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Estimation of sub-pixel land cover composition in the presence of untrained classes

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Abstract

Remotely sensed data are an attractive source of land cover data over a wide range of spatial and temporal scales. The realisation of the full potential of remote sensing as a source of land cover data is, however, restricted by numerous factors. One commonly encountered problem is the presence of mixed pixels, which cannot be appropriately accommodated in conventional image classification techniques used in thematic mapping from remotely sensed data. This problem has generally been resolved through the adoption of a soft or fuzzy classification from which the fractional coverage of classes in the image pixels may be mapped. In this type of approach, the strength of membership, a pixel displays to a class, is used as a surrogate for the fractional coverage of that class. The accuracy of the resulting land cover representation is, therefore, dependent on the relationships between class membership strength and associated class fractional coverage. Since class membership can only be measured in relation to the classes defined in the training stage of the classification, untrained classes may influence the accuracy of the class composition estimation. For example, a pixel representing an area of an untrained class can only display membership to the trained classes. The effect of an untrained class on the accuracy of sub-pixel class composition estimation will depend on how the class membership strength is calculated. Here, the effect of untrained classes on sub-pixel land cover composition estimation using algorithms that produce relative and absolute measures of class membership was assessed. The algorithms investigated were the widely used fuzzy *c*-means (FCM) and its possibilistic counterpart, the possibilistic *c*-means (PCM), algorithms which derive relative and absolute measures of class membership strength, respectively. Both algorithms were able to provide accurate estimates of sub-pixel land cover composition. When all classes had been defined in training a classification, the FCM generally provided the most accurate class composition estimates. The presence of an untrained class, however, could substantially degrade the accuracy of the sub-pixel land cover composition estimates derived from the FCM but had no effect on those from the PCM. Since untrained classes are commonly encountered it may be more appropriate to use approaches such as the PCM in addition to, or instead of, the FCM to enhance the extraction of land cover information from remotely sensed data.

Keywords: Pixel unmixing; Remote sensing; Fuzzy memberships

1. Introduction

Land cover is a significant variable in both the physical and human environments. Despite its signifi-

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cance, however, data sets on land cover are often of poor quality, particularly in terms of currency and accuracy (Rhind and Hudson, 1980; DeFries and Townshend, 1994; Estes and Mooneyhan, 1994). Of the various sources of land cover data, satellite remote sensing is particularly attractive. Remote sensing provides map-like imagery at a range of spatial and temporal scales that may be used to derive land cover data of interest to a variety of users. The considerable potential of remote sensing as a source of land cover data has, however, yet to be fully realised (Townshend et al., 1991). One of the main reasons for this situation relates to the methods used in mapping land cover from the remotely sensed imagery.

Classification techniques are generally used in mapping land cover from remotely sensed imagery. Since the classes of interest are generally known a priori, supervised image classification techniques are widely used (Campbell, 1996). With these techniques, the aim is to allocate each pixel (or other defined spatial unit such as a field or parcel of land) to the land cover class with which it has the greatest similarity from the set defined in the training stage of the classification. Thus, for example, the commonly used maximum likelihood classification allocates each pixel to the class with which it has the highest posterior probability of membership. This type of classification may be described as being ‘hard’, with full membership of a single class assumed. The conventional ‘hard’ classification is, therefore, suitable when the pixel represents an area of homogeneous coverage of one of the classes defined by the analyst. Unfortunately, ‘hard’ classifications may often be inappropriate for the classification of remotely sensed imagery as the area represented by a pixel often contains more than one land cover class (Campbell, 1996). The fundamental problem is that any class allocation derived from a ‘hard’ classification for a pixel of mixed class composition must to some extent be erroneous. Therefore, the utility of a ‘hard’ classification for land cover mapping will decline with an increase in the proportion of mixed pixels in the imagery, which is in turn a function of the land cover mosaic and pixel size.

