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Research Article

A Parameterized Representation of Uncertain Conceptual Spaces

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Abstract
Most conceptual modeling in geographic information science to date has used a symbolic approach with little or no recognition of the semantic uncertainty often found in geographic concepts. This work describes a concept model based on parameterized concept descriptions that uses a spatial metaphor, the conceptual space, as an organizing structure (Gärdenfors 2000). This cognitive theory of conceptual spaces is combined with a formal representation of semantic uncertainty based on rough fuzzy sets. The conceptual space then represents each concept as a collection of rough fuzzy property definitions with associated salience weights, where a property itself can be treated as a special case of a concept. Instead of explicitly defining concept hierarchies, we can allow different conceptual structures to emerge through measures of concept inclusion and similarity. A land use/land cover example demonstrates how the model represents concepts, concept similarity, hierarchical structures and the context dependence of concepts. The final section of the paper points to the need for further studies of context effects, concept similarity measures, and uncertainty representation using the proposed model.

1 Background

Recent efforts to develop collaborative GIS (Goodchild et al. 1999, Jankowski and Nyerges 2001, Nyerges et al. 2002), human centered GIS (Miller 2003) and studies of the interaction between (GI) science, technology, and society (Harvey 2000), demonstrate some increasingly important dimensions of GIS research. This paper contributes to this research as one in a series of investigations to develop tools to express more of the theories, models, intentions and motivations behind the concepts used in geographic information.

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Many efforts to implement conceptual models in GIScience have been grounded in some cognitive principles, and various representational approaches have been proposed (Nyerges 1991, Livingstone and Raper 1994, Usery 1996, Bishr 1997, Mennis 2003). This paper also recognizes the cognitive dimension of concept modeling, hence a need to provide a deeper understanding of information stored in databases. Highlighting some important aspects related to prototype theory (Rosch 1975), concept similarity (Hahn and Chater 1997), generalization (Shepard 1987), and context dependence (Medin et al. 1993), this work intends to bring together the cognitive theory of conceptual spaces (Gärdenfors 2000) with a formal representation of the semantic uncertainties we frequently find in geographic data (Ahlqvist et al. 2003).

1.1 Conceptual spaces

Although space is frequently used as a metaphor for exploring and visualizing data (see for example Kohonen 1995, Fabrikant and Buttenfield 2001), there has been limited work on using space as a structuring metaphor for concept modeling and representation. Recognizing symbolic (Newell and Simon 1976) and connectionist (such as artificial neuron networks) approaches as the two dominating cognitive frameworks to model concept representation, Gärdenfors (2000) argues that a third approach based on conceptual spaces, which uses spatial structures, is needed to bridge between them, and to effectively model the important aspects of concept similarity, formation and understanding.

A conceptual space is constructed from a set D_n of n quality dimensions. These are normally divided into sets of *properties* relating to certain qualities, for example a quality dimension "Tree crown closure" can, depending on the application, be divided into properties such as {closed, semi-closed, semi-open, open} or {0–10%, 10–20%, . . . , 90–100%}. A single (quality) dimension or a small number of integral dimensions forms a *domain* (a compound quality dimension), in which each property is defined as a point or region, for example an interval of percent values that corresponds to "closed".

Properties are usually specified using some measurement scale, but they need not be described through a quantitative metric, any semi-structured ordering imposed on the ordinal, interval, or ratio domain is possible, as in the previous example. Some examples of other geographic quality domains might be temperature, altitude, direction, population density, soil grain size, or soil color.

Often there are several domains involved in defining a concept. Thus, a concept is defined by a vector, $v = \langle d_1, \ldots, d_n \rangle$, that holds the collection of property values d in each domain that together define a position in a conceptual space.

However, a large problem with any property based categorization is that we can expect certain properties to be more important characteristics of a concept than others. It is also likely that the importance of properties will differ depending on the context, in other words, viewing a concept from different perspectives. To represent this, a commonly suggested solution is to assign attention weights to the different properties to model the *salience* of each domain in a concept definition (Medin and Schaffer 1978, Gärdenfors 2000). Many decision support methods, such as multi-criteria evaluation apply the same strategy (Malczewski 1999).

So far, this framework resembles implementations of the well known frame theory (Minsky 1975) with slots for different *features* but, as we shall see, a conceptual space can account for closeness, and is therefore capable of distinguishing more or less central,

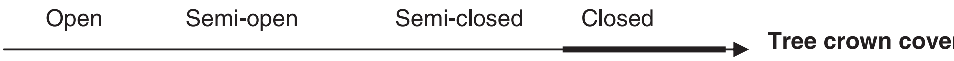


Figure 1 The tree crown cover domain separated into the properties "Open", "Semi-open", "Semi-closed" and "Closed"

or typical, parts of the concept definition, as well as different aspects of similarity between concepts.

1.2 Similarity, distance and overlap in conceptual spaces

Gärdenfors (2000) argues that assessment of similarity between different concepts plays a key representational role in human categorization and concept learning. From psychology two common approaches to estimate concept similarity are property-based (Tversky 1977) and spatial (e.g. Nosofsky 1986) models. These are often presented as conflicting theories, rather than as complementary views of similarity.

Using Tversky's (1977) approach of measuring similarity as a function of common properties, this paper follows the assumption that it is important to estimate if, for example, a forest is closed or not. However, for graded properties such as crown closure, answering closed or not closed becomes a matter of inclusion, that is, how much of the found property value is part of the property value that determines the concept. But even if there is no inclusion we still view for example "closed" as more similar to "semi-open" than to "open". Concept similarity in this sense is clearly related to the difference between quality properties, and follows the spatial model (Figure 1). Henceforth, the term "similarity" is used to mean spatial-distance-based similarity, and the term "overlap" to mean common-property-based similarity.

Since a conceptual space is inherently spatial, similar concepts will be found close to each other. A spatial measure of distance is defined along each domain that is used to define a certain concept, in an n-dimensional quality space. The distance value would depend on the type of spatial metric used. Examples of well-known distance metrics are Euclidian, Diagonal, and Mahalanobis distances (Burrough and McDonnell 1998).

If we look at conceptual spaces from the perspective of prototype theory (Rosch 1975), categories share common properties and some objects can be described as more or less central or typical for a concept. Certain points in a conceptual space can represent prototypes of concepts (Figure 2, left). We may then measure the similarity as distances between instances and concepts in the conceptual space.

Performing a Voronoi tessellation divides the conceptual space into regions based on the minimum distance from concept prototypes. We can view each point in the concept space as a member of a concept based on these regions, where distance/similarity to the prototype point reflects its typicality. We may also define prototypes as regions where all points within such a region are equally good examples of a concept. In these cases a Voronoi tessellation is based on region perimeters (Figure 2, right).

