Soft Comput (2008) 12:243–249
DOI 10.1007/s00500-007-0201-z

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# Fuzzy sets in remote sensing classification

**Daniel Gomez · Javier Montero**

Published online: 16 May 2007


**Abstract** Supervised classification in remote sensing is a very complex problem and involves steps of different nature, including a serious data preprocessing. The final objective can be stated in terms of a classification of isolated pixels between classes, which can be either previously known or not (for example, different land uses), but with no particular shape nither well defined borders. Hence, a fuzzy approach seems natural in order to capture the structure of the image. In this paper we stress that some useful tools for a fuzzy classification can be derived from fuzzy coloring procedures, to be extended in a second stage to the complete non visible spectrum. In fact, the image is considered here as a fuzzy graph defined on the set of pixels, taking advantage of fuzzy numbers in order to summarize information. A fuzzy model is then presented, to be considered as a decision making aid tool. In this way we generalize the classical definition of fuzzy partition due to Ruspini, allowing in addition a first evaluation of the quality of the classification in this way obtained, in terms of three basic indexes (measuring covering, relevance and overlapping of our family of classes).

**Keywords** Fuzzy classification systems · Remote sensing · Fuzzy graph

D. Gomez (✉)
Faculty of Statistics, Complutense University,
Madrid, Spain
e-mail: dagomez@estad.ucm.es

J. Montero
Faculty of Mathematics, Complutense University,
Madrid, Spain
e-mail: javier_montero@mat.ucm.es

## 1 Introduction

Since the beginning of the history of Fuzzy Sets Theory Zadeh (1965), classification has been a key issue, both from a theoretical and a practical point of view (see, e.g., Bezdek and Harris 1978; Gath and Geva 1989; Ruspini 1970). Many problems within this field are naturally formalized by considering fuzzy concepts. In fact, most concepts that users have in mind are fuzzy in nature, in the sense that they allow degrees of verification. This is the case in remote sensing classification problems, where the introduction of crisp classes represent an unrealistic oversimplification of reality, leading to obviously wrong interpretations of direct observation.

Remote sensing is a wide topic that has been a tremendous development in the last decades. Remote sensing is defined in Cogalton and Green (1999) as "the art and science of obtaining information about an object without being in direct physical contact with the object". As pointed by remote sensing researches, classification is probably the most important issue in remote sensing. And fuzzy sets play a very important role in their classification problems, since they allow to model non-statistical imprecision that appears in the definition of Nature classes (see also Binahi and Rampini 1993; Bensaid et al. 1996; Fisher and Pathirana 1990; Foody 1999, 1996; Foody and Cox 1994; Wasilakos et al. 1990; Wang 1990).

Earth's surface is amazingly complex, and different and difficult pre-processing techniques should be applied before facing the classification problem. Some of them can be summarized in the following items (see Cogalton and Green 1999, for more details):

(a) *Sensor determination* taking into account the objects or classes under study, an adequate sensor has to be chosen in order to discriminate classes. Fuzzy sets theory appears in a natural way when the preferences and aims of the decision maker are modeled.

(b) *Management data and transformation* errors and fuzziness are often present in data acquisition process. Furthermore, sometimes a reduction of the amount of information is needed when the image is extremely complex (for example more that 100 spectral bands).

(c) *Training site and pattern recognition* in order to know the main features associated with each class, a previous unsupervised classification or expert classification is needed (for example, in Sect. 2 of this paper we expose a segmentation technique that takes into account the neighborhood, in addition to spectral information of each pixel).

(d) *Supervised classification algorithm* this step ends with a classification (crisp or fuzzy) of the image. Each pixel or unit sample is classified into crisp or fuzzy classes, taking into account the information given from the training site.

(e) *Post classification* in order to smooth the classification and improve the classification accuracy, some learning process will be needed. Some logical rules should be considered in order to improve and smooth results.

(f) *Analysis of results* once classification and post classification are finished, accuracy of the process and classifiers must be determined. For example, by considering different agreement measures between the reference data set and the final classification. Recent researches give to the fuzzy sets and important role in this analysis.