The solution of the mixed pixel problem in mapping land cover from remotely sensed imagery has generally been achieved by allowing pixels to have multiple and partial class membership and mapping the fractional coverage of the classes in the pixels (Smith et al., 1990; Wang, 1990). In this way, both pure and mixed pixels may be accommodated in the analysis. The production of an accurate land cover map, however, requires the accurate estimation of the land cover composition of each pixel. A commonly used approach for estimating the class composition of pixels for land cover mapping applications is the use of a soft or fuzzy classification. With this type of approach, a measure of the strength of membership to each class derived from the classification is used as a surrogate for the fractional coverage of the associated classes in the area represented by the pixel (e.g., Foody et al., 1992; Foody, 1996; Bastin, 1997). There are many approaches that may be used to derive this type of soft classification. The conventional maximum likelihood classification may, for example, be softened and provide for each pixel the probability of membership to every defined class (Foody et al., 1992). One particularly popular approach for soft classification is the fuzzy *c*-means (FCM) algorithm. The FCM outputs class membership values that may be interpreted as (posterior) probabilities or degrees of sharing, which are attractive for the unmixing the class composition of pixels, and allows the fuzziness of the output to be adjusted by the analyst.

The FCM has been applied successfully in a range of environments for the estimation and mapping of sub-pixel land cover composition (Fisher and Pathirana, 1990; Foody, 1996; Atkinson et al., 1997). In common with many other approaches, however, class membership is calculated with respect to all defined classes and hence the values of class membership strength are relative rather than absolute. As a result, the magnitude of the class membership values and accuracy of sub-pixel land cover estimates may be sensitive to the nature of the classes defined. Only the classes that have been defined and characterised in the training stage of a supervised image classification contribute to the calculation of class memberships and the class composition of each pixel must be divided-up among only these classes. Often, however, the image may contain areas of classes that were excluded from the training stage. Thus, although it is often assumed that the set of classes defined in the training stage of the classification is exhaustive and encompasses all the classes found at a site, untrained classes are commonly encountered. For example, tracts of urban land cover may be encountered in a crop mapping investigation but ignored by the analyst in training the classification. Clearly a pixel representing an area of any class that has been excluded from the training stage of the classification cannot be classified appropriately or accurately as it can only display membership to the set of trained classes; basic processes such as the allocation of pixels that are extremely atypical of all classes to an ‘unknown’ or ‘other’ class are not considered here, particularly as they are relatively uninformative. If a relative measure of the strength of class membership is calculated, as in the FCM, then a pixel of even a spectrally distinctive untrained class may display high membership to one or more trained classes. Moreover, due to the probabilistic constraint imposed by the FCM and other approaches, the full membership of a pixel of an untrained class will be partitioned in some manner among the set of trained classes; the total membership over all defined class will sum to unity. Consequently, the relationship between the proportional coverage of a class and the strength of membership to that class will be degraded. The presence of untrained classes may, therefore, significantly degrade the accuracy of sub-pixel land cover composition estimation and the accuracy of soft classifications.

An alternative approach to techniques such as the FCM for sub-pixel land cover composition estimation which is insensitive to untrained classes is to base the estimation on an absolute measure of the strength of class membership that indicates the degree of belonging or typicality to a class. One such measure may be derived from a possibilistic counterpart of the FCM, the possibilistic *c*-means (PCM) algorithm. The aim of this paper is to evaluate the effect of untrained classes on the accuracy of sub-pixel land cover composition estimation with the FCM and PCM.

2. Supervised FCM and PCM

This section highlights the salient features of the FCM and PCM. Attention is restricted to the calculation of the class membership values from supervised versions of these algorithms. Further details on the algorithms are provided in the literature, notably Bezdek et al. (1984) and Cannon et al. (1986) on the FCM and Krishnapuram and Keller (1993) FCM and Krishnapuram and Keller (1996) on the PCM.