Concepts are often organized hierarchically in a tree-like structure, with more inclusive and abstract concepts at the top and more detailed concepts further down the hierarchy, a structure that can be modeled with set inclusion (IS-A) relations. Conceptual spaces do not represent hierarchical arrangement of categories explicitly, but we may look at the amount of overlap of property regions in each domain for the concepts of interest. For example, the "closed" concept could be sub-divided into lower level

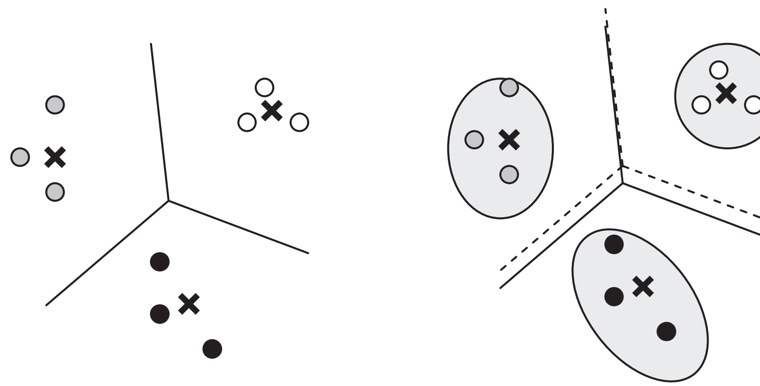


Figure 2 Two-dimensional concept spaces with prototype points (crosses) and separated into concept areas by Voronoi tesselation (solid lines). In the right figure prototype regions (shaded ellipses) cause a change in the Voronoi tessellation, adjusting the concept borders (hatched lines)

categories "fairly closed" and "very closed". In a concept space the IS-A relationship is recognized if the crown closure interval for "very closed" is included within the interval making up the "closed" concept. In many cases though, concept hierarchies do not form trees, instead sub-concepts straddle the boundaries of upper and lower level categories (Freksa and Barkowsky 1996). In most of these cases, the spatiality of a conceptual space will interpret a partial concept overlap as a graded IS-A relationship. This is a generalized way of evaluating concept inclusion and construction of hierarchies. Concepts defined as regions in a conceptual space can still use set inclusion relations, but the relations will typically have a graded or partial character (Baldwin et al. 2000).

In summary then, a concept is represented as a set of regions in a number of domains, a conceptual space, and every domain is attributed a salience weight according to the importance of that property in the concept definition. Concept similarity and hierarchical relationships are evaluated based on spatial distance and overlap metrics in the conceptual space.

1.3 Semantic uncertainty

So far, points or regions in a conceptual space have been viewed as crisp entities with no uncertainty about them. However, empirical, cognitive and even social sources make uncertainty an inseparable companion of almost any information (Klir and Wierman 1998, Couclelis 2003). Although uncertainty may have various sources, randomness and imprecision are two major types that are of importance in spatial knowledge representation and inference (Leung 1997). Randomness typically relates to well-defined concepts where measurement uncertainty is involved. Imprecision, on the other hand, arises normally at a cognitive level, when concepts are difficult to define.

Imprecision is assumed here to be closely related to semantic accuracy, which refers to the quality with which (geographical) objects are described in accordance with the selected model (Salgé 1995). A major problem with quality evaluations is their inherently relative character (Goodchild 1995, Salgé 1995, Veregin 1999), and the problems of specifying a reference for the quality assessment. A major consequence of this emerges

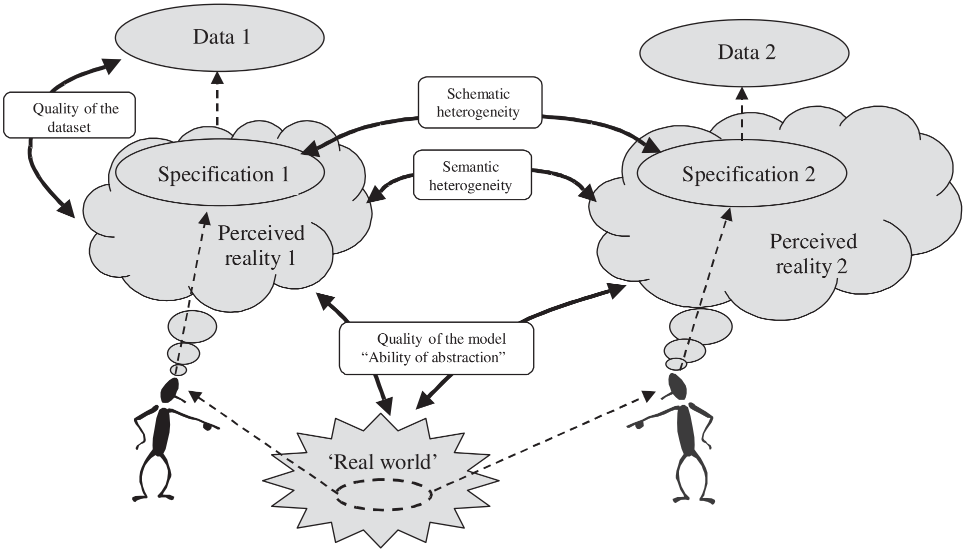


Figure 3 Aspects of semantic uncertainty in a multi-user context

as problems with schematic and semantic heterogeneity in multi-user environments (Figure 3) (Bishr 1997, Ahlqvist 2000). Although a large literature addresses efforts to resolve these problems (Aslan and McLeod 1999), most tend not to account for imprecision (Yazici and Akkaya 2000).

Two important aspects of semantic imprecision, vagueness, and indiscernibility were previously problematic from a representational viewpoint, but work on fuzzy (Zadeh 1965) and rough (Pawlak 1991) extensions of traditional set theory have provided viable techniques to handle those uncertainty types. There are several examples that describe fuzzy conceptual models for GIS (Usery 1996, Cross and Firat 2000, Yazici and Akkaya 2000, Morris 2003) and a recent overview of fuzzy set theory applications in GIS can be found in Robinson (2003). In addition, rough sets and the concept of rough classification have demonstrated promising applications for geographic information handling (Schneider 1995, Worboys 1998, Ahlqvist et al. 2000).

As Klir and Wierman (1998) note, uncertainty has a multidimensional character, and one type of uncertainty seldom comes alone. Combining different types of uncertainty is therefore of interest and the remainder of this paper intends to build a conceptual space representation capable of including both vagueness and indiscernibility in geographic concepts.

2 Representing Conceptual Spaces

The next section will start by introducing a formal representation, a rough fuzzy set, which can handle combinations of semantic imprecision and vagueness. This is followed by a description of how this can be used to represent semantic uncertainty of concept definitions in a conceptual space. The final two sections develop metrics to assess concept similarity and hierarchical relationships among collections of concepts.