In this paper some advantages of fuzzy approaches in some of these stages will be shown. This paper is organized as follow: in Sect. 2, an algorithm that uses a fuzzy graph modelization for searching possible classes is described. Once the decision maker has a first idea about the possible classes in the image, a fuzzy (or crisp) supervised classification can be achieved. Although most classifiers use to produce a fuzzy partition (in the sense of Ruspini 1969), implied conditions of this model are very difficult to fulfill, requiring in practice a long learning process. In order to allow such a learning process, some appropriate tools are given in Sect. 4, introducing concepts like covering, relevance and redundancy for each classification. These concepts are analyzed taken into account some aggregation operators, leading altogether into a fuzzy classification system (Sect. 5), which generalizes the fuzzy partition proposed by Ruspini (1969). Finally, in Sect. 6 some remarks are presented, pointing out among other things the relevance of painting algorithms in any decision aid tool.

## 2 Searching for fuzzy classes in remote sensing

As already pointed out, before classifying pixels into known classes (for example, a forest of coniferous or oaks), a complex data pre-processing is always required. Once the pre-processed image is obtained, the standard immediate objective is the identification of homogeneous regions. These regions will allow decision maker to identify training sites for the further classification.

Most techniques for determining homogeneous regions are based on statistical methods that only take into account the spectral information of each isolated pixel. In this way clusters are obtained. But since they are obtained taking into account only spectral information of each pixel, it may happen that the proposed cluster is not related to the real classes decision maker is interested in. Consequently, there is a need to include contextual information in those algorithms. With this aim, in this section we present a coloring algorithm that takes into account the neighborhood of each pixel (see http://www.mat.ucm.es/fcs for more details, but also Pardalos et al. 1998).

In order to represent the dissimilitude between pixels, we will model the image as a fuzzy graph where the nodes are crisp and the edges are fuzzy. Mathematically, a remote sensing image can be defined as a set

$$P = \{p_{ij}/1 \le i \le r, 1 \le j \le s\}$$

of $r \times s$ information units -pixels-, where $p_{ij} = \left(p_{ij}^1, p_{ij}^2, \ldots, p_{ij}^k\right)$ is the pixel associated with the coordinate *(i,j)*.

In order to find homogeneous regions in the image we model this image by a fuzzy planar graph. The graph is planar in the sense that two pixels $p_{i,j}$ and $p_{i',j'}$ are not connected if $|i - i'| + |j - j'| > 1$ (see Fig. 1).

The fuzzy graph $\tilde{G} = (P, \tilde{A})$ is then defined by the image pixel $s$ and the set of fuzzy arcs $\tilde{A}$ will be characterized by the matrix:

$$\mu_{\tilde{A}} = \left(\mu_{p_{ij}, p_{i'j'}}\right)_{p_{ij}, p_{i'j'} \in \bar{P}}$$

where $\overline{P}$ is the set of connected pixels, i.e.,

$$\bar{P} = \Big\{(p_{ij}, p_{i'j'}) \in P^2 : |i - i'| + |j - j'| = 1; 1 \le i \le r, 1 \le j \le s\Big\}$$

The coloring algorithm for fuzzy graphs proposed here (see also Amo et al. 2004; Calvo et al. 2002) is a sequence of binary coloring processes. The first binary coloring analyzes the set $P$, assigning to each pixel either the color "0" or the color "1". The second binary coloring is then applied separately to the sub graph generated by those pixels colored

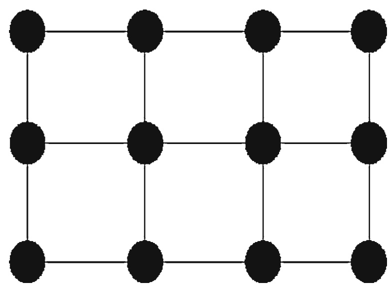

**Fig. 1** Planar graph with $r$=3 and $s$=4

as “0”, to obtain the color classes “00” and “01”, and to the sub graph generated by those pixels colored as “1”, to obtain the color classes “10” and “11”. This hierarchical process of binary coloring procedures is repeated a number of iterations, until a significative segmentation is obtained (see Gómez et al. 2007, 2006). Possible homogeneous regions will be associated to connected pixels with the same color. Once the algorithm finishes, the segmentation information can be included in any of the standard unsupervised classification algorithm, improving the overall accuracy of the classical algorithms that only take into account the spectral information of each pixel.