The FCM is a clustering algorithm that has commonly been adapted for supervised classification of remotely sensed imagery (e.g., Key et al., 1989; Foody, 1996; Atkinson et al., 1997). The modification from unsupervised to supervised classification simply involves the specification of the class centroids and requires only a single pass of the data through the algorithm. From the supervised FCM, the strength of membership of each pixel to every class can be derived. The fuzzy membership value of pixel i to class j, u_{ij}, may be derived from,

$$u_{ij} = \left[\sum_{k=1}^{c} \left(\frac{d_{ij}^2}{d_{ik}^2} \right)^{1/(m-1)} \right]^{-1} \quad (1)$$

where d_{ij}^2 is the distance the pixel lies from the centroid of the class, c is the number of classes and m a user defined fuzziness parameter (Bezdek et al., 1984). The fuzzy membership values output from the algorithm indicate the strength of membership of the pixel to the defined classes. The partitioning of the total membership between the classes is taken to indicate the sub-pixel land cover composition.

The derivation of the fuzzy membership values in the FCM is subject to the probabilistic constraint that for each pixel the memberships derived over all classes sum to unity. Consequently, the fuzzy membership values derived are similar in nature to posterior probabilities of class membership and may be interpreted as probabilities or degrees of sharing. Note also that the calculation is made with respect to all classes and the values are, therefore, relative measures of the strength of class membership.

The PCM is a modification of the FCM, in which the probabilistic constraint has been relaxed. As with the FCM, it is a clustering algorithm which can be adapted to operate in a supervised mode through the provision of class centroids. The fuzzy membership values output from the PCM are derived with respect to each class independently of all others. Thus, the fuzzy membership value of a pixel to a class indicates the degree of its belonging or typicality to the specified class (Krishnapuram and Keller, 1993) and may be derived from,

$$u_{ij} = \left[1 + \left(\frac{d_{ij}}{\eta_j} \right)^{1/(m-1)} \right]^{-1} \quad (2)$$

where η_j is the bandwidth parameter which specifies the distance at which the membership to a class equals 0.5. As with the FCM, m is a parameter which controls the fuzziness of the analysis but its optimal value and interpretation differs between the two algorithms (Krishnapuram and Keller, 1996).

Unlike the FCM, calculation of fuzzy membership values with the PCM is with respect to a single class independent of all others. The membership values derived are, therefore, absolute measures of the strength of class membership and are in effect measures of typicality.

3. Data and methods

An airborne thematic mapper (ATM) image of part of the western outskirts of the city of Swansea (see Fig. 1), acquired with a Daedalus 1268 sensor was used. This image was acquired in eleven spectral wavebands with a spatial resolution of approximately 1.5 m. Attention focused on a small region immediately to the west of the University campus. This test-site was comprised of mainly three land cover classes, trees, grass and asphalt (car park) which could be readily identified from the fine spatial resolution ATM imagery. For the purpose of this investigation, each pixel in this fine spatial resolution image was assumed to be pure and classified visually into the three classes. This classification was verified in the field and used as ground/reference data on the distribution of the three land cover classes. Since ATM data are often threedimensional in character, with the dimensions relating to reflectance in the visible, near- and shortwave-infrared wavelengths, the analyses were simplified by using only one band from each of those spectral regions. The data selected for the analyses were those acquired in the 605–625 nm, 695–750 nm and 1550–1750 nm wavebands (see Fig. 2), which, from previous studies (e.g., Foody and Cox, 1994), were known to provide a high level of inter-class separability. The ATM image was then spatially degraded with an 11×11 low pass (mean) filter to provide a crude simulation of imagery with a relatively coarse spatial resolution (Foody and Cox, 1994). For each pixel in this simulated coarse spatial resolution image the proportion of three land cover classes contained within it could be derived from the classification of the original, spatially undegraded, image. These estimated class proportions formed the ground/reference data on the actual class composition of pixels in the simulated coarse spatial resolution image.

The centroid of each class was estimated using five pure pixels of each class drawn from the simulated coarse spatial resolution image. These class centroids were then used to classify an independent testing set using the FCM and PCM. The analyses were based on Euclidean distance measurements between sampled pixels and the centroids of the classes defined with the parameter m set, after a series of trials, at 2.0 and 3.0 for the FCM and PCM classifications, respectively. The parameter η_j in the PCM was defined in relation to the mean pixel-to-class centroid distance for each class (Krishnapuram and Keller, 1993).