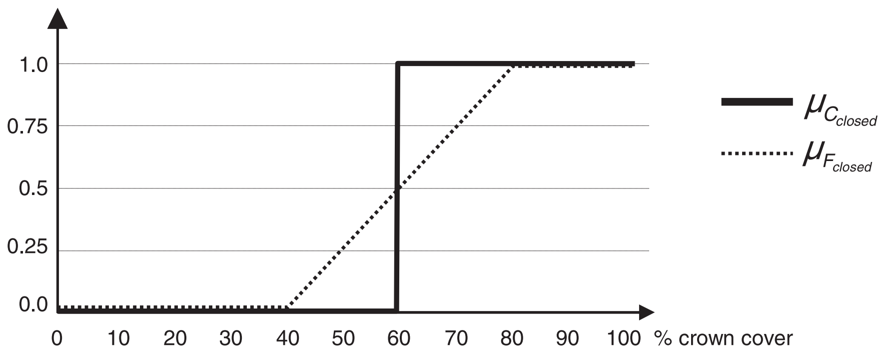


Figure 4 Membership function $\mu(x)$ to define the fuzzy set "closed" (dotted line) and corresponding crisp membership function (solid line)

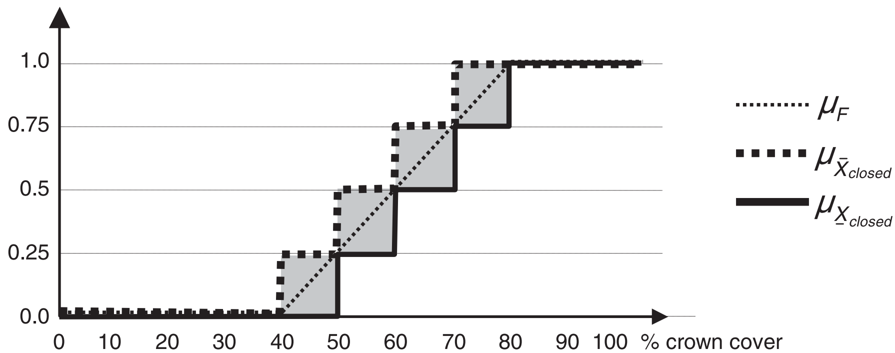
2.1 A formal representation of concept vagueness and indiscernibility

Information about our social and physical environment is constantly dissected into discrete units such as; high, low, rich, poor, warm, cold. Still, a lot of those units are essentially subdivisions of a continuum. The concept "closed" is a typical example of such a subdivision. Instead of following a crisp and rather artificial definition that an area with tree crown cover more than, say 60%, is "closed", (Figure 4, dotted line) we can define "closed" as a continuous function of crown closure % (Figure 4, solid line), where membership values $\mu(x)$ indicate the degree of being a member of the fuzzy set "closed".

In many cases available data also have a limited resolution in terms of, for example, data categories or the spatial resolution. Data on crown closure (Figure 4) might come in classes of crown closure ranges such as "0–10%", which reduces the resolution, and will cause problems if we want to apply a graded concept such as "closed". Pawlak (1991) introduced rough set theory as a general framework for treating effects of limited resolution. Fuzzy and rough set theories have since been further generalized by Dubois and Prade (1990) into rough fuzzy sets, a joint representation for vague and resolution limited information.

A rough fuzzy set is based on an approximation space (U, θ) defined on a universe of discourse U with an equivalence relation θ. The granularity imposed by θ on U results in a set of equivalence classes $U/\theta = \{E_i\}$, the quotient set. An equivalence class that contains $x \in U$ is denoted $[x]$. Now, let U be the universe of all closure percentages, and F a fuzzy set of percentage values considered as "closed" for tree crown closure defined by a membership function μ_F: $U \rightarrow [0, 1]$ that indicate the degree of membership of a certain percentage in F. If we use data that come in interval classes, it imposes the granularity θ on U, and gives us equivalence classes such as [0–10] (%). The quotient set $U/\theta = \{E_i\}$, are all interval classes.

So, how do we represent F by means of U/θ, that is, how can we form a subset of "closed" areas using the crown cover classes? We follow Ahlqvist et al. (2003), and introduce the idea of a rough fuzzy definable set, defined by $\mu: U \rightarrow [0, 1]$ such that $\forall x \forall y \in [x]$: $\mu(x) = \mu(y)$. A rough fuzzy set (RF-set) then is a pair of rough fuzzy definable sets (L, U), such that $L \subseteq U$, and it approximates the fuzzy set F if $L \subseteq F \subseteq U$. We will often write an RF-set definition as $X = (\mu_{\underline{X}}, \mu_{\bar{X}})$ where $\mu_{\underline{X}}$ and $\mu_{\bar{X}}$ are the membership functions that define L and U respectively.



U/θ	[0–10]	[10–20]	[20–30]	[30–40]	[40–50]	[50–60]	[60–70]	[70–80]	[80–90]	[90–100]
$\mu_{\underline{X}_{closed}}$	0.0	0.0	0.0	0.0	0.0	0.25	0.50	0.75	1.0	1.0
$\mu_{\bar{X}_{closed}}$	0.0	0.0	0.0	0.0	0.25	0.50	0.75	1.0	1.0	1.0

Figure 5 Fuzzy "closed" membership function, μ_F, and rough fuzzy "closed" membership functions, ($\mu_{\underline{X}_{closed}}$, $\mu_{\bar{X}_{closed}}$), that use a 10% units interval granularity on the % crown cover universe, U/θ

If we look for "closed" areas in data that is described by the exemplified interval classes we get a rough fuzzy set, $X_{closed} = (\mu_{\underline{X}_{closed}}, \mu_{\bar{X}_{closed}})$, (Figure 5).

The RF-set "closed" approximates the fuzzy set "closed" (thin dotted line) leaving an area of uncertainty (gray areas) due to the limited resolution of the census classes. We can see for example that each instance of the [60–70%] class is a member of the RF-set "closed" with membership degrees (0.25, 0.50), and each instance of the [80–90%] class is a member of the RF-set "closed" with membership degrees (0.75, 1.0).

2.2 Creating a parameterized concept definition

A domain, as defined in section 1 above, is formalized through an approximation space $S = (U, \theta)$. U represents the domain made up of one or several integral quality dimensions, and a property region in that domain is defined by an RF-set $P = (\mu_{\underline{x}}, \mu_{\bar{x}})$ of the approximation space.

An extension of the scope of a conceptual space is proposed here to allow for nominal domains, with properties specified by semantic variables. In such cases we can still assume that there may be one or more underlying quantifiable domains, sometimes unknown at the time of definition, but possible to substitute by further specification of the semantic variables. For example, a tree-type domain may be separated into two crisp, qualitative values; "deciduous" and "evergreen". Further specification of these could involve use of quantitative proportions of deciduous and evergreen species, creating fuzzy partitions of a semantic space, similar to the crown cover example.

A concept C is formalized using three vectors (S_i, P_i, W_i) holding the defining approximation spaces S_i together with accompanying property values P_i given as RF-set definitions, $P_i = (\mu_{\underset{\sim}{x}}, \mu_{\tilde{x}})$. A weight vector W_i holds the salience values (importance) of each approximation space in the concept definition context.

In summary, a conceptual space is represented by a collection of approximation spaces, and corresponding salience weights together with RF-sets that define domain properties.