### 2.1 Basic binary coloring algorithm

A binary coloring of a graph $G = (V, E)$ is a particular case of a 2-coloring, $col : V \rightarrow \{0, 1\}$. The binary coloring procedure that we propose as the basic procedure, colors two adjacent pixels as “0” or “1” depending on the fuzzy dissimilitude between them, when compared to a prescribed threshold. Notice that a standard crisp approach assigns the same class to any two pixels whenever such a distance is small, no matter if they are not adjacent.

In order to define the first binary coloring procedure, a value $\alpha$ is fixed. We can start, for example, with pixel (1,1) in the top-left corner of the image, and then pixels can be colored from left to right and from up to down, in the following way:

$$col(i+1, j) = \begin{cases} col(i, j) & if\ d_{p_{ij}, p_{i'j'}} \tilde{\geq} \alpha \\ 1 - col(i, j) & if\ d_{p_{ij}, p_{i'j'}} \tilde{\leq} \alpha \end{cases} \quad \text{for all } (i, j) \in \{1, \ldots r\} \times \{1, \ldots s-1\}$$

$$col(i, j+1) = \begin{cases} col(i, j) & if\ d_{p_{ij}, p_{i'j'}} \tilde{\geq} \alpha \\ 1 - col(i, j) & if\ d_{p_{ij}, p_{i'j'}} \tilde{\leq} \alpha \end{cases} \quad \text{for all } (i, j) \in \{1, \ldots r-1\} \times \{1, \ldots s\}$$

In order to determine if the fuzzy number $d_{p_{ij}, p_{i'j'}}$ is greater than alpha, a ranking function will be applied (see, e.g., Dubois and Prade 1983). Therefore, given a colored pixel $p_{ij}$, the adjacent pixels $p_{i+1j}$ and $p_{ij+1}$ can be colored in a similar way or not. However, it has to be noticed that, since pixel $p_{i+1j+1}$ can be colored either from pixel $p_{i+1j}$ or from pixel $p_{ij+1}$, both coloring processes may not lead to the same color, showing what we can call an *inconsistent* coloring situation. Obviously, our binary coloring procedure is dependent on the particular order we proceed for coloring.

Anyway, we can look for a value $\alpha^*$ assuring consistency, which always exists, and then consider the subsequent hierarchical classification where pixels are classified either as color class “0” or “1”, each one divided into a more precise color (class “0”, for example, will switch either into “00” or “01”), and so on. Such a process is performed separately, by alternatively activating only one of the classes already colored in the previous stage. The same process will be applied in subsequent stages, in such a way that the above binary coloring process is carried out to the activated pixels under consideration, i.e., a subset $P'$ of pixels contained within $P$.

As any fuzzy representation technique, the tool presented in this section gives decision maker an additional understanding of the image, hopefully allowing a more accurate description of images involving fuzzy classes. Our hierarchical output offers a systematic sequence of colored images that can be carefully analyzed by decision makers for a better understanding of the image (depending on the decision maker objectives and abilities). In this sense, we want to stress the absolute need for developing manageable descriptive tools in order to show fuzzy uncertainty. In fact, the information given by this algorithm has been included in different classical unsupervised classification methods in order to improve the training site description (see Ruspini 1969).

## 3 Fuzzy classification

The first objective, once a family $C$ of available classes has been defined, is to determine the degree $\mu_C(p)$ to which a pixel or unit sampling $p$ belongs to the class $c \in C$, for every pixel $p$ under consideration, $p \in P$. In this way, a membership function $\mu_c : P \rightarrow [0, 1]$ is defined for each class $c \in C$ (see, e.g., Amo et al. 2004).

Notice that we consider the whole family of classes when a pixel has to be classified, as usually done in remote sensing. Classification procedures use to need the whole view of possibilities (classes). In this sense, most classification methods are in general extremely dependent on the family of classes the user is forced to consider. Even in a crisp context, users frequently have a look at all possible choices before choosing a particular class for a given object. Hence, a key concept in classification is the notion of partition, which is quite often a structured family of classes.