A series of classifications were performed. In these classifications, either all three classes were defined or one was left untrained (i.e., the training pixels for the untrained class were deleted from the training set prior to the classification). The test set used to evaluate the accuracy of the sub-pixel land cover composition estimates comprised 35 pixels of variable class composition, ranging from pure pixels to mixed pixels representing an area comprising all three classes; areas containing any class in addition to the three defined (e.g., sand, parked vehicles etc.) were excluded from the analyses. The accuracy of the classification outputs derived were assessed relative to the class composition of the testing pixels. Specifically, accuracy was assessed by comparison of the predicted or estimated coverage of a class in a pixel, indicated by the fuzzy membership value to the class derived from the classification, with the proportional coverage observed in the reference data. This was quantified by measuring the correlation and RMS error between the two data sets on class fractional coverage for each class.

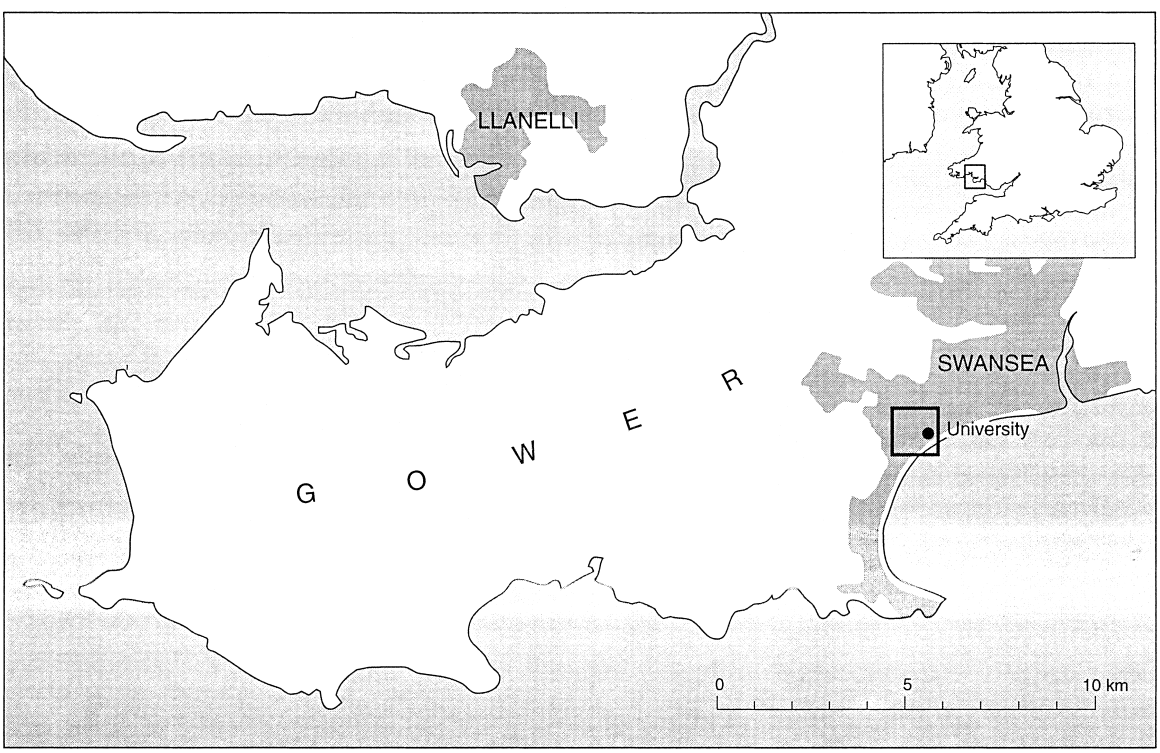

Fig. 1. Location of test site.

Fig. 2. ATM imagery of test site. (A) 605–625 nm, (B) 695–750 nm and (C) 1550–1750 nm wavebands.