2.3 Measuring concept similarity

In cases where concept properties are imposed on ordinal, interval or ratio scales, concept similarity can be expressed as an exponentially decaying function of distance (Shepard 1987):

$$s = e^{-c*d} \tag{1}$$

Here c is a general sensitivity parameter, and d is the distance between two concepts C_A and C_B. There are several ways to set up a distance measure in an i-dimensional concept space depending on how we want to account for salience weights, and what type of spatial metric we want to use. In this work, we will use the following:

$$d(C_A, C_B) = \sqrt{\sum_{i}^{|U|} W_{B_i}(P_{A_i} - P_{B_i})^2} \tag{2}$$

where $P_{A_i} - P_{B_i}$ is the distance between properties A and B in domain i. The distances are weighted according to the salience of each domain, W. The use of a root mean squared weighted distance evaluates to the Euclidean distance in a multidimensional concept space. In this way Equation (2) evaluates how distant C_A is from C_B by summing the squared distances in each of the domains, and then applying the perspective of concept B by multiplying domain salience weights, W_B, where $\sum_i W_{B_i} = 1$. This means that the distance from A to B does not have to be the same as the distance from B to A. This property is intended to support the idea of context effects such as the concept asymmetry we can find in statements such as "a hospital is more similar to a building than a building is to a hospital" (Rodriguez and Egenhofer 2004). Several other formalizations to include context effects in a similarity measure are possible (see for example Nosofsky 1986, Feng and Flewelling 2003, Song and Bruza 2003).

With properties defined as fuzzy sets we can employ existing distance measures defined for comparison of fuzzy numbers (Kaufman and Gupta 1985). There are a number of candidate measures (Tran and Duckstein 2001) and the following example will use the fuzzy dissemblance index (Kaufman and Gupta 1985):

$$\delta(A, B) = 1/2(\beta_2 - \beta_1) \int_{\alpha=0}^{1} (|a_1^\alpha - b_1^\alpha| + |a_2^\alpha - b_2^\alpha|)d\alpha \tag{3}$$

Here A and B are two fuzzy numbers and $[\beta_1, \beta_2]$ an interval of real numbers given values in order to surround $A^{\alpha=0}$ and $B^{\alpha=0}$ (Figure 6), $\delta(A, B)$ takes values [0, 1], where $A = B \Rightarrow \delta(A, B) = 0$ and $\delta(A, B) = 1 \mid \forall \alpha \in [0, 1]$: $A^\alpha = [\beta_1, \beta_1]$, $B^\alpha = [\beta_2, \beta_2]$. In other words, very similar fuzzy numbers would produce low values of δ and very different numbers would produce high values of δ.

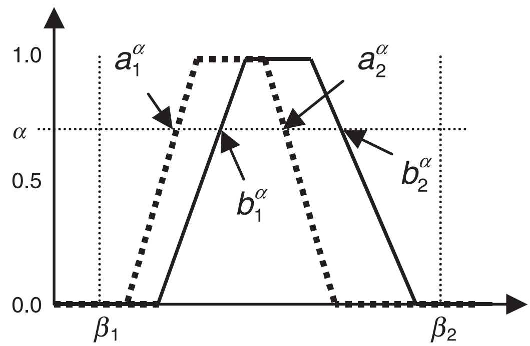


Figure 6 Illustration of the fuzzy distance based dissemblance index (Equation 3) calculation

Thus, the concept distance measure from Equation (1) becomes:

$$d(C_A, C_B) = \sqrt{\sum_i^{|U|} W_{B_i} * \delta(P_{A_i}, P_{B_i})^2} \tag{4}$$

Note again that $d(C_A, C_B)$ is not symmetric since W can be different for C_A and C_B, and the distance measure uses W_B to evaluate C_A from the perspective of C_B. For properties defined as RF-sets, upper and lower approximations of the dissemblance index are calculated separately, giving an upper and lower approximation of the distance. For ordinal domains a simple rank order measure is suggested as an approximation of the concept distance. Nominal domains get zero distance if the values are identical, otherwise the upper and lower approximation of the distance is assigned 0 and 1 respectively, acknowledging that the distance between two different nominal values can be anything from (infinitesimally close to) similar to totally dissimilar.

Concept similarity (Equation 1) can now be estimated using the distance (Equation 4) measured from an object or a prototype region to another object or prototype region. Again, RF-sets give rise to upper and lower approximations of the similarity estimate.

2.4 Measuring concept overlap – hierarchical relationships in conceptual spaces

Hierarchical concept relationships, such as if concept A is included in B, were suggested in section 1.2 to relate to Tversky's (1977) account of measuring similarity. The similarity measure defined above is not sensitive to concept inclusion as it can give identical similarity values for full and partial overlap between properties P_A and P_B (Figure 7).

A number of candidate measures for evaluating concept overlap can be found in Bouchon-Meunier et al. (1996). For the example below, a measure was chosen that can be interpreted as a degree of concept overlap:

$$o(p_A, p_B) = \int \min(f_{p_a}(x), f_{p_b}(x))dx \Big/ \int f_{p_b}(x)dx \tag{5}$$

This metric takes values between 0 (for disjoint concepts) and 1 (where B is a subset of A). In Figure 7 we can see that the overlap measure is capable of separating between the right and left case where the similarity measure fails to do so. By exploring concept relations using the overlap measure we can infer hierarchical taxonomies based on graded set/subset relations.

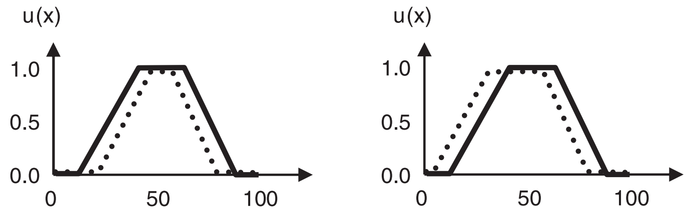


Figure 7 Domain value p_A (solid line) and p_B (dotted line) defined as fuzzy numbers. Concept (value) similarity (s) and concept overlap (o), are $s = 0.95$, $o(p_A, p_B) = 1.0$, $o(p_B, p_A) = 0.5$ in the left case, and $s = 0.95$, $o(p_A, p_B) = 0.75$, $o(p_B, p_A) = 0.75$ in the right case

For measures of overlap on non-ordered, qualitative domains there are some similar candidate measures (Bouchon-Meunier et al. 1996). Here, a modification of Equation (5) can be used as follows:

$$o(p_A, p_B) = \sum \min(p_A, p_B) \Big/ \sum p_B \qquad (6)$$

This measure shares the same properties as the previous overlap measure for quantitative domains (Equation 5). A combined measure of concept overlap is calculated in a similar way as in Equation (4) previously:

$$O(C_A, C_B) = \sum_{i}^{|U|} W_{B_i} * o(P_{A_i}, P_{B_i}) \qquad (7)$$

3 An Example Conceptual Space

This section will illustrate the suggested approach using the U.S. Geological Survey's Anderson vegetation classification system (Anderson et al. 1976) as an example of how land use/land cover concepts can be defined in a conceptual space, using an existing classification system specification as a guideline.

Figure 8 shows a simplified version of the first two Anderson levels where important characteristics that separate the classes in the hierarchy were picked out from the text specification. These characteristics are used as domains of concepts and the respective properties in these domains define the six concepts at level 1 and 2. Each domain is initially given a salience value of 1.

