(\lambda x_1 + (1-\lambda)x_2) \geq \min [\mu_A(x_1), \mu_A(x_2)]$, for all $x_1 \varepsilon U$, $x_2 \varepsilon U$, for all $\lambda \varepsilon [0, 1]$.
4. *Inclusion.* A fuzzy subset A is included in a fuzzy subset B, denoted as $A \subset B$, if and only if $\mu_A(x) < \mu_B(x)$, for all $x \varepsilon U$.
5. *Equality.* Fuzzy subsets A and B are equal, denoted as A = B, if and only if $\mu_A(x) = \mu_B(x)$, for all $x \varepsilon U$.
6. *Complementation.* Fuzzy subsets B is the complement of fuzzy subset A, if and only if $\mu_B(x) = 1 - \mu_A(x)$, for all $x \varepsilon U$.
7. *Intersection.* The intersection of fuzzy subsets A and B, denoted as $A \cap B$, is defined by $\mu_{A \cap B}(x) = \min [\mu_A(x)_1, \mu_B(x)]$, for all $x \varepsilon U$.
8. *Union.* The union of fuzzy subsets A and B, denoted as $A \cup B$, is defined by $\mu_{A \cup B}(x) = \max [\mu_A(x), \mu_B(x)]$, for all $x \varepsilon U$.
9. *Algebraic product.* The algebraic product of fuzzy subsets A and B, denoted as $A \hat{+} B$, is defined by $\mu_{A \cdot B}(x) = \mu_A(x) \cdot \mu_B(x)$, for all $x \varepsilon$ U.
10. *Algebraic sum.* The algebraic sum of fuzzy subsets A and B, denoted as A+, is defined by $\mu_{A \hat{+} B}(x) = \mu_A(x) + \mu_B(x) - \mu_A(x) \cdot \mu_B(x)$, for all $x \varepsilon U$.
11. *Fuzzy relation.* A n-ary fuzzy relation is a fuzzy subset in $U_1 \times U_2 \times \ldots \times U_n$ defined by $\mu_R(x_1, x_2, \ldots, x_n) \varepsilon [0, 1]$, $x_i \varepsilon U_i$, for i = 1, 2, ..., n. Specifically, a binary fuzzy relation is a fuzzy subset in $U_1 \times U_2$.
12. *Linguistic variable.* A linguistic variable is a variable whose values are words or sen-

tences in our everyday language. It may be designated by a quintuple (X, T(X), U, G, M). The symbol X is the name of the variable, e.g. distance. T(X) is the term set of X which consists of terms like *short* and *long*. U is the universe of discourse. The term in the term set are generated by a syntactic rule G. A semantic rule M gives each term its meaning M(X), a fuzzy subset in U.

13. *Possibility distribution.* A possibility distribution π on a universe of discourse U is a mapping from U to [0, 1]. In application, it can be treated as a fuzzy subset of U.

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