Fuzzy partitions were introduced by Ruspini (1969). In his definition, given a discrete family $C$ of classes, it is assumed that for every object under consideration $\sum_{c \in C} \mu_c(p) = 1$.

Hence, each pixel $p$ may belong to several classes -to a certain degree-, and the total degree of membership is distributed among all classes. In this way, the classical crisp partition concept was generalized.

From our point of view, Ruspini's proposal may in principle represent a desirable situation, in the sense that it models a typical situation were every pixel seems to be explained, in some way, with the minimum amount of information. But again, it seems to be a too restrictive definition within fuzzy modeling. In most cases, the membership function of the fuzzy classes under consideration does not verify the conditions of such a Ruspini fuzzy partition (the decision maker will not meet those requirement if not artificially forced). This hypothesis is unrealistic from a remote sensing point of view, since the users are not able to assure that every pixel is fully explained without superfluous information. Moreover, although we can assume that a standard fuzzy classification system may pursue a Ruspini partition, we must point out that this is not always the case (some decision makers are willing for massive overlapping classes). Moreover, the whole procedure must be meaningful for decision maker (nice mathematical models may not be useful for decision makers if they fully loose direct intuition about results).

Based on the information given in the unsupervised stage, a standard procedure is to propose a particular family of classes, and then check degrees of membership. Obviously, we cannot expect that those classes are from the very beginning just the perfect ones we were seeking for our particular purposes. It is not trivial at all to get a Ruspini's partition as the first stage. Although some practical difficulties of Ruspini's partitions can be partially overcome by a weaker approach proposed by some other authors (see, e.g., Thiele 1996a,b), it is clear that Ruspini's classification system cannot always be used (see an example in Amo et al. (2001), concerning a general computer security problem).

In the next section we propose to analyze classification systems by means of aggregative models (see Amo et al. 2004), which should present Ruspini's partition as a particular additive solution.

## 4 Aggregation analysis: covering, relevance and redundancy

In this section, the concepts of covering, relevance and redundancy are described in order to get a basic description of the quality of each classification, allowing in this way some hints about the possible post-classification improvements, i.e., the first step for a learning process about membership functions and the classification system itself. All these concepts are based on aggregation operators that first have to be introduced.

An aggregation process is present in remote sensing classification every time we need to amalgamate the information obtained about each pixel or unit sampling. Usually, the information associated to each pixel is given by means of degrees of membership for each spectral band. We can find in the literature, for example, that additive rules and some t-conorms (see, e.g., Calvo et al. 2002; Klement et al. 2000; Iancu 1999) are useful for forming conjunctive rules. Information is in this way aggregated into one single index. Since we usually do not know in advance the number of chunks of information we should aggregate, classical approaches assume the existence of a basic binary operator being associative, in such a way that a sequential application of such an operator will give us the aggregated information, no matter the dimension of information.

But we know that not every operator is associative (OWA operators Yager (1993), for example, need the dimension of the data set to be fixed in advance). Otherwise we may have problems in defining what a rule is (see Cutello and Montero (1975) for a possible solution for OWA operators). Other possibility is to consider the recursive approach introduced in Cutello and Montero (1999), which generalizes the concept of associativity, and the particular representation results obtained in Amo et al. (2001).

**Definition** A recursive rule $\phi$ is a family of aggregation functions $\{\phi_n : [0, 1]^n \to [0, 1]\}_{n\geq 2}$ such that there exists an ordering rule $\pi$ and two sequences of binary operators

$$\left\{L_n : [0, 1]^2 \to [0, 1]\right\}_{n\geq 2} \text{ and } \left\{R_n : [0, 1]^2 \to [0, 1]\right\}_{n\geq 2}$$

such that

$$\begin{aligned}
&\phi_n(\pi(x_1), \pi(x_2), \ldots, \pi(x_n)) \\
&\quad = L_n(\phi_{n-1}(\pi(x_1), \pi(x_2), \ldots, \pi(x_{n-1})), \pi(x_n)) \\
&\quad = R_n(\pi(x_1), \phi_{n-1}(\pi(x_2), \pi(x_3), \ldots, \pi(x_n))).
\end{aligned}$$