Thus, four approximation spaces formally define a conceptual space for the simplified hierarchy:

$S_1 = (U_1, \theta_1)$, U_1 = intensity_of_use, $U_1/\theta_1 = \{high, low\}$

$S_2 = (U_2, \theta_2)$, U_2 = food_and_fiber_production, $U_2/\theta_2 = \{high, low\}$

$S_3 = (U_3, \theta_3)$, U_3 = crown_closure, $U_3/\theta_3 = \{Low, Medium, High\}$

$S_4 = (U_4, \theta_4)$, U_4 = tree_species, $U_4/\theta_4 = \{deciduous, evergreen\}$

All concepts are defined by specifying what properties they have for each domain. We represent these properties as rough fuzzy sets of the approximation space, as shown in Table 1. Linguistic properties such as "low" and "high" can be further specified similar to the "deciduous"/"evergreen" tree-type and "closed" tree crown cover examples in

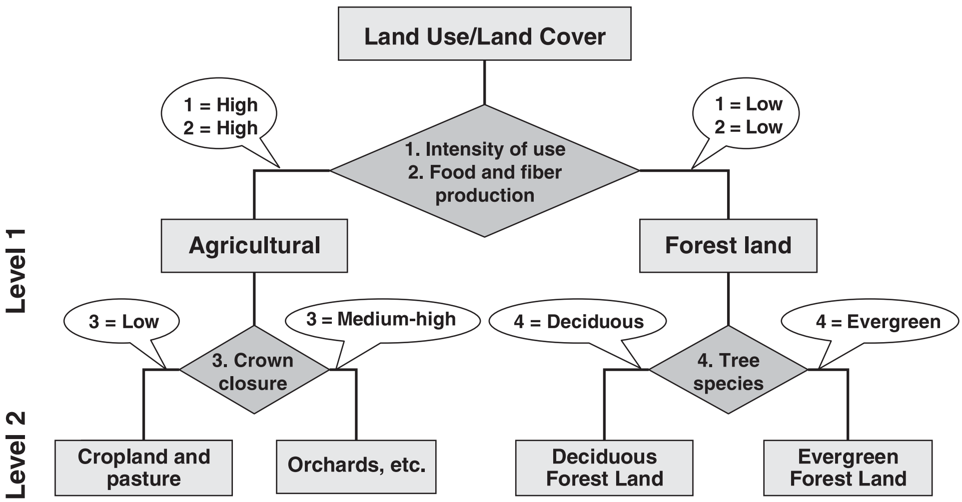


Figure 8 Simplified Anderson vegetation class hierarchy with examples of decision attributes that can be defined as quality dimensions of a conceptual space

section 2. To do this we create additional concept definitions of the properties we want to specify in more detail. In this example, the new concepts define an approximation space on the same underlying domain, but use different granulations of that domain. Table 2 shows an example of how the "low" food and fiber production property has been defined as a concept.

It is preferred to create a definition that uses a domain and granulation that correspond to some measurable characteristic, such as percent tree crown cover (Figure 5). But, as Table 2 demonstrates, we can also use an arbitrary granulation on the domain, *U*, as long as the measurement scale can be agreed on. After defining as many linguistic variables as possible we can substitute the values in the original LULC definitions to provide more specific concept definitions (Figure 9).

Table 1 Concept definition of Anderson LULC category 'Evergreen forest land'

	Wi	Domain				
S_1	1	Intensity of use	U_1/θ_1	[low]	[high]	
			$\mu_{\underline{x}}$	1	0	
			$\mu_{\bar{x}}$	1	0	
S_2	1	Food and fiber production	U_2/θ_2	[low]	[high]	
			$\mu_{\underline{x}}$	1	0	
			$\mu_{\bar{x}}$	1	0	
S_3	1	Crown closure %	U_3/θ_3	[low]	[med.]	[high]
			$\mu_{\underline{x}}$	0	1	1
			$\mu_{\bar{x}}$	0	1	1
S_4	1	Tree species	U_4/θ_4	[deciduous]	[evergreen]	
			$\mu_{\underline{x}}$	0	1	
			$\mu_{\bar{x}}$	0	1	

Table 2 Concept definition of category 'low' (food and fiber production)

	Wi	Domain												
S_2	1	Food and fiber production	U_2/θ_5	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]
			$\mu_{\underline{x}}$	0.95	0.85	0.75	0.65	0.55	0.45	0.35	0.25	0.15	0.05	0.0
			$\mu_{\bar{x}}$	1.0	0.95	0.85	0.75	0.65	0.55	0.45	0.35	0.25	0.15	0.05

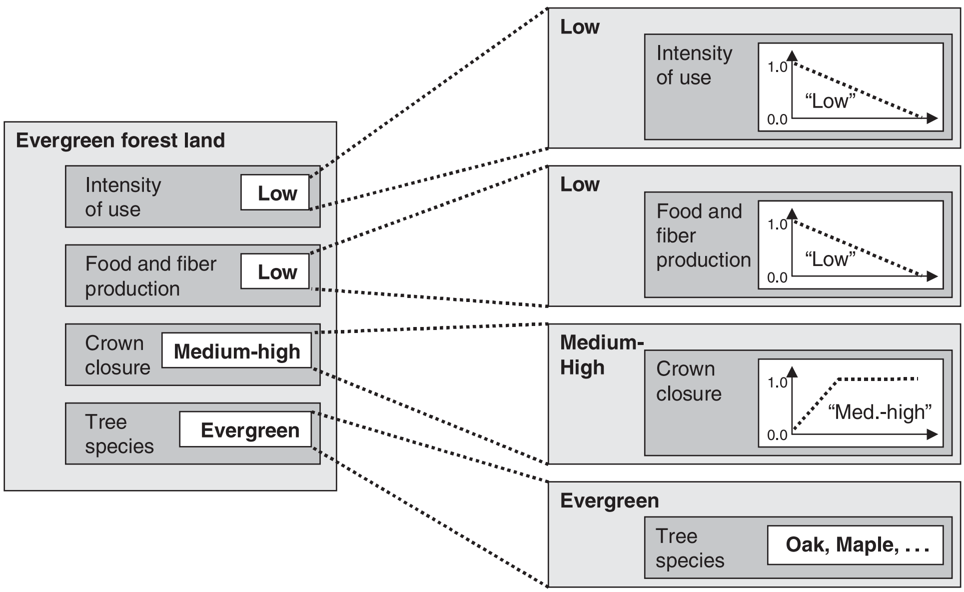


Figure 9 Concept definitions of linguistic property values that substitute values in "Evergreen forest land" concept definition

After substitution, Evergreen forest land is described by the same qualities but using domains that allow for explicit representation of vague values. For example, the value "Medium-high" crown closure value is replaced by a membership function over a %-tree-crown-cover domain. The "Evergreen" tree species value is replaced by a list of tree species categories that have either full or partial membership to the "Evergreen" category. This stage of the process is relying on user knowledge about the source and target classes, as well as a procedure for specifying membership functions for the mappings. Further details on this are provided in the discussion.