4. Results

The sub-pixel land cover composition estimates derived from the FCM trained with all three classes were in close agreement with the actual coverage of the classes (see Table 1). This concurs with the results of other studies that have used the FCM for the estimation of sub-pixel land cover composition. The exclusion of any one class from the training stage, however, resulted in a reduction in the correspondence between the predicted and actual coverage of the classes defined in the analysis (Table 1). Every permutation of two-class classifications was undertaken to illustrate the effect of a single untrained class on the fuzzy membership values derived and their relationship with the coverage of the relevant class on the ground. Although significant correlations between the predicted and actual coverage of each defined class were obtained from each classification, the relationships, and hence the degree of correspondence between the two data sets, were weaker than when all three classes had been included in the training stage. The magnitude of change in the correspondence between the estimated and actual coverage was a function of the spectral contrast of the trained and untrained classes. The sensitivity of the classes to an untrained class was, therefore, variable. For instance, the correlation between estimated and actual coverage of the trees class dropped from 0.89 to as low as 0.54 whereas for the grass class the largest decrease in correlation was from 0.93 to 0.90. Note that in this instance, the results reflect the high degree of spectral distinctiveness of grass relative to the other two land cover classes (see Fig. 3). Nonetheless, the accuracy with which sub-pixel land cover composition was estimated for a class with the FCM was sensitive to the presence of an untrained class (see Fig. 4).

The analyses were repeated using the PCM. With all three classes defined in the training stage, significant correlations between the estimated and actual coverage of each class were obtained (Table 1). However, the correspondence between the estimated and actual class compositions was weaker than that derived from the similar analysis with the FCM; the use of different parameter settings, in each algorithm, could alter the strength of the relationships observed. Since the fuzzy membership values derived from the PCM were calculated for each class independently of all others their magnitude was insensitive to the effect of untrained classes. Thus, the fuzzy membership value for a class remains constant if one, or more, of the other classes is excluded from the training stage of the analysis. While the presence of an untrained class can lessen the accuracy of sub-pixel land cover composition estimation

Table 1
Correspondence between estimated and actual class composition of testing set pixels[a]

Algorithm	Untrained class	Correlation coefficient			RMS error		
		Trees	Grass	Asphalt	Trees	Grass	Asphalt
FCM	None	0.89	0.93	0.84	0.14	0.13	0.18
	Trees	–	0.92	0.43	–	0.13	0.46
	Grass	0.82	–	0.81	0.22	–	0.27
	Asphalt	0.54	0.90	–	0.43	0.15	–
PCM	N/A	0.75	0.92	0.70	0.35	0.27	0.33
$PCM_{rescaled}$	N/A	0.75	0.92	0.70	0.20	0.12	0.24

[a] Correlation coefficient and RMS error were calculated for each class, with fuzzy membership values output from classification algorithm taken to represent fractional coverage of classes. As PCM outputs are measures of typicality and not constrained to sum to 1.0 their interpretation is more difficult. Consequently, fuzzy membership values derived from PCM were also rescaled ($PCM_{rescaled}$) to express percentage cover of classes to enable realistic assessment of RMS error in estimation of sub-pixel class composition).

with the FCM it, therefore, has no effect on the estimates derived from the PCM. Moreover, the degree of correspondence between the estimated and actual class compositions derived from the PCM can be higher than that from the FCM in the presence of an untrained class. Stronger correlations were, for example, obtained between the predicted and actual coverage of the trees and grass classes from the PCM than FCM when the asphalt class had been excluded from the training stage of the classification (Table 1).

The interpretation of the fuzzy membership values derived from the PCM is more difficult than with the FCM where they can be directly interpreted as the proportional coverage of a class. A calibration relationship between the magnitude of the fuzzy membership value and proportional coverage of a class is required for sub-pixel class composition estimation. A crude estimate of this was derived for each class using the training data set. These relationships were used to rescale the fuzzy membership values to indicate the percentage cover of the classes. These rescaled PCM outputs showed a strong correspondence to the actual class coverages (see Fig. 5). This rescaling also provided values that were closely related to the ground coverage and consequently reduced the RMS errors calculated (Table 1).

