# ROCK SLOPE ENGINEERING

1. MODES OF FAILURE OF ROCK SLOPES

2. SINGLE PLANE SLIDING MODE

3. WEDGE SLIDING WITH TWO FREE SURFACES

## 1. MODES OF FAILURE OF ROCK SLOPES

The modes of failure of slopes in rock are more complex than slopes in soils since, in most cases, they are controlled by discontinuities. Several modes of failure can be observed in rock masses.

*Modes of Failure in Soft, Weathered or Jointed Rock Masses*

These include raveling, slumping or sliding through the body of rock itself. They are common in soft formations such as shale and in deeply weathered, jointed or broken rock masses (see Figure 1).

*Modes of Failure in Hard Rock Masses*

These are related to movement along rock mass discontinuities such as joints, faults, bedding and cleavage planes. Several failure modes can take place. The three basic ones are: PLANE SLIDING, WEDGE SLIDING, TOPPLING.

- Plane sliding: Movement of a block along a plane of weakness (Figure 2a). The conditions for plane sliding include: (1) the plane of weakness must daylight, and (2) lateral block release must be available through joints, topography, excavation, or intact rock failure.

- Wedge sliding: Movement of a block along two planes of weakness (Figure 2b). The conditions for wedge sliding include: (1) the line of intersection of planes must daylight, and (2) lateral block release is required.

- Toppling: Overturning of rock layers inclined steeply into the hillside (Figure 2c). A mechanism is needed to start the process of toppling such as erosion, excavation at the toe of the slope. Several types of toppling mechanisms have been identified by geologists (see Figures 3 and 4).

*Higher Modes of Failure*

These are exhibited by complex jointed and bedded rock masses in which plane sliding, wedge sliding, and toppling occur simultaneously or successively (see Figures 4 and 5).

**References**

Goodman, R.E. and Bray, J. W. (1976) Toppling of rock slopes, in *Proc. Specialty Conf. on Rock Engineering for Foundations and Slopes*, ASCE, Boulder, pp. 201-234.

Goodman, R.E. (1989). *Introduction to Rock Mechanics*, Wiley, 2nd Edition

Hoek, E. and Bray, J.W. (1977) *Rock Slope Engineering*, IMM, London, 2nd Edition.

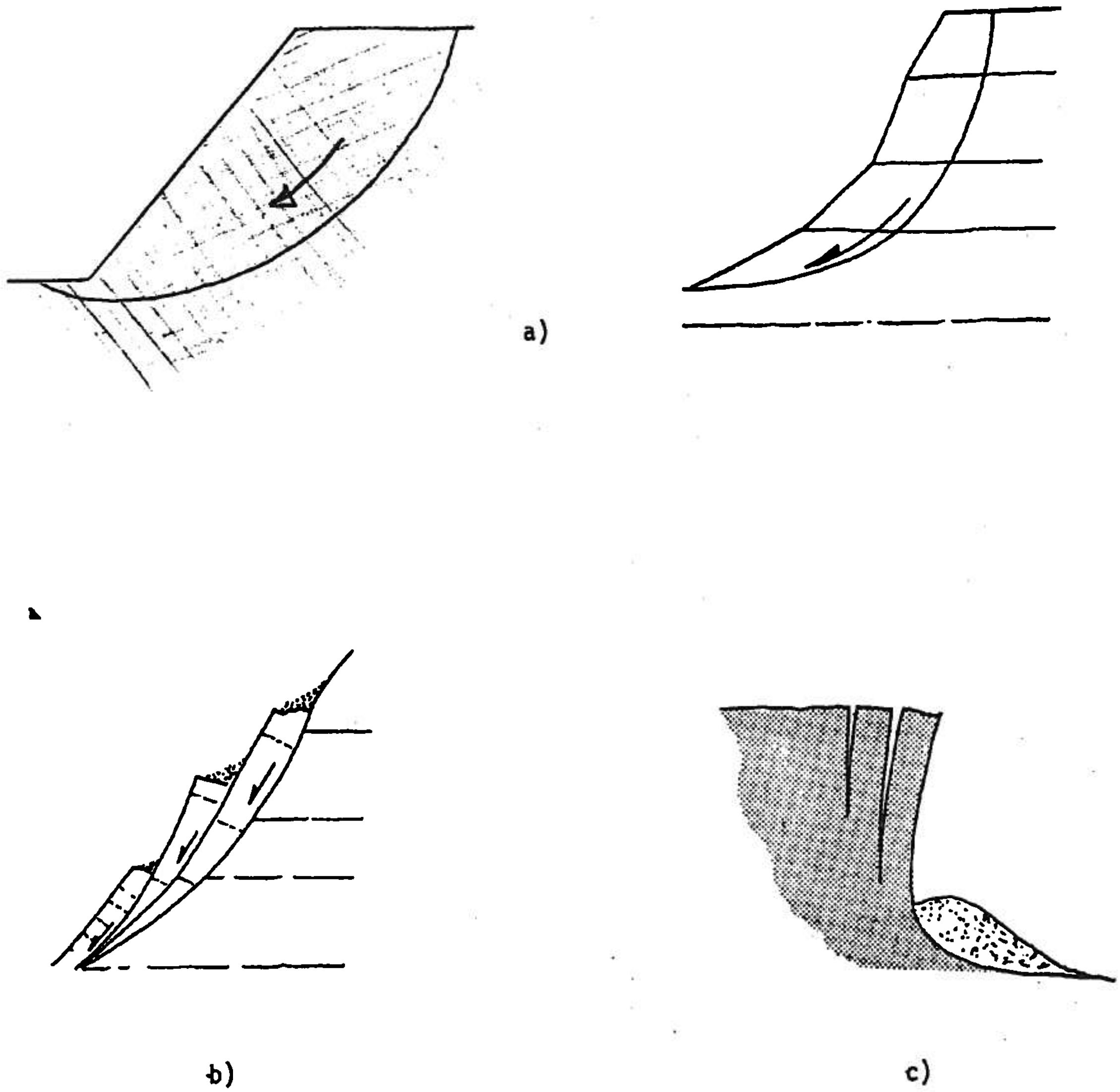



Figure 1. Modes of failure in soft, weathered, or jointed rock masses. (a) Rotational (soil type), (b) Raveling, (c) Tension crack toppling.