Concept similarity values (Table 3) and hierarchical relationships measured as concept overlap (Table 4) were calculated using Equations (1) through (7).

We can use Table 3 to evaluate the concept similarity between the Anderson LULC concepts. There is, of course, an absolute similarity between a concept and itself (the table diagonal) and we find higher values for concept pairs that are similar. For example, comparing the two agricultural concepts "Cropland" and "Orchard" with "Forest

Table 3 Concept similarity $s = e^{-c*d}$ calculated from upper and lower approximations of concept distance (Equation 3), with $c = 5$

	Deciduous forest land		Evergreen forest land		Cropland and pasture		Orchards, groves, etc.		Forest land		Agricultural land	
	$\underline{s}$	$\bar{s}$	$\underline{s}$	$\bar{s}$	$\underline{s}$	$\bar{s}$	$\underline{s}$	$\bar{s}$	$\underline{s}$	$\bar{s}$	$\underline{s}$	$\bar{s}$
Deciduous forest land	1.00	1.00	0.29	1.00	0.10	0.51	0.13	0.60	0.29	1.00	0.13	0.60
Evergreen forest land	0.29	1.00	1.00	1.00	0.10	0.51	0.13	0.60	0.29	1.00	0.13	0.60
Cropland and pasture	0.10	0.51	0.10	0.51	1.00	1.00	0.69	0.78	0.10	0.51	0.83	0.86
Orchards, groves, etc.	0.13	0.60	0.13	0.60	0.69	0.78	1.00	1.00	0.13	0.60	0.97	0.99
Forest land	0.29	1.00	0.29	1.00	0.10	0.51	0.13	0.60	1.00	1.00	0.13	0.60
Agricultural land	0.13	0.60	0.13	0.60	0.83	0.86	0.97	0.99	0.13	0.60	1.00	1.00

Table 4 Concept overlap calculated with Equation 7, where Equation 5, and Equation 6 are employed for quantitative and qualitative concept definitions respectively. Values indicate to what degree a row-concept is a sub-concept of a column-concept

	Deciduous forest land		Evergreen forest land		Cropland and pasture		Orchards, groves, etc.		Forest land		Agricultural land	
	$\underline{O}$	$\bar{O}$	$\underline{O}$	$\bar{O}$	$\underline{O}$	$\bar{O}$	$\underline{O}$	$\bar{O}$	$\underline{O}$	$\bar{O}$	$\underline{O}$	$\bar{O}$
Deciduous forest land	1.00	1.00	0.75	0.75	0.50	0.59	0.68	0.73	1.00	1.00	0.73	0.78
Evergreen forest land	0.75	0.75	1.00	1.00	0.50	0.59	0.68	0.73	1.00	1.00	0.73	0.78
Cropland and pasture	0.42	0.57	0.45	0.60	1.00	1.00	0.85	0.90	0.53	0.68	1.00	1.00
Orchards, groves, etc.	0.56	0.61	0.58	0.63	0.78	0.81	1.00	1.00	0.67	0.72	1.00	1.00
Forest land	0.86	0.86	0.89	0.89	0.50	0.59	0.68	0.73	1.00	1.00	0.73	0.78
Agricultural land	0.52	0.60	0.55	0.63	0.80	0.82	0.93	0.95	0.63	0.71	1.00	1.00

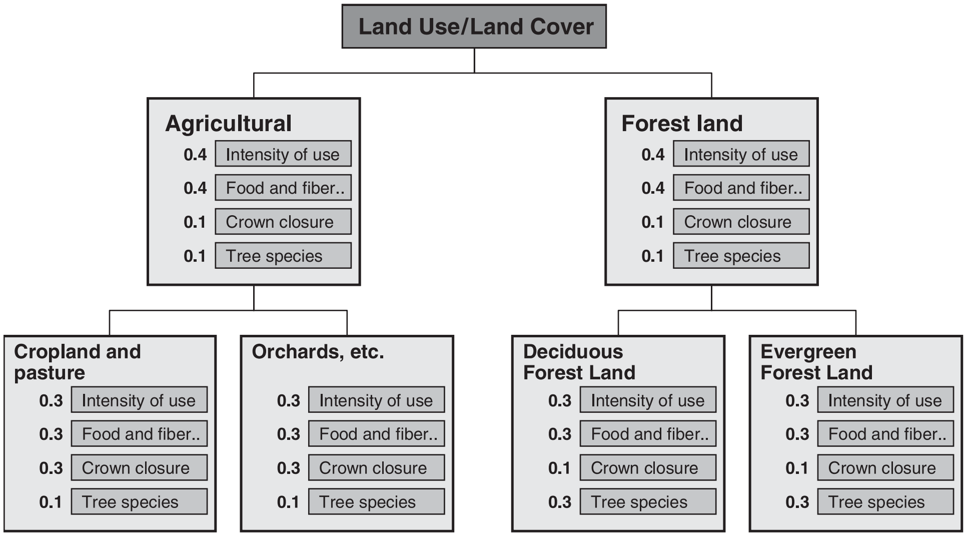


Figure 10 Different salience weights assigned to the domains of concept definitions

land", "Orchard" has the highest similarity values. Note that Table 3 is symmetric only because the salience weights are the same for all approximation spaces in the concepts. The overlap values in Table 4 can be used to find out whether one concept is a sub-concept of another. For example, the second row concept "Evergreen forest land" is a proper sub-concept of "Forest land" (column values 1.00) and is partially included (0.73, 0.78) in the "Agricultural land" concept. Note that Table 4 is not symmetric.

It is important to remember that each evaluation of concept similarity and overlap is performed from a certain perspective. Thus, evaluating how similar "deciduous forest" is to "agricultural land" uses the weights assigned to the approximation spaces in the "agricultural land" concept definition. Values in Tables 3 and 4 assumed equal salience weights for all approximation spaces in the concepts. From the perspective of defining "Forest land" and "Agricultural land", one may pay greater attention to the "Intensity of use" and "Food and fiber production" qualities and similarly pays different attention to the qualities from the perspective of the concepts at level 2. Figure 10 show an example of how different salience weights in the concept hierarchy can be set to account for the different attention paid to certain characteristics from each concept perspective. Applying the weights from Figure 10 to the concept definitions will result in different similarity and concept overlap values (see Tables 5 and 6 for examples).

The effects of applying salience weights differ depending on the property uncertainty and how the domains are weighted. The qualitative domains can have an especially large influence on the overall level of uncertainty, since similarity can only be measured roughly in these domains, taking either one of two possible RF-values; (0, 1), (1, 1). In Table 6, the use of different weights has reduced the amount of overlap between all the Anderson LULC categories, making the concept/sub-concept relations more distinct to each other. There are at the same time slight changes (both positive and negative) in the absolute area of uncertainty, that is, the differences between the upper and lower approximation, of the overlap measure.