5. Summary and conclusions

Mixed pixels are a major problem in the analysis of remotely sensed imagery. In land cover mapping applications, the solution to the mixed pixel problem has often been based on the use of soft or fuzzy classification techniques that allow for multiple and partial class membership. For these techniques to be of value, the fuzzy classification output must accurately indicate

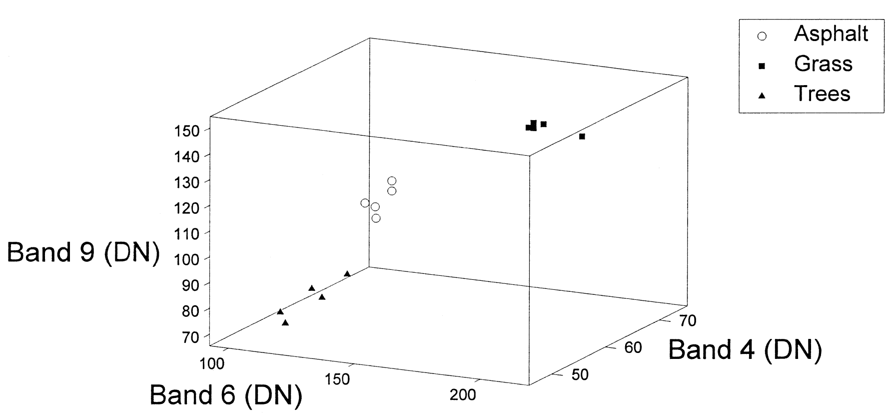

Fig. 3. Location of training samples in feature space. Note that grass is most spectrally distinct class and hence least sensitive to presence of untrained classes when using relative fuzzy membership values derived from FCM to indicate sub-pixel class composition.