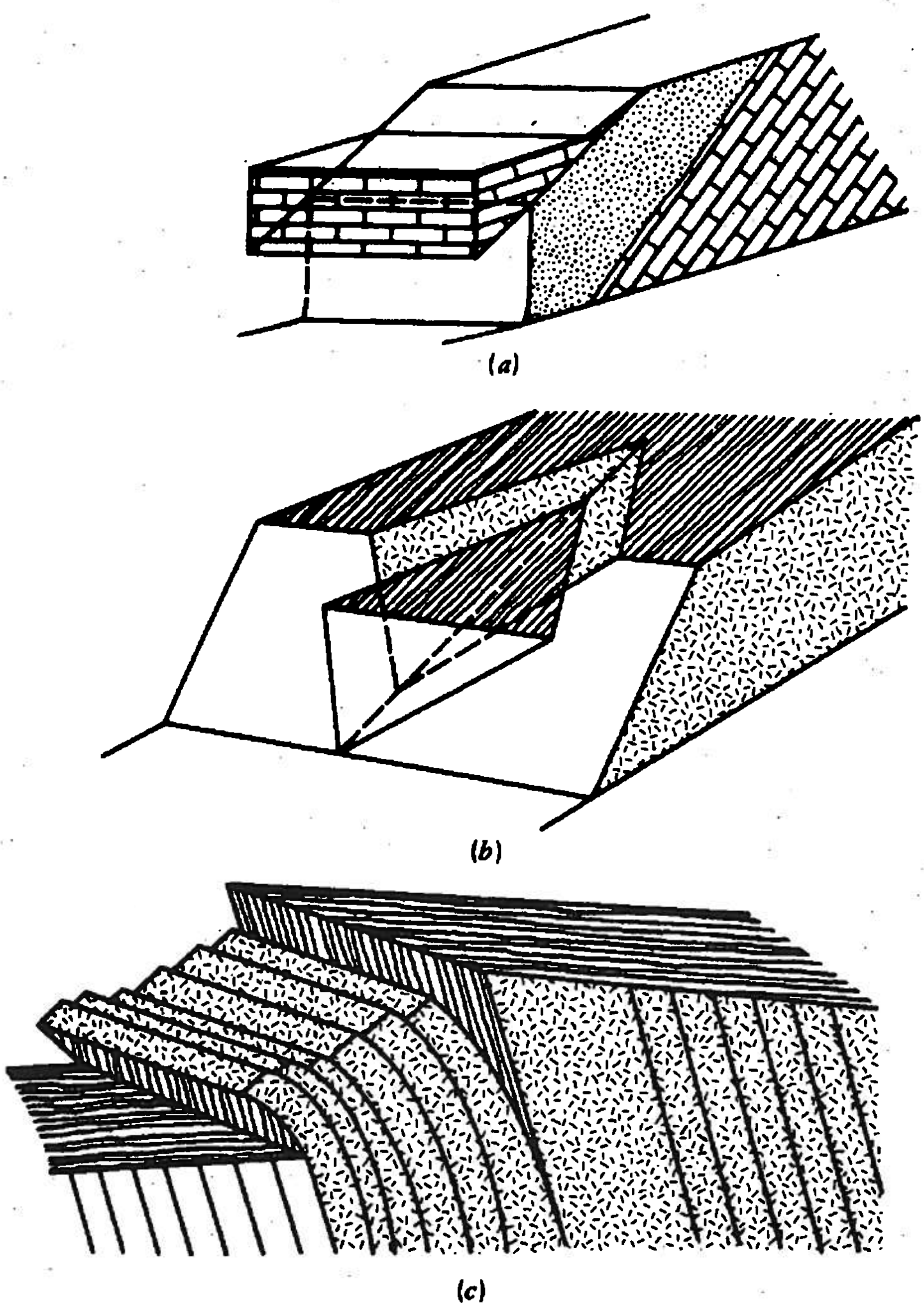



Figure 2. Modes of failure in hard rock masses (a) Single plane sliding, (b) Wedge sliding, (c) Toppling (after Goodman, 1989).

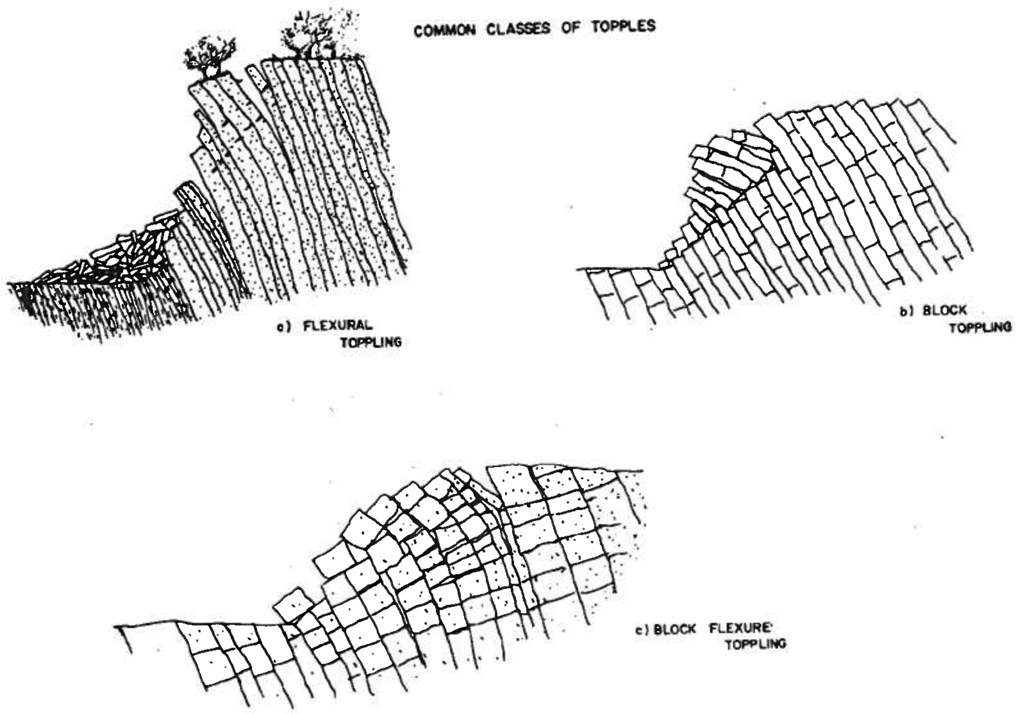



Figure 3. Three classes of toppling mechanisms (after Goodman and Bray, 1976).

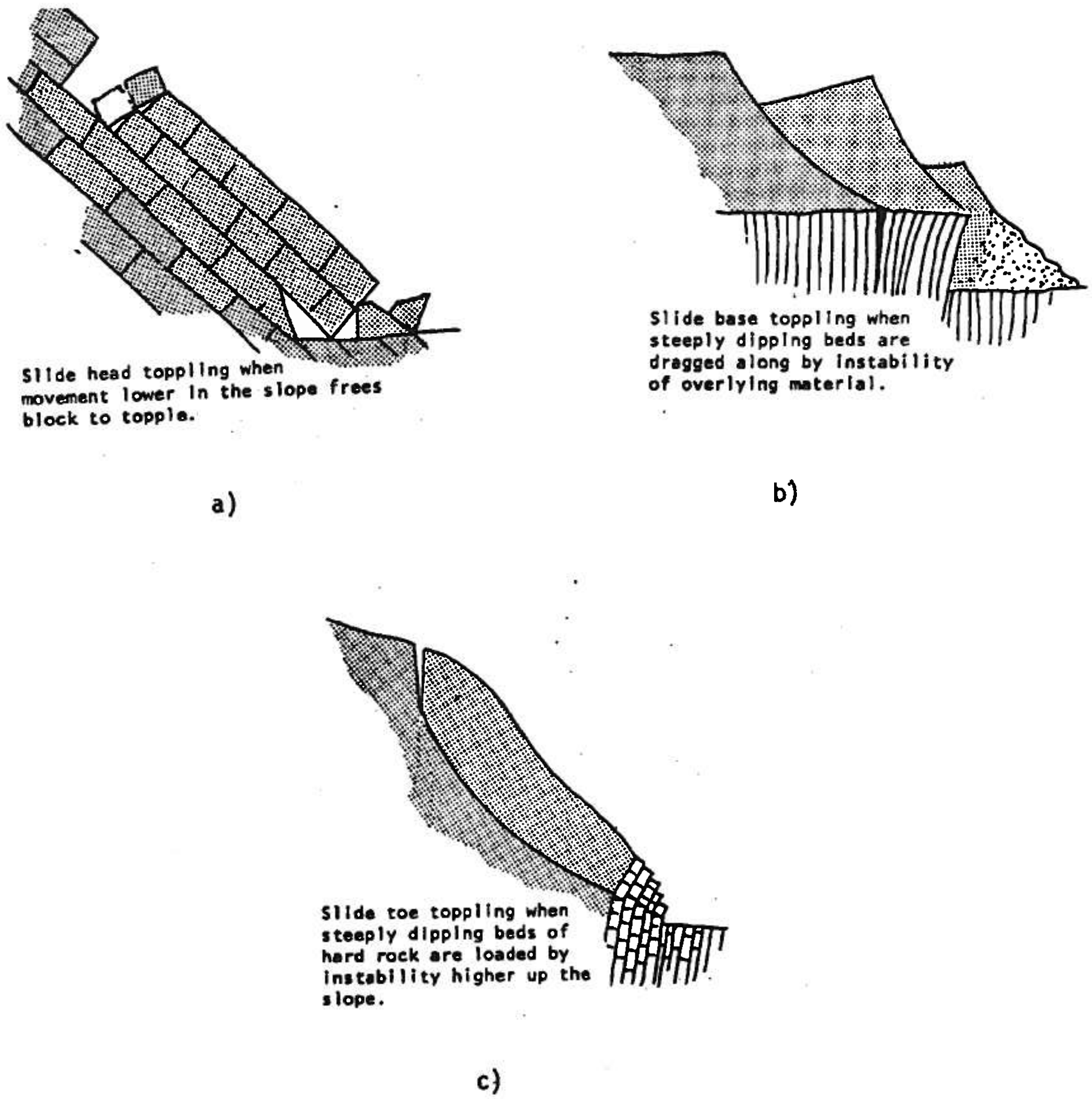



Figure 4. Higher modes of slope failure. (a) Slide head toppling, (b) Slide base toppling, (c) Slide toe toppling (after Hoek and Bray, 1977).