Recursiveness is a property of a sequence of operators $\{\phi_n\}_{n\geq 2}$ allowing the aggregation of any number of items: $\phi_2$ tells us how to aggregate two items, $\phi_3$ tells how to aggregate three items and so on, in such a way that being faced with $n$ items we shall aggregate them by means of $\phi_n$. Recursiveness assures consistency of such a family of operators by assuming that such a rule is operational, in the sense that it can be evaluated both from left and right by means of a sequence of binary operators. In general, each new data implies a modification of the binary operator, in such a way that these binary operators evolve. A recursive rule is a family of operators allowing a sequential reckoning by means of a successive application of binary operators, once data have been properly ordered: the ordering rule assures that new data do not introduce modifications in the relative position of items already ordered (see Amo et al. 2004). Of course, the underlying linear structure of data assumed in such a recursive approach should be modified in the future in order to take into account the spatial structure of data (see Gómez and Montero 2004).

### 4.1 Covering

Once the aggregation system and the recursive aggregation model have been defined, from the degree to which a pixel $p \in P$ belongs to each class $c \in C$ an aggregated value $\mu_C(p) = \varphi\{\mu_c(p)c \in C\}$ can be obtained (depending on the particular disjunctive recursive rule). This aggregated value can be understood as the degree to which the pixel $p$ is explained by such a family of classes $C$. The higher all these values, the better. In case $\mu_C(p) = 1 \, \forall \, p \in P$, it can be understood that the whole family of objects is fully covered by our family of fuzzy classes $C$. The lower such an aggregated covering value, the stronger claim for a new class.

Obviously, covering is not enough in order to get a nice classification system. The family $C$ of classes should explain as much as possible variation in all the objects (such a family of classes must represent a nice covering of objects), but it should be also as compact as possible, taking into account not only relevant classes, reducing at the same time possible overlapping information (redundancy). Let us discuss these two extra notions (relevancy and redundancy) in the next subsections.

### 4.2 Relevance

In order to analyze relevancy, we propose to compare behavior of each nonempty class $A$ with behavior of the remaining $C - A$ classes being kept in the model, taking into account

$$\varphi\{\mu_c(p)c \in C\} \tag{1}$$

$$\varphi\{\mu_k(p)k \in A\} \tag{2}$$

$$\varphi\{\mu_k(p)k \in C - A\} \tag{3}$$

for every pixel $p$, being $A$ a fixed nonempty family of classes, $A \subset C$. For example, when (1) is significantly higher than (3), then we can in principle suggest that $A$ is a relevant family of classes, even if (2) is not high. And whenever (1) is not significantly different than (3), we can in principle suggest that $A$ is a non relevant family of classes, even when (2) is not low.

The relevancy issue can be therefore addressed as a dimensionality reduction problem, in such a way that other statistical and non statistical representation models can be introduced (see, e.g., Amo et al. 2001, 2004). Obviously, a class which gives no additional information at all about how to classify our objects does not deserve to be kept in the model.

### 4.3 Redundancy

Once relevancy has been studied at a first stage (on the basis of our $\varphi$ disjunction rule), class overlapping can be studied by means of a conjunction rule $\phi$. In fact, the value $\phi\{\mu_c(p), \mu_d(p)\}$ can be understood as the degree of overlapping between classes $c$ and $d$ with respect to the pixel $p$.

For the special case in which the classification is a crisp partition, we have:

- $\varphi\{\mu_c(p)/c \in C\} = 1 \, \forall \, p \in P$
- $\phi\{\mu_c(p), \mu_d(p)\} = 0 \, \forall \, p \in P, \, \forall \, c \neq d \in C$

Overlapping can be evaluated also for every subset of classes $A \subset C$ with at least two elements, allowing the possibility of a better insight into the structure and relationships among those classes.

Again, we expect that redundancy for pairs of classes will be enough in many practical problems, at least at a first stage. Additional redundancy indexes will be evaluated only in case results suggest decision maker to try a more accurate classification system.

## 5 Fuzzy classification systems

A fuzzy classification system will be given by a set of classes together with a disjunctive and a conjunctive rule, and a set of objects (pixels in our remote sensing context). We can denote a classification system as $(C, \phi, \varphi, P)$. Of course, a fuzzy classification system will be meaningless in some cases, whenever it does not allow any discrimination.