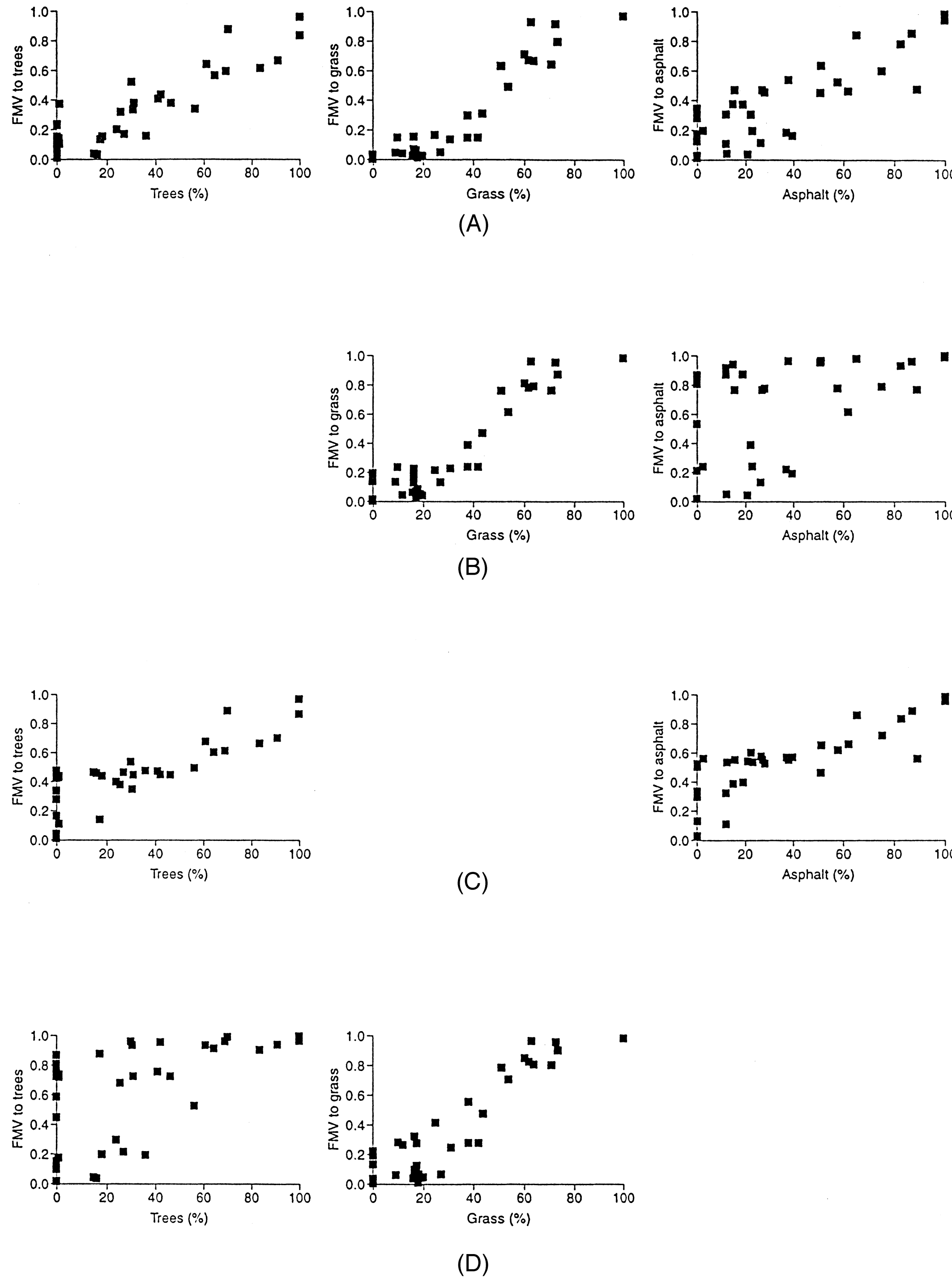

Fig. 4. Relationships between fuzzy membership value (FMV) to class and its corresponding coverage on ground derived with FCM. (A) all classes trained, (B) tress untrained, (C) grass untrained and (D) asphalt untrained.

the class composition of pixels in the imagery. A range of methods have been used for this sub-pixel class composition estimation in mapping land cover from remotely sensed imagery. In many, a measure of the strength of class membership is used to indicate the class composition, with its magnitude taken as a surrogate for the proportional coverage of the class within the area represented by the pixel. For example, the FCM, which is subject to the probabilistic constraint that the class memberships sum to unity and generates relative measures of the strength of class membership, has been used widely. This type of approach has been found to provide accurate estimates of sub-pixel land cover composition. However, these assessments have typically stemmed from investigations that were constrained to include only the classes defined in training throughout the analyses. Commonly, the remotely sensed imagery contain further, untrained, classes. Pixels representing an area of an untrained class, in full or in part, can, however, only display membership to the trained classes. As a consequence of this, the accuracy with which sub-pixel land cover composition is estimated and mapped with a technique such as the FCM may be hypothesised to be sensitive to the presence of untrained class(es).

The results of the analyses presented showed that the FCM may be used to derive accurate estimates of sub-pixel land cover composition when all classes have been defined and included in the training stage of the classification. However, the presence of an untrained class degraded the accuracy of sub-pixel class composition estimation with the FCM. The correlation between the predicted and actual class of membership, for example, varied markedly between situations in which all classes were included in training and when one was left untrained. This was not the situation with the PCM. The PCM calculates a measure of typicality to each class that is insensitive to the existence of untrained classes. Although, the sub-pixel land cover estimates derived from the PCM were generally slightly less accurate than those from the FCM when all classes were defined they could, however, sometimes be more accurate when an untrained class was present.

While the case study used to illustrate the effect of untrained classes on the accuracy with which sub-pixel land cover composition may be estimated was based on a relatively simple situation, it highlighted the different information conveyed by relative and absolute measures of class membership and is of broad applicability. In essence, estimates of sub-pixel land cover