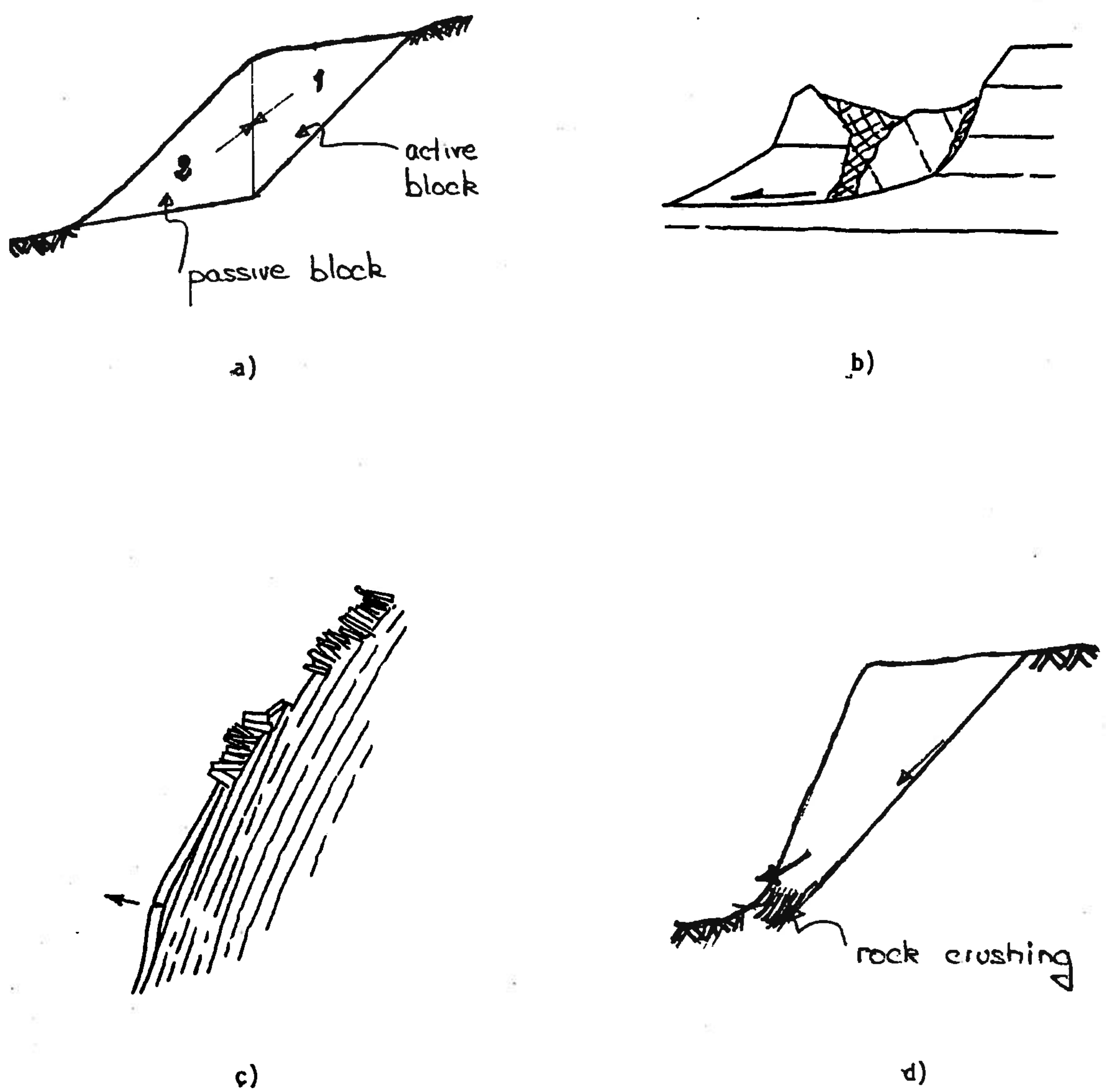



Figure 5. Higher modes of slope failure. (a) Two block gliding, (b) Block gliding with rotated graben, (c) Buckling, (d) Rock crushing and sliding (adapted from Hoek and Bray, 1977).

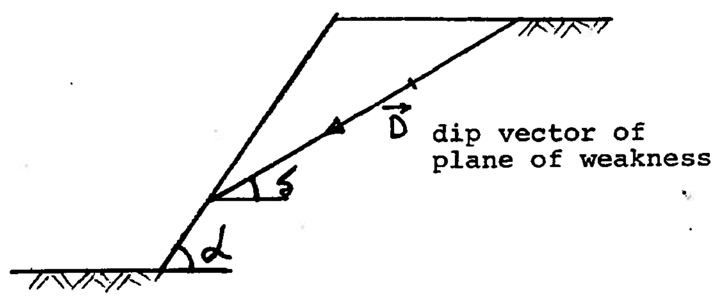
## 2. SINGLE PLANE SLIDING MODE

### 2.1 Fundamentals

Sliding of a block along a plane of weakness in a rock mass requires that the block is free to move kinematically and that friction is mobilized along the plane of weakness. The plane on which sliding occurs must strike parallel or nearly parallel (within ± 20°) to the slope face.

Plane sliding under gravity alone, takes place in a direction parallel to the dip vector of the plane of weakness. Let $\delta$ be the dip angle of that plane, and $\alpha$ be the *apparent* dip angle of the slope face in the direction of the dip vector of the plane of weakness. The shear strength of the plane of weakness is assumed to be described by a Coulomb criterion with friction angle $\phi$.

Sliding takes place when $\alpha > \delta > \phi$.



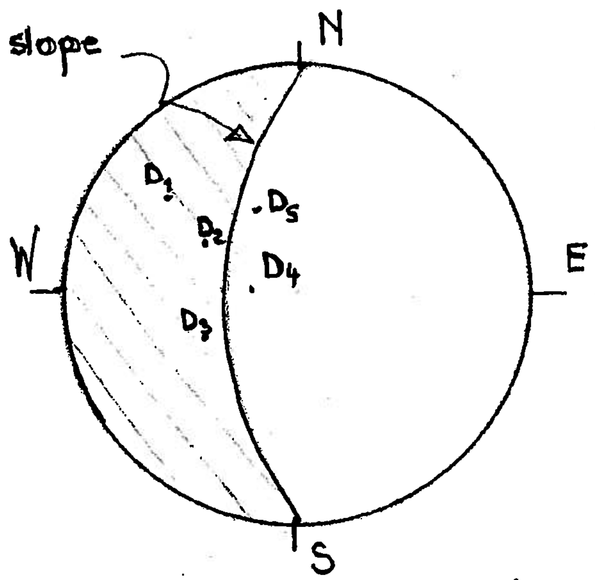

*Illustrative example*: Consider a rock slope with orientation N 0° E 60° W (60/270). The slope is cut by four major joint sets with the following orientations:

| | | |
|---|---|---|
| Plane 1: | N 30° E 40° NW | (40/300) |
| Plane 2: | N 47° W 30° SW | (30/223) |
| Plane 3: | Horizontal | |
| Plane 4: | N 0° E 40° W | (40/270) |

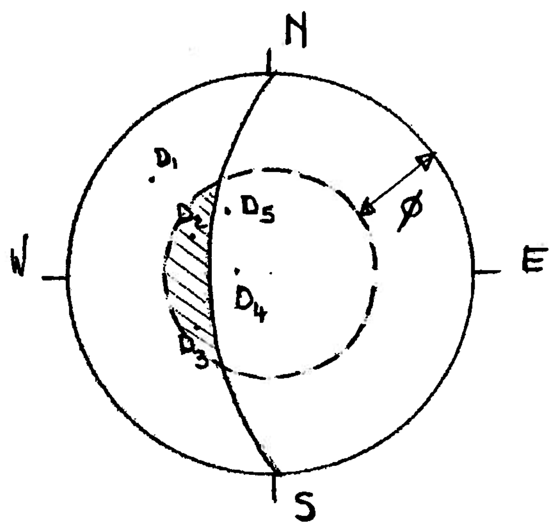
The planes of weakness have same friction angle $\phi = 30°$. What are the planes that could allow a slide?