Table 5 Concept similarity when domains are weighted according to Figure 10. Numbers indexed $^{-}$ are lower than for non-weighted similarity (Table 3) and numbers indexed $^{+}$ are higher

	Deciduous forest land		Evergreen forest land		Cropland and pasture		Orchards, groves, etc.		Forest land		Agricultural land	
	$\underline{s}$	$\bar{s}$	$\underline{s}$	$\bar{s}$	$\underline{s}$	$\bar{s}$	$\underline{s}$	$\bar{s}$	$\underline{s}$	$\bar{s}$	$\underline{s}$	$\bar{s}$
Deciduous forest land	1.00	1.00	0.22^{-}	1.00	0.18^{+}	0.44^{-}	0.24^{+}	0.54^{-}	0.61^{+}	1.00	0.18^{+}	0.44^{-}
Evergreen forest land	0.22^{-}	1.00	1.00	1.00	0.18^{+}	0.44^{-}	0.24^{+}	0.54^{-}	0.61^{+}	1.00	0.18^{+}	0.44^{-}
Cropland and pasture	0.08^{-}	0.51	0.08^{-}	0.51	1.00	1.00	0.65^{-}	0.74^{-}	0.16^{+}	0.42^{-}	0.93^{+}	0.94^{+}
Orchards, groves, etc.	0.09^{-}	0.54^{-}	0.09^{-}	0.54^{-}	0.65^{-}	0.74^{-}	1.00	1.00	0.18^{+}	0.44^{-}	0.99^{+}	1.00^{+}
Forest land	0.22^{-}	1.00	0.22^{-}	1.00	0.18^{+}	0.44^{-}	0.24^{+}	0.54^{-}	1.00	1.00	0.18^{+}	0.44^{-}
Agricultural land	0.09^{-}	0.54^{-}	0.09^{-}	0.54^{-}	0.80^{-}	0.83^{-}	0.97	0.99	0.18^{+}	0.44^{-}	1.00	1.00

Table 6 Concept overlap when domains are weighted according to Figure 10. Numbers indexed $^{-}$ are lower than for non-weighted overlap (Table 4)

	Deciduous forest land		Evergreen forest land		Cropland and pasture		Orchards, groves, etc.		Forest land		Agricultural land	
	$\underline{O}$	$\bar{O}$	$\underline{O}$	$\bar{O}$	$\underline{O}$	$\bar{O}$	$\underline{O}$	$\bar{O}$	$\underline{O}$	$\bar{O}$	$\underline{O}$	$\bar{O}$
Deciduous forest land	1.00	1.00	0.70^{-}	0.70^{-}	0.40^{-}	0.51^{-}	0.61^{-}	0.68^{-}	1.00	1.00	0.56^{-}	0.64^{-}
Evergreen forest land	0.70^{-}	0.70^{-}	1.00	1.00	0.40^{-}	0.51^{-}	0.61^{-}	0.68^{-}	1.00	1.00	0.56^{-}	0.64^{-}
Cropland and pasture	0.42	0.52^{-}	0.45	0.55^{-}	1.00	1.00	0.82^{-}	0.88^{-}	0.48^{-}	0.60^{-}	1.00	1.00
Orchards, groves, etc.	0.47^{-}	0.53^{-}	0.50^{-}	0.56^{-}	0.73^{-}	0.77^{-}	1.00	1.00	0.54^{-}	0.62^{-}	1.00	1.00
Forest land	0.83^{-}	0.83^{-}	0.87^{-}	0.87^{-}	0.40^{-}	0.51^{-}	0.61^{-}	0.68^{-}	1.00	1.00	0.56^{-}	0.64^{-}
Agricultural land	0.46^{-}	0.53^{-}	0.49^{-}	0.56^{-}	0.76^{-}	0.79^{-}	0.91^{-}	0.94^{-}	0.52^{-}	0.61^{-}	1.00	1.00

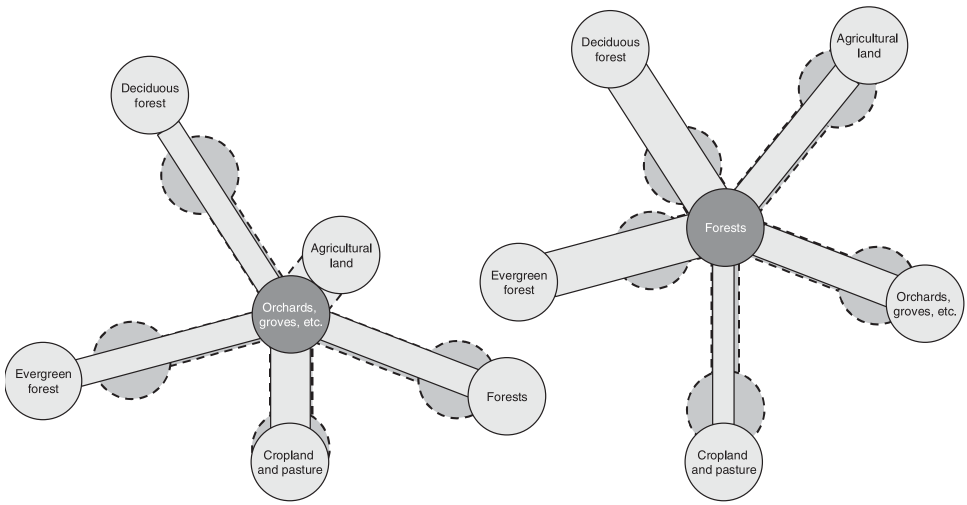


Figure 11 Concept similarity and overlap illustrated from the "Orchards, groves, etc." viewpoint (left) and the "Forests" viewpoint (right). Lengths of bars separating concept nodes reflect concept distance and width of the bar reflect concept overlap. Light shaded features represent the upper approximations and dark shaded, dashed features represent the lower approximations

Due to the multidimensionality of the concept space, it is hard to visualize the interrelationships between the concepts. Figure 11 uses nodes to represent concepts and the connecting bars illustrate concept distance and overlap as lengths and widths respectively. The lower approximation is gray shaded and dashed outlined features represent the upper approximation.

Concept relationships can also be visualized spatially as in Figure 12, where an example map using Anderson level 2 categories (a) is turned into a map showing the concept overlap of each category with the level 1 concept "Forest land" (b), and the

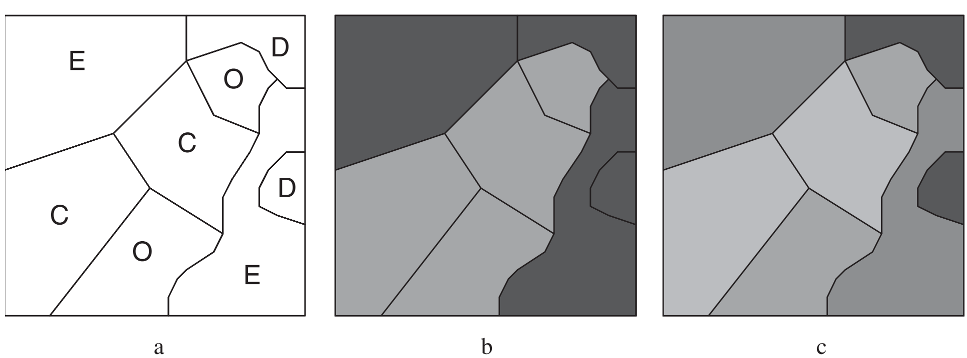


Figure 12 Example land cover map with categories E = evergreen forest, D = deciduous forest, C = cropland, O = orchards. Generalized into (b) illustrating concept overlap with category "Forest land" and (c) illustrating concept overlap with category "Deciduous forest". Increasingly darker shades indicate larger concept overlap

concept overlap with level 2 category "Deciduous forest" (c). Increasingly darker shades indicate larger concept overlap.