**Definition 5.1** A fuzzy classification system $(C, \phi, \varphi, P)$ will be *meaningful* if and only if:

- $\varphi\{\mu_c(p)/c \in C\} \succ 0 \, \forall \, p \in P$
- For all $A \subset C$ with at least two classes exist a pixel $p$ such that $\phi\{\mu_c(p)/c \in A\} \prec 1$.

Once a classification system has been fixed, whenever $\varphi\{\mu_c(p)/c \in C\} = 0$ we should be searching for a new class. If $\phi\{\mu_c(p), \mu_d(p)\} = 1 \, \forall \, p \in P$, these two classes are fully redundant and a modification of the set of classes must be done. As a general criteria, the lowest value $\phi\{\mu_c(p), \mu_d(p)\}$ and the highest value of $\varphi\{\mu_c(p)/c \in C\}$, the better classification system we have.

**Definition 5.2** A fuzzy partition is a fuzzy classification system $(C, \phi, \varphi, P)$ that:

- $\varphi\{\mu_c(p)/c \in C\} = 1 \, \forall \, p \in P$

- $\phi\{\mu_c(p)/c \in A\} = 0$ for each pixel and for each subset of classes with at least two elements.

Taking into account these definitions, a fuzzy partition is just a completely explanatory and non redundant fuzzy classification system that could be the final objective for many decision makers (but not necessarily a declared objective). Anyway, a natural learning process will search for a better classification system (in a sense to be fixed by each user), taking into account at least the above covering, relevancy and redundancy arguments. We should always try to explain as much as we can, avoiding as much as possible irrelevant and redundant classes. Of course each one of those three key arguments (covering, relevancy and redundancy) will allow degrees of verification, and decision makers should make their mind up about the right levels they are willing to accept in each particular case.

## 6 Final comments

In this paper we have described how fuzzy sets can be applied in the phase of *training site and pattern recognition*, and in the phase of *supervised classification* for remote sensing image classification.

In the first phase, an interactive algorithm for searching an initial subset of classes that allows an initial classification has been proposed. In order to do this, a coloring algorithm for fuzzy graphs is described in Sect. 2 (see http://www.mat.ucm.es/fcs and Gómez et al. (2006) for computational results). Such a coloring algorithm offers several possible pictures of the image. Each one of these pictures is obtained by means of an automatic coloring procedure, where each pixel is colored taking into account color similarity degrees in its neighborhood. Our coloring process is based upon a basic binary procedure which is again and again applied, leading to a hierarchical structure of colors. Each colored picture can be analyzed by decision makers in a posterior classification procedure: certain homogeneous regions can be identified, and a subsequent comparison may lead to a fuzzy classification.

The classification model proposed in this paper shows a promising behavior. On one hand, covering and relevancy are analyzed by means of a disjunctive rule, leading to deletion of classes or a search for extra classes. On the other hand, redundancy is analyzed by means of a conjunctive rule, leading to a search for new mixtures of classes. Together with the above indexes for covering, relevancy and redundancy, we can also take into account some global indexes in order to get a more complete evaluation of the quality of each classification.

Possible improvements of our classification system have to be made by at least considering covering, relevancy and redundancy. Learning process will stop when decision maker considers that a good enough classification system has been obtained. But this decision about which one is the chosen final classification system should be taken keeping in mind that our main objective should be a better comprehension of the problem (sometimes we forget that complex information may be not manageable by the decision maker). In this sense, we do not expect that an objective optimal number of fuzzy classes exist, because it will depend on each particular decision maker. Once the decision maker has a certain idea about possible fuzzy classes and regions in the image, we need to develop tools for managing fuzzy classes.

Anyhow, coloring techniques (like the one proposed in this paper) should play a key role in the future in order to help decision makers to capture a global view of complex images, including those where regions do not show clear boundaries. Without appropriate representation techniques, decision makers may look at some sophisticated models as black boxes, and therefore reject them in practice.

**Acknowledgments** This research has been partially supported by the Government of Spain, grants MTM2005-08982-C04 and TIN2006-0619, and by a Del Amo joint project within Complutense University and the University of California.

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