*Inverse problem*: Given the orientation of a potentially trouble-some plane of weakness, what is the steepest safe slope for a rock cut of given orientation?

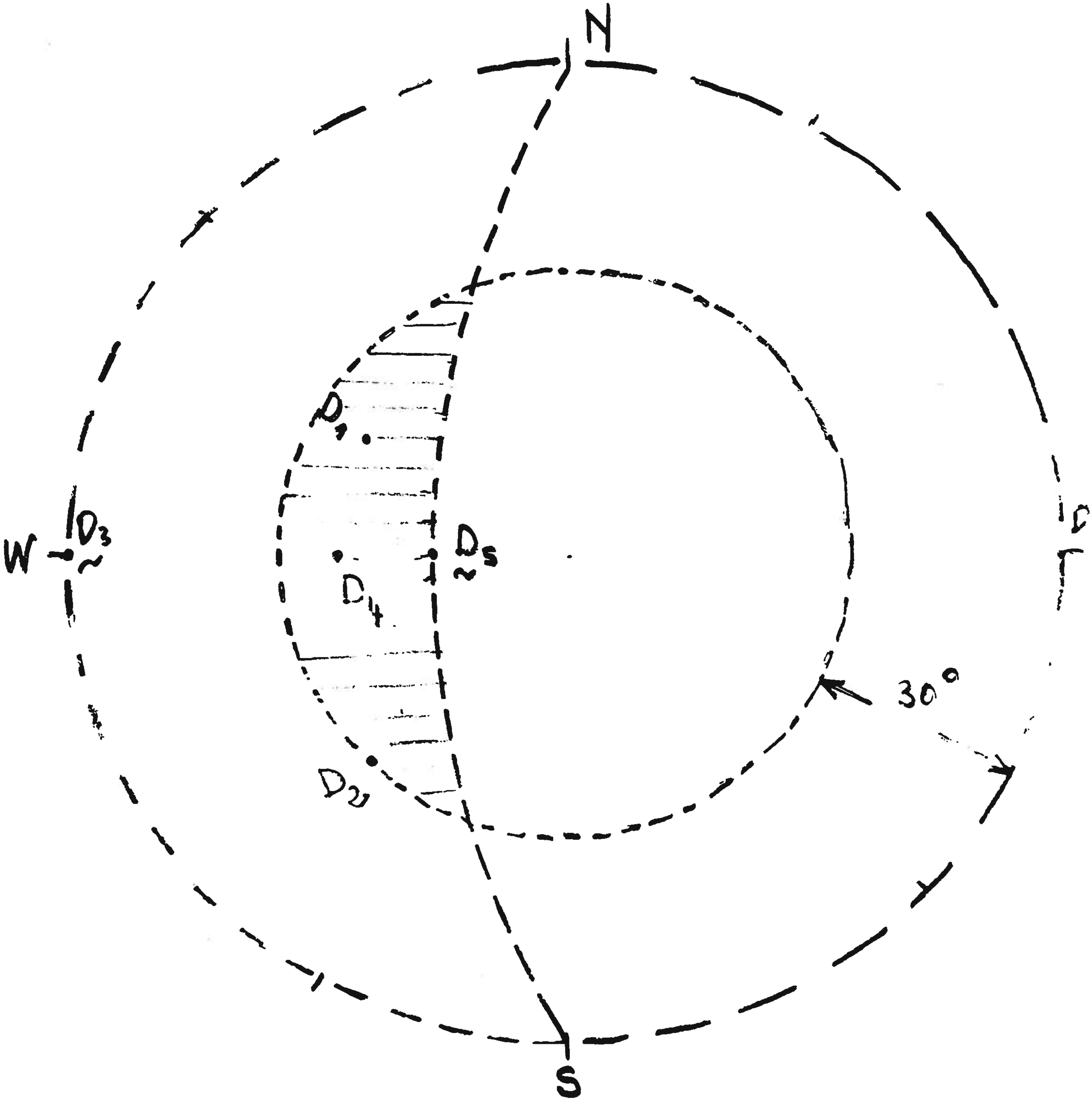
Example: A slope contains a critical plane of weakness with orientation N 0° E 40° W. Find the steepest safe slopes for a rock cut whose orientation varies between N 0° E and N 90° E.




Kinematic condition $\alpha > \delta$ is satisfied in the ruled area.



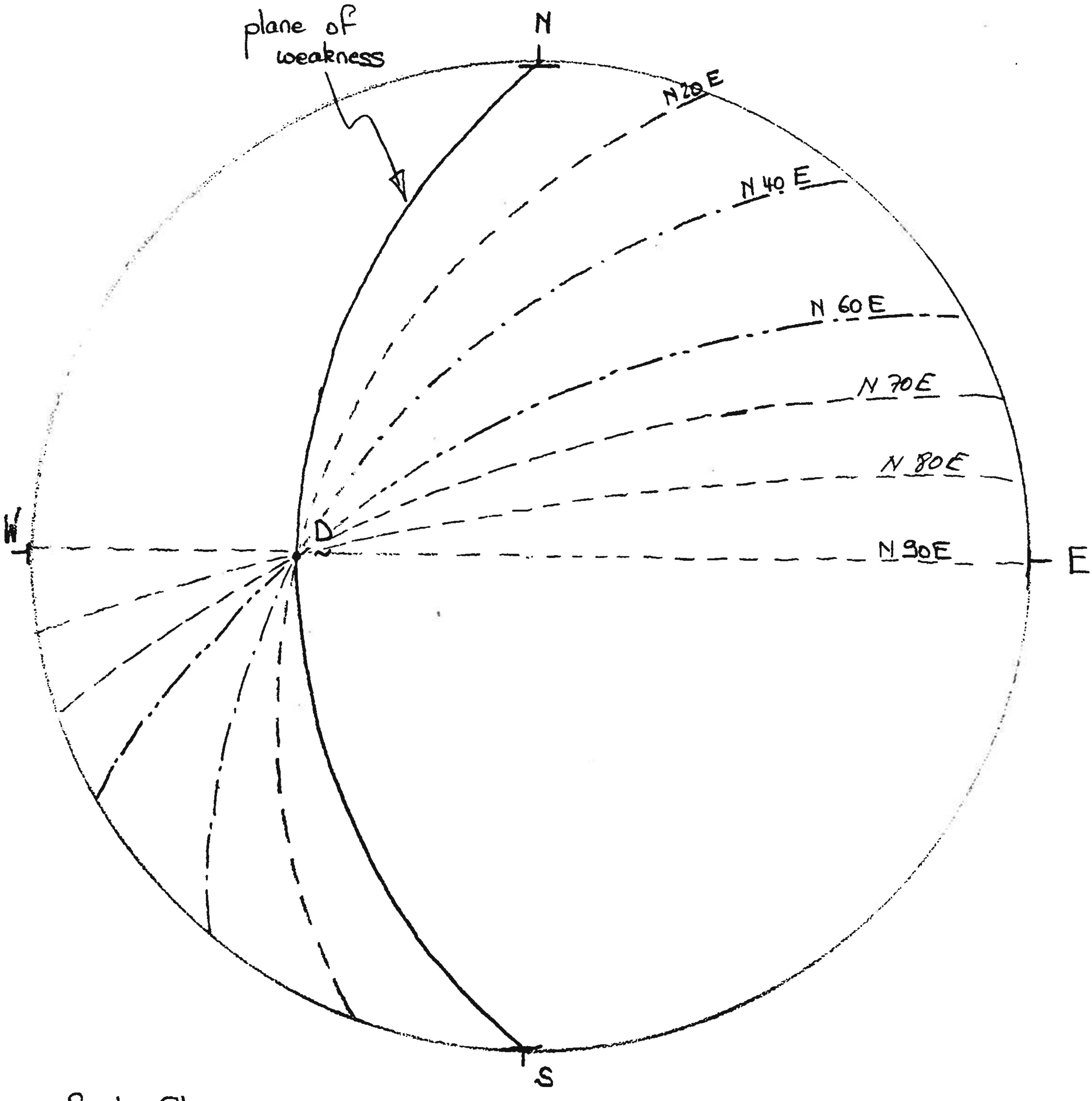

The condition $\alpha > \delta > \phi$ is satisfied in the ruled area.