These last two figures demonstrate how each concept can be viewed from different perspectives, contextualizing the view to be based on, for example, the "Forest land" view of the world, or the "Orchard, groves, etc." view of the world.

4 Discussion and Conclusions

This paper has made more of a first proposal than it has presented a coherent framework for concept modeling. Nevertheless there are certain conclusions that can be drawn in this last section together with comments on some of the weaknesses and issues that need deeper scrutiny.

As Figure 12 demonstrated, a conceptual space representation can be visualized for instances of land cover concepts in a map like fashion. In spatially continuous representations, the rough fuzzy concepts definitions would not create a discrete delineation of land cover categories; rather a combined continuous and discrete spatial output would be expected similar to what has been demonstrated by Ahlqvist et al. (2003). Other visualizations are also possible but a main challenge is to simultaneously combine different similarity metrics, handle different perspectives on concepts and being able to compare each concept with every other concept, suggesting dynamic solutions similar to those described by MacEachren et al. (2004).

The spatial expression of concept instances (Figure 12) also points at the importance of the spatial configuration in which a certain phenomena exists (Gurney and Townshend 1983); hence spatial relations, such as distance, direction, and connectivity, are likely to be salient qualities in definitions of, for example, landforms and geographic regions. However, representing relations between concepts is one of the largest problems with spatial and feature-based models of similarity (Hahn and Chater 1997). Thus, studies of how spatial and temporal quality dimensions can be incorporated into the suggested model are therefore an important part of further work.

Several approaches to construct bridges between ontologies have been suggested where, for example, common ontologies (Bishr et al. 1999, Gahegan 1999) or metaclasses (Raper and Livingstone 1995) were put forward. In addition, a few land use/land cover studies have developed parameterized concept definitions to create bridges between concepts in different nomenclatures (Feng and Flewelling 2004, DiGregorio and Jansen 2000). These studies used standard set theoretic representations without recognizing a semantic space underlying the concept representation, thus limiting the possibilities to measure semantic similarity based on concept distance and to accommodate vague categories. The proposed solution is currently evaluated against such crisp set approaches in a study of translations between land cover taxonomies. Preliminary findings indicate a better separation between categories as well as increased possibilities to explore semantic relationships between different ontologies, within ontologies and for ontologies used in different contexts.

The most prominent difference between this and other approximate concept modeling approaches (e.g. Faucher 2001, Morris 2003) is the use of an underlying psychological concept space. This makes it possible to use fuzzy arithmetic to measure concept distance and concept inclusion, and the use of rough fuzzy sets, creates an explicit representation of indiscernibility in the concept similarity measures. There are a number of different suggested metrics for calculating the similarity between fuzzy numbers (Tran and Duckstein 2001, Chen and Chen 2003) but evaluations of the advantages of one measure over the other are very hard to perform, and depend not only on the similarity measure, but also on the aggregation method of multiple dimensions as well as the preceding parameterization of linguistic values that will be affected by interpersonal differences. We need to acknowledge that determining the relevant domains involved in the explanation of a concept is a prime scientific activity. Depending on the goal there are two cognitive interpretations of a conceptual space, one is *phenomenal* and concerns cognitive structures such as memory and perception, and the other is *scientific* where the conceptual space dimensions are treated as part of a scientific theory (Gärdenfors 2000). This work is concerned primarily with the latter case, where we as scientists are responsible of choosing quality dimensions of the conceptual space according to some underlying theory. However, geography engages a combination of both phenomenal and scientific conceptual spaces, and we can expect that a toolbox of several different approaches will be needed to elicit parameterized concept definitions. Some example approaches are:

- **The SRAS technique** (Robinson 2000) that uses a computer based questionnaire to elicit user judgments on the applicability of certain concepts to specified situations. This approach may prove useful to develop individual and group based membership functions.
- **The Analytic Hierarchy Process** (Saaty 1990) has been used to elicit both membership values and importance weights for a multi-criteria evaluation in a geographic information setting (Banai 1993). For more complex non-hierarchical relationship structures the Analytic Network Process (Saaty 2001) may provide additional support.
- **The Delphi method** (Cornish 1977) provides a set of guidelines to semi-quantitatively structure group communication and reveal patterns of thought, areas of consensus as well as disagreement. The integration of computer-based Delphi exercises, and other complementary approaches such as concept graphs (Sowa 2000), and self organizing maps (Kohonen 1995) demonstrates promising capabilities to explore and develop complex concept structures (Pike and Gahegan 2003).

Although only briefly mentioned in connection to Figure 2, it is possible to represent an object as a point in a conceptual space. But as soon as there is any uncertainty about an object's properties the object would be represented as an area, instead of a point. Thus, the difference between an object and a concept becomes blurred, as they both will be represented by regions in the conceptual space. This feature of the conceptual space representation provides an opportunity to explore solutions where concept intensions and extensions are modeled in parallel, similar to solutions proposed by MacEachren et al. (2004). Future research will investigate this further, especially the links related to connectionist approaches and methods, for example self-organizing maps (Kohonen 1995) and other exploratory techniques.

Another issue for future work is that the used dissemblance measure has some undesired mathematical properties, and we are currently developing new measures that better conform to the requirements of a conceptual space. One major problem with a distance-based similarity measure is that it only works on quantitative or semi-ordered domains. This obstacle was dealt with by using a rough representation that declares maximum distance in the lower approximation and 0 distance in the upper approximation. Although this often increases the uncertainty of the total similarity value, it makes it possible to include qualitative domains in a concept definition, and weigh the uncertainty according to the salience of that property.

This work is part of a larger framework to develop an infrastructure to support the work of four research sites that make up the Human Environment Regional Observatory (HERO) network. The goal is to enable researchers to share and reconcile their concepts in a conceptual space, and create mappings between their different notions. We also anticipate using conceptual space representations to keep track of changes as concepts evolve, similar to Gahegan and Brodaric (2002). Much human-environment research recognizes some form of uncertainty, mainly randomness or vagueness (see for example de Kok et al. 2000, Phillis and Andriantiatsaholiniaina 2001, Holling 2001). This paper has made an effort to simultaneously handle two important types of uncertainty, vagueness and indiscernibility. Future enhancements need also include uncertainty related to randomness in which previous work on evidence sets (Rocha 1999) may provide viable approaches. It remains an open question though how randomness may fit within the theory of conceptual spaces.

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