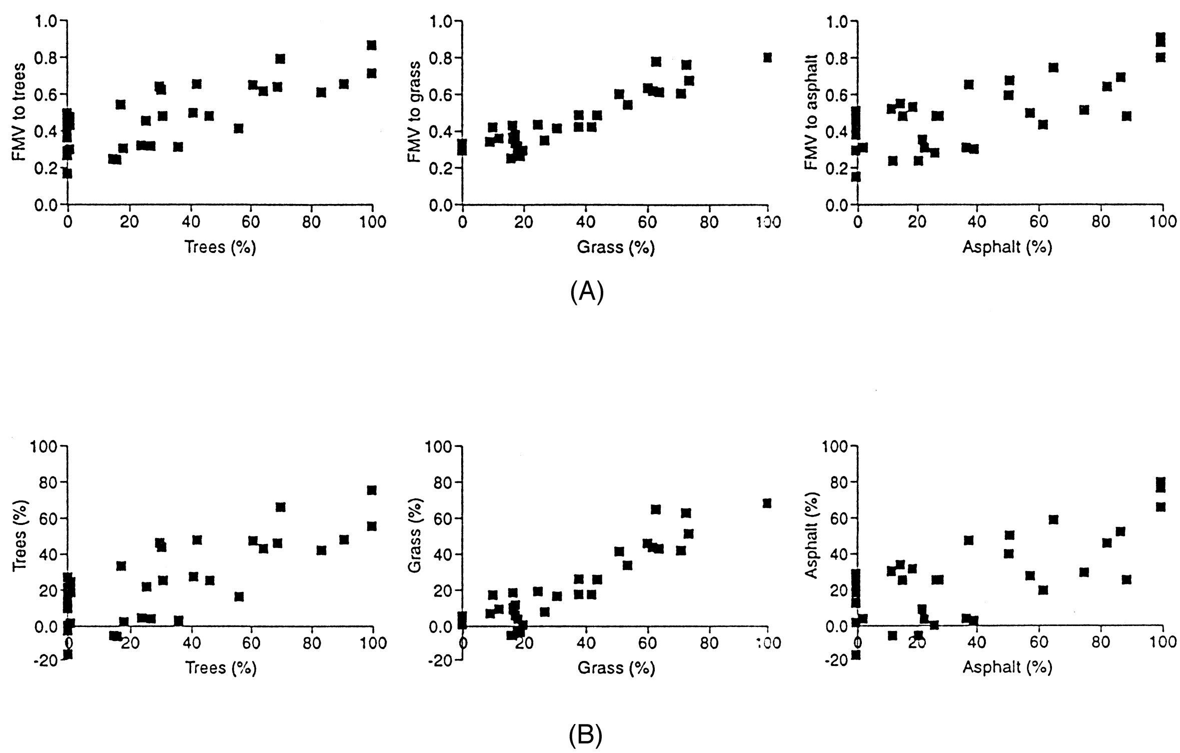

Fig. 5. Relationships between fuzzy membership value (FMV) to class and its corresponding coverage on ground derived with PCM. (A) Based directly on FMVs output and (B) FMVs converted to estimate of percentage class coverage.

composition derived on the basis of a relative measure of the strength of class membership will be sensitive to the presence of an untrained class. The degree to which the estimates of sub-pixel class coverage vary with the presence of an untrained class will be a function of the spectral contrast of the classes involved. Estimates of sub-pixel class composition derived on the basis of an absolute measure of the strength of class membership, however, will be insensitive to the presence of untrained classes. The set of classes defined in the training stage of such an analysis therefore need not be exhaustive. The trends observed in the case study therefore indicate the degree of sensitivity of estimates of sub-pixel class composition for classes of differing spectral distinctiveness in situations in which the standard assumption of an exhaustive classification scheme has not been satisfied.

Since untrained classes are commonly encountered, it may be inappropriate to use techniques that provide a relative measure of class membership, such as the FCM, for the estimation of sub-pixel class composition. Approaches such as the PCM which provide an absolute measure of the strength of class membership indicating typicality may be more appropriate when untrained classes are present. The calculation of memberships from the PCM is simple, based on the distance between a pixel and the class centroid, and could be produced alongside, perhaps as a by-product of, a standard FCM analysis. The analyst may find that the use of class membership values derived from both the FCM and PCM convey useful and complementary information, in a manner similar to posterior probabilities and typicalities which can be derived from the maximum likelihood classification (Foody et al., 1992). Consequently, the analyst may elect to use both measures of class membership to enhance the level of information extraction from the remotely sensed imagery. Indeed the use of both relative and absolute measures of the strength of class membership can help resolve problems associated with the presence of untrained classes (Foody, 1998).

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