ROCK CUT SLOPE : N 0 E 60 W

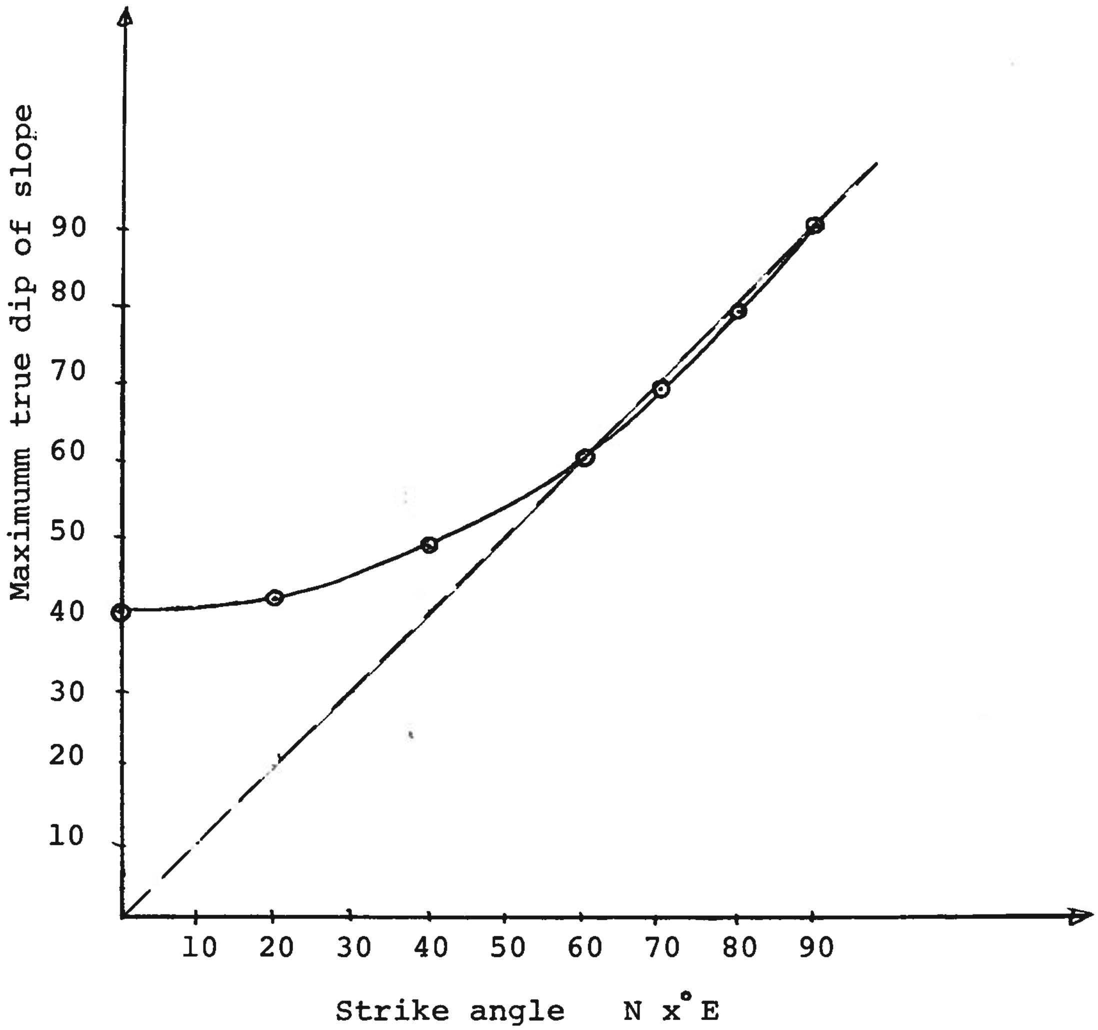
PLANE 1: N 30 E 40 NW
PLANE 2: N 47 W 30 SW
PLANE 3: Horizontal
PLANE 4: N 0 E 40 W

FRICTION ANGLE : 30 degrees

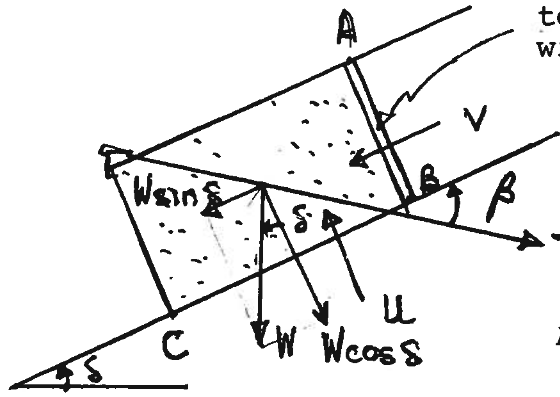



Rock Slope

| Strike N α° E | Max. True Dip. | Strike N α° E | Max True Dip. |
|---|---|---|---|
| 0 | 40 | 70 | 69 |
| 20 | 42 | 80 | 79 |
| 40 | 48 | 90 | 90 |
| 60 | 60 | | |


Maximumm true dip of slope
90
80
70
60
50
40
30
20
10
10
20
30
40
50
60
70
80
90
Strike angle N x° E

## Analysis of Plane Slide




A: base area of the block

Applied normal force $N = W\cos\delta - U + T_b \sin\beta$ (1)

Applied shear force $T = W\sin\delta + V - T_b\cos\beta$ (2)

Resisting shear force $cA + N\tan\phi$

Safety factor $$F = \frac{cA + (W\cos\delta - U + T_b\sin\beta)\tan\phi}{W\sin\delta + V - T_b\cos\beta}$$ (3)

The expression for the water force components U and V depends on the water pressure distribution between points A, B and C.

Solve equation (3) for $T_b$

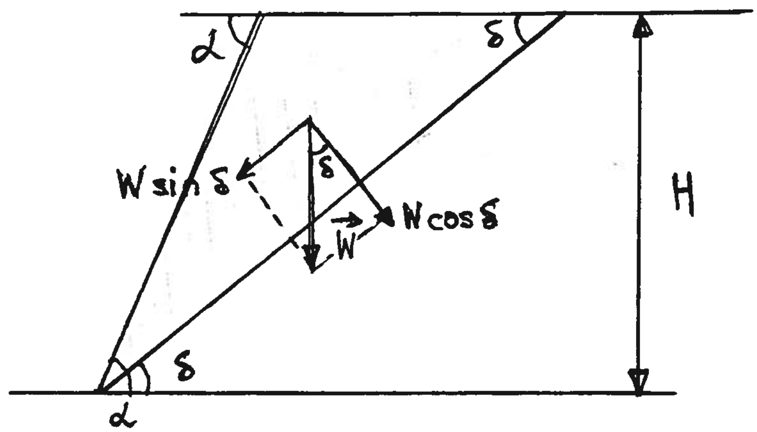
$$T_b = \frac{-cA + (U - W\cos\delta)\tan\phi + F(W\sin\delta + V)}{F\cos\beta + \tan\phi\sin\beta}$$ (4)

The minimum value of $T_b$ is such that $dT_b/d\beta = 0$ $\Longrightarrow$

$$\tan\beta_c = \frac{\tan\phi}{F}$$

The corresponding value of $T_b$ is obtained by substituting $\beta = \beta_c$ in eq. (4).

CRITICAL SLOPE HEIGHT VERSUS SLOPE ANGLE RELATIONSHIP




- plane of weakness is parallel to the rock slope face
- consider a unit length of slope
- cohesion, c, and friction angle along the plane of weakness

$$W = \frac{1}{2} \gamma H^2 \left[ \frac{1}{\tan\delta} - \frac{1}{\tan\alpha} \right] \quad (1)$$

For any fixed value of the safety factor F, we have

$$H = \frac{2c}{\gamma \cos\delta \, (F \sin\delta - \cos\delta \tan\phi)\left(1 - \frac{\tan\delta}{\tan\alpha}\right)} \quad (2)$$

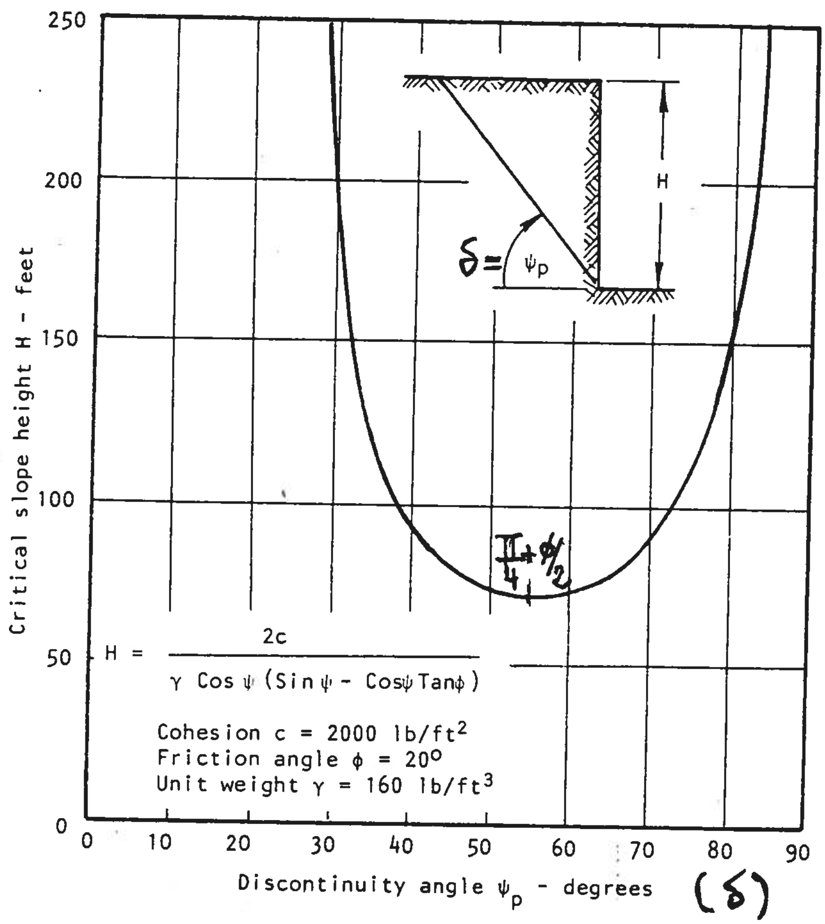
For a vertical slope ($\alpha$=90), the previous equation becomes

$$H = \frac{2c}{\gamma \cos\delta \, (F \sin\delta - \cos\delta \tan\phi)} \quad (3)$$

The minimum value of H takes place when $dH/d\delta = 0$, i.e. when $\delta = \delta_c$ with

$$\tan 2\delta_c = F \tan(\pi/2 + \phi)$$

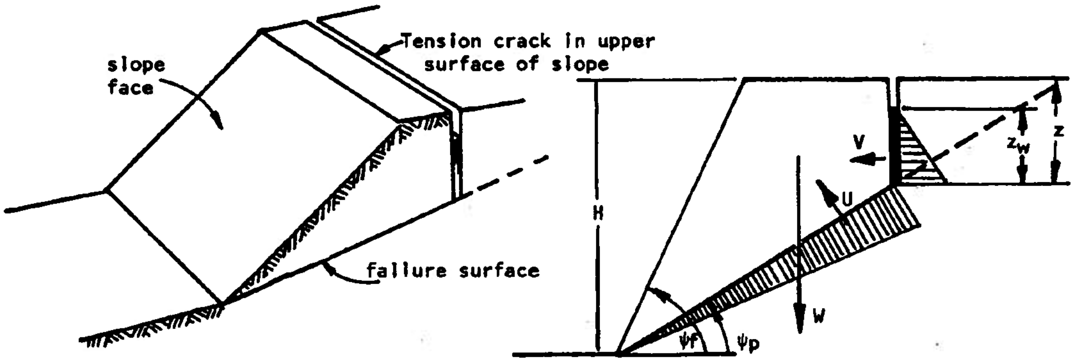
when F=1 $$\delta_c = \pi/4 + \frac{\phi}{2}$$




(Hoek and Bray, 1977)

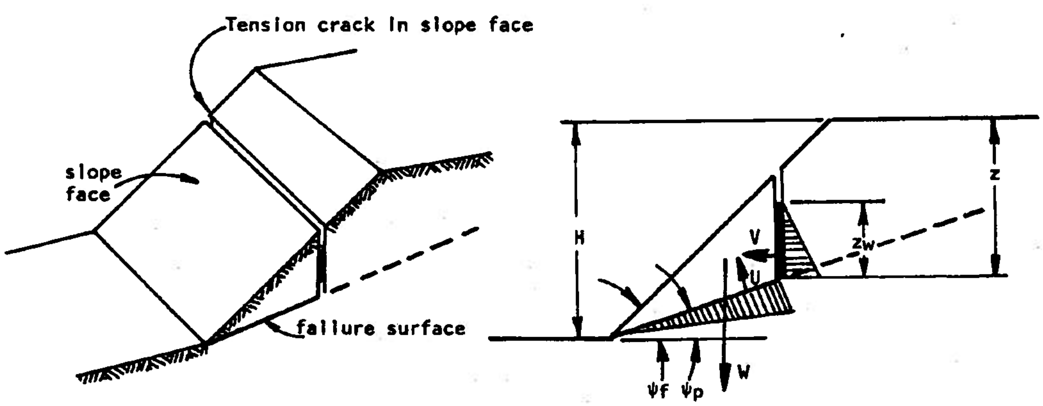
## 2.2 Slope with tension crack (from Hoek and Bray, 1977)

*Tension crack in upper surface of the slope*




$$W = \frac{1}{2}\gamma H^2 \left[(1 - (z/H)^2)\cot \psi_p - \cot \psi_f\right]$$

*Tension crack in the slope face*




$$W = \frac{1}{2}\gamma H^2 \left[(1 - (z/H)^2)\cot \psi_p(\cot\psi_p \tan\psi_f - 1)\right]$$

In all cases,

$$V = \frac{1}{2}\gamma_w z_w^2 \; ; \; U = \frac{1}{2}\gamma_w z_w A \; ; \; A = \frac{(H-z)}{\sin\psi_p}$$

The safety factor against sliding is then equal to

$$F = \frac{cA + (W\cos\psi_p - U - V\sin\psi_p)\tan\phi}{W\sin\psi_p + V\cos\psi_p}$$

or

$$F = \frac{(2c/\gamma H).P + [Q.\cot\psi_p - R.(P + S)]\tan\phi}{Q + R.S.\cot\psi_p}$$

where P, Q, R and S are dimensionless parameters that depend on the geometry and not on the size of the slope. They are equal to

$$P = (1 - z/H)/\sin\psi_p \; ; \; R = \frac{\gamma_w}{\gamma}.\frac{z_w}{z}.\frac{z}{H} \; ; \; S = \frac{z_w}{z}.\frac{z}{H}.\sin\psi_p$$

and

$$Q = [(1 - (z/H)^2)\cot \psi_p - \cot \psi_f]\sin\psi_p$$

when the tension crack is in the upper surface of the slope, and

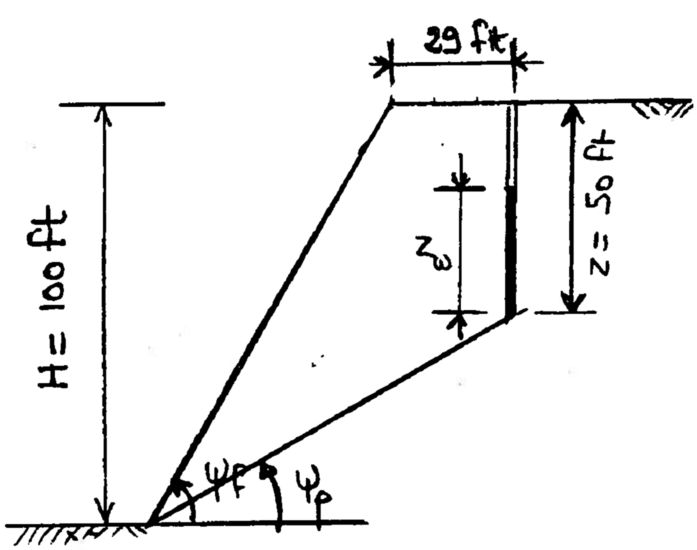
$$W = [(1 - (z/H)^2)\cot \psi_p(\cot\psi_p\tan\psi_f - 1)]$$

when the tension crack intersects the slope surface.

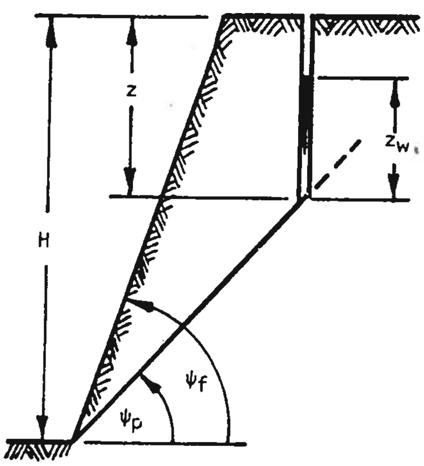
*Numerical example:*

c = 1000 lb/ft$^2$
$\phi$ = 30°
$\psi_f$ = 60
$\psi_p$ = 30

For z/H = 0.5, the charts give P = 1 and Q = 0.36. The safety factor F is found to be equal to 1.34, 1.10 and 0.77 when $z_w/z$ = 0 (dry tension crack), 0.5 and 1.0, respectively.



(Hoek and Bray, 1977)



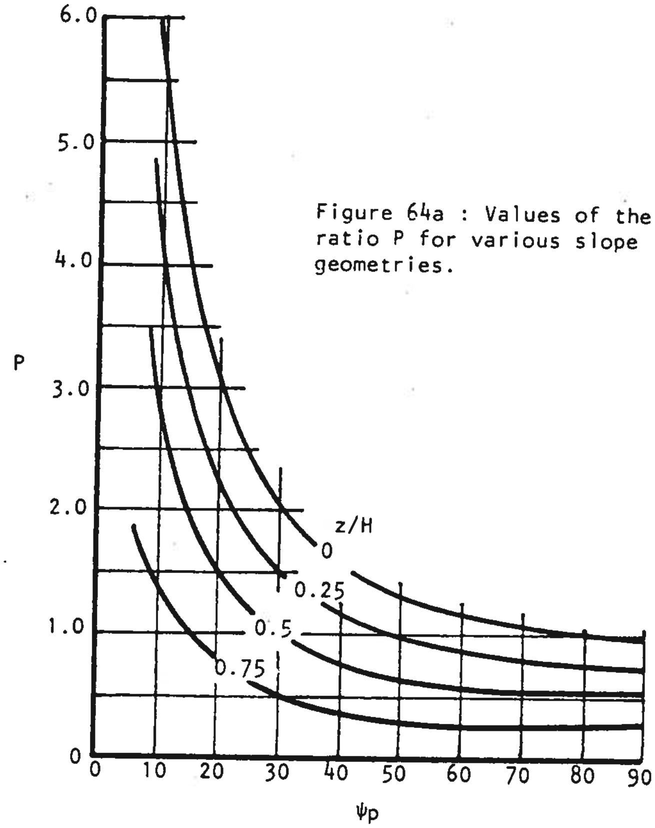




Figure 64a : Values of the ratio P for various slope geometries.

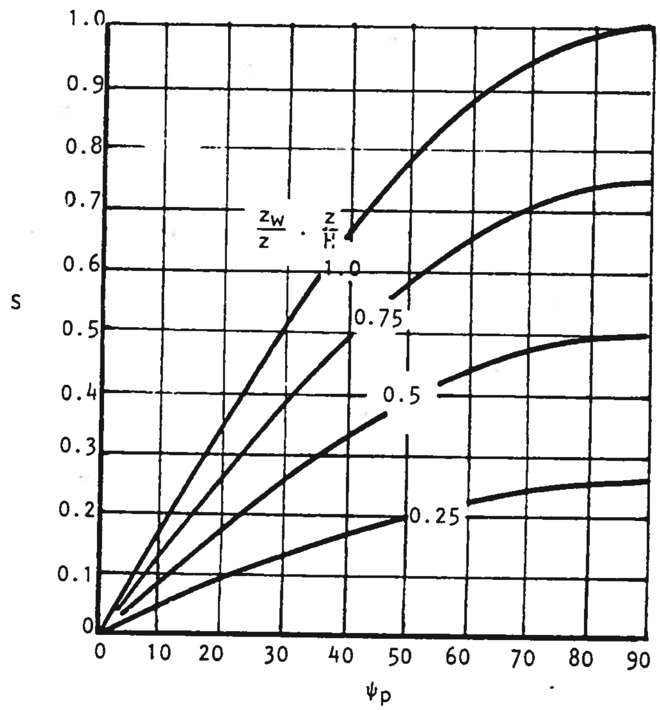



Figure 64b: Values of the ratio S for various geometries

(Hoek and Bray, 1977)

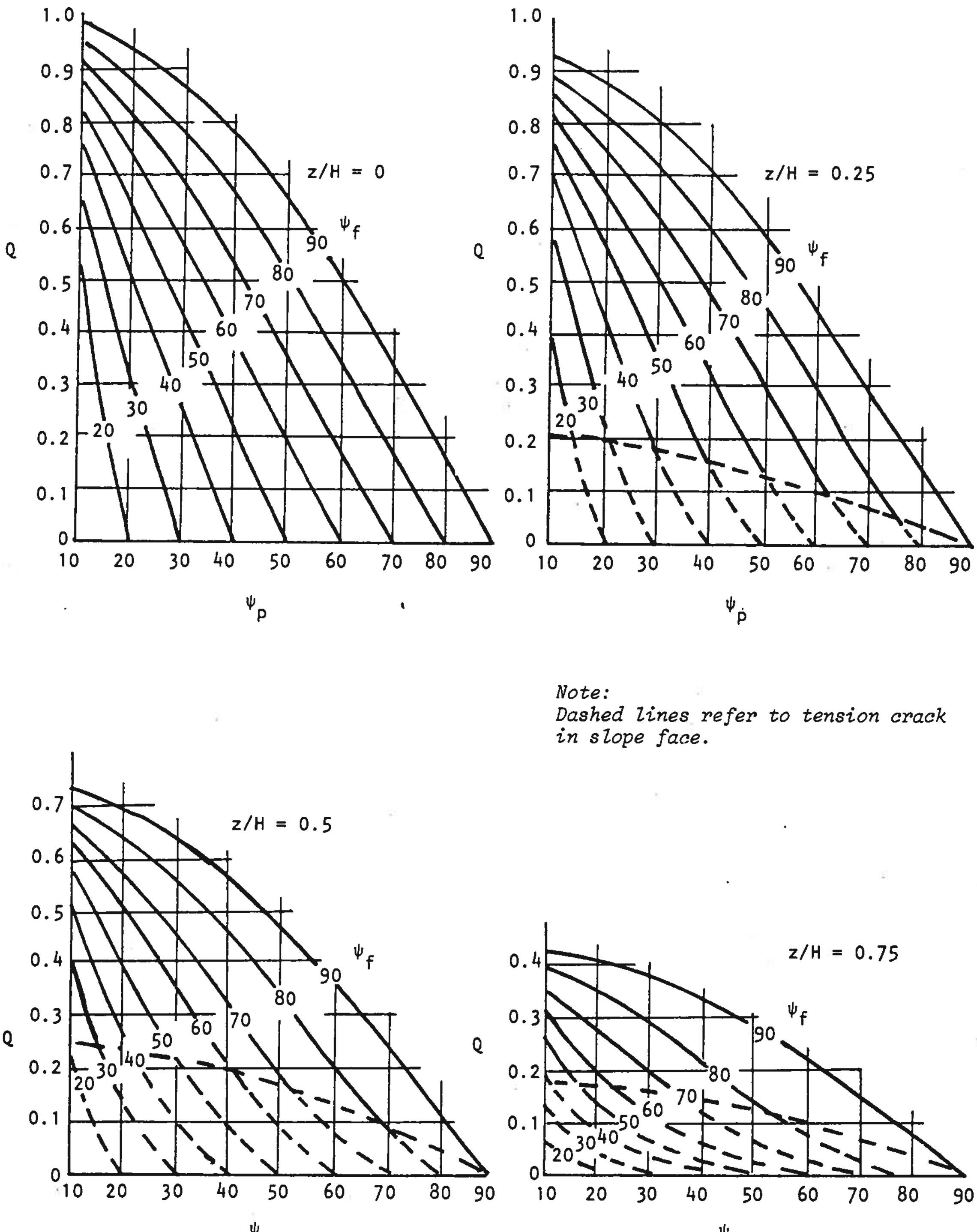
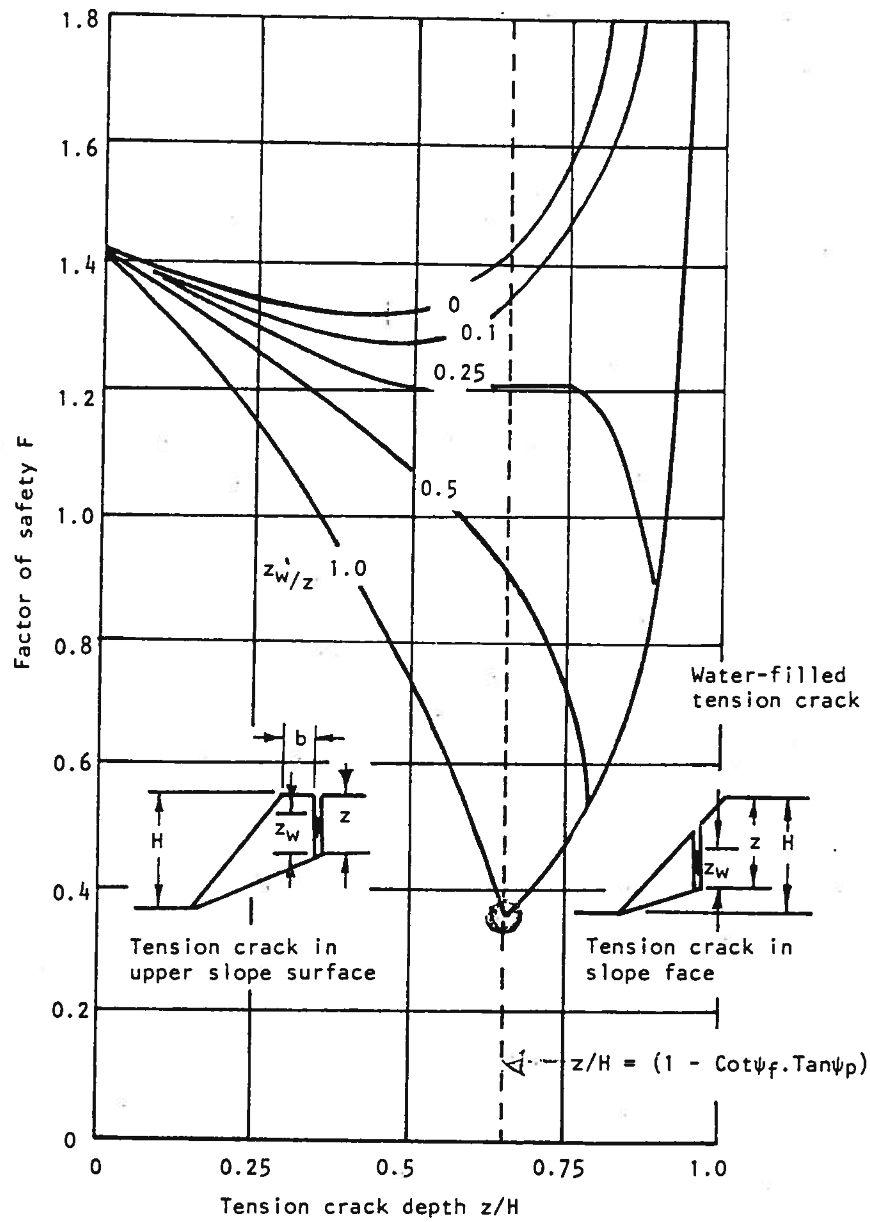



Figure 64c : Value of the ratio Q for various slope geometries. –



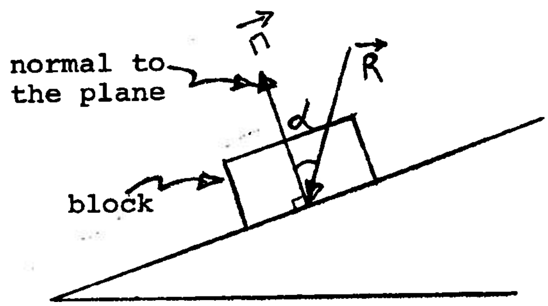

tension crack
at the crest
of the slope

Figure 66 : Influence of tension crack depth and of depth of water in the tension crack upon the factor of safety of a slope. (Slope geometry and material properties as for example on page 154).

(Hoek and Bray, 1977)

## 2.3. Analysis of block sliding using the stereographic projection

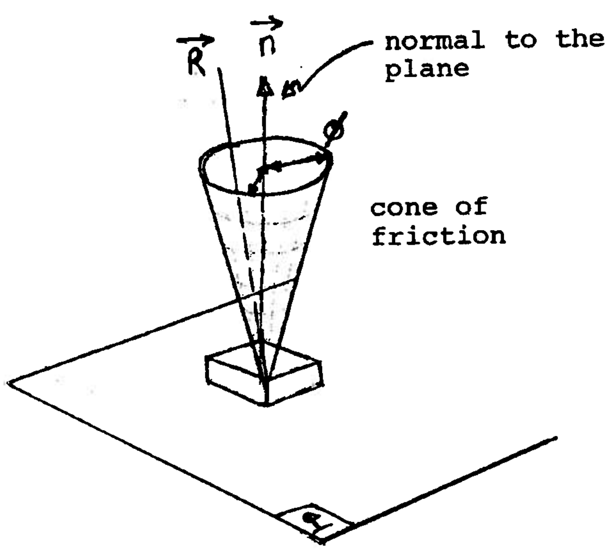
The stereographic projection can be a valuable tool to conduct stability analysis of rock slopes. Consider a block resting on a planar surface. Let $\phi$ be the friction angle between the block and the surface. The block will remain at rest on the surface if the resultant of all forces acting on the block is inclined with the normal to the surface at an angle less than $\phi$. If the block is free to move in any direction, the envelope of all allowable resultant forces on the block is a cone of vertex angle $2\phi$ centered around the normal of the planar surface. This cone of static friction will be projected on the stereographic projection as a small circle of radius $\phi$ about the normal $\vec{n}$ to the planar surface.

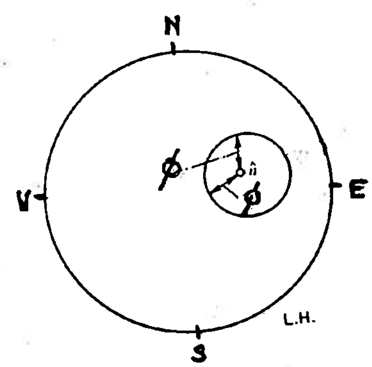



$\alpha < \phi$ block at rest

$\alpha = \phi$ limiting equilibrium

$\alpha > \phi$ sliding




Lower stereographic projection and firction circle.

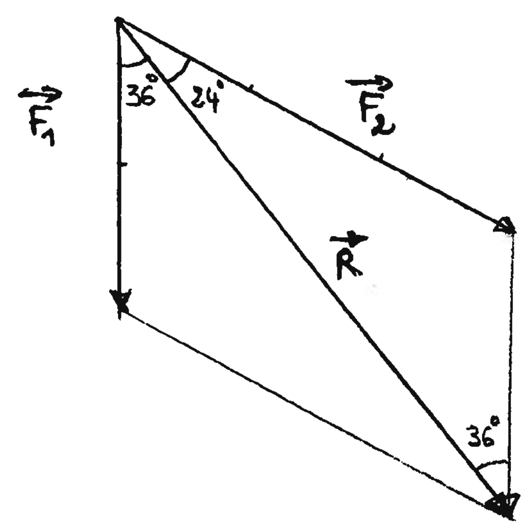
Force vectors can be represented as points on the stereographic projection. Depending on the orientation of a vector, the lower or upper hemisphere projection should be used for the projection of the vector.

How to find the orientation and magnitude of the resultant $\vec{R} = \vec{F}_1 + \vec{F}_2$ of two forces $\vec{F}_1 = F_1 \vec{f}_1$ and $\vec{F}_2 = F_2 \vec{f}_2$ knowing their orientation and magnitude? $F_1$ and $F_2$ are the magnitudes of the two force vectors and $\vec{f}_1$ and $\vec{f}_2$ are the unit vectors parallel to the two forces.

*Numerical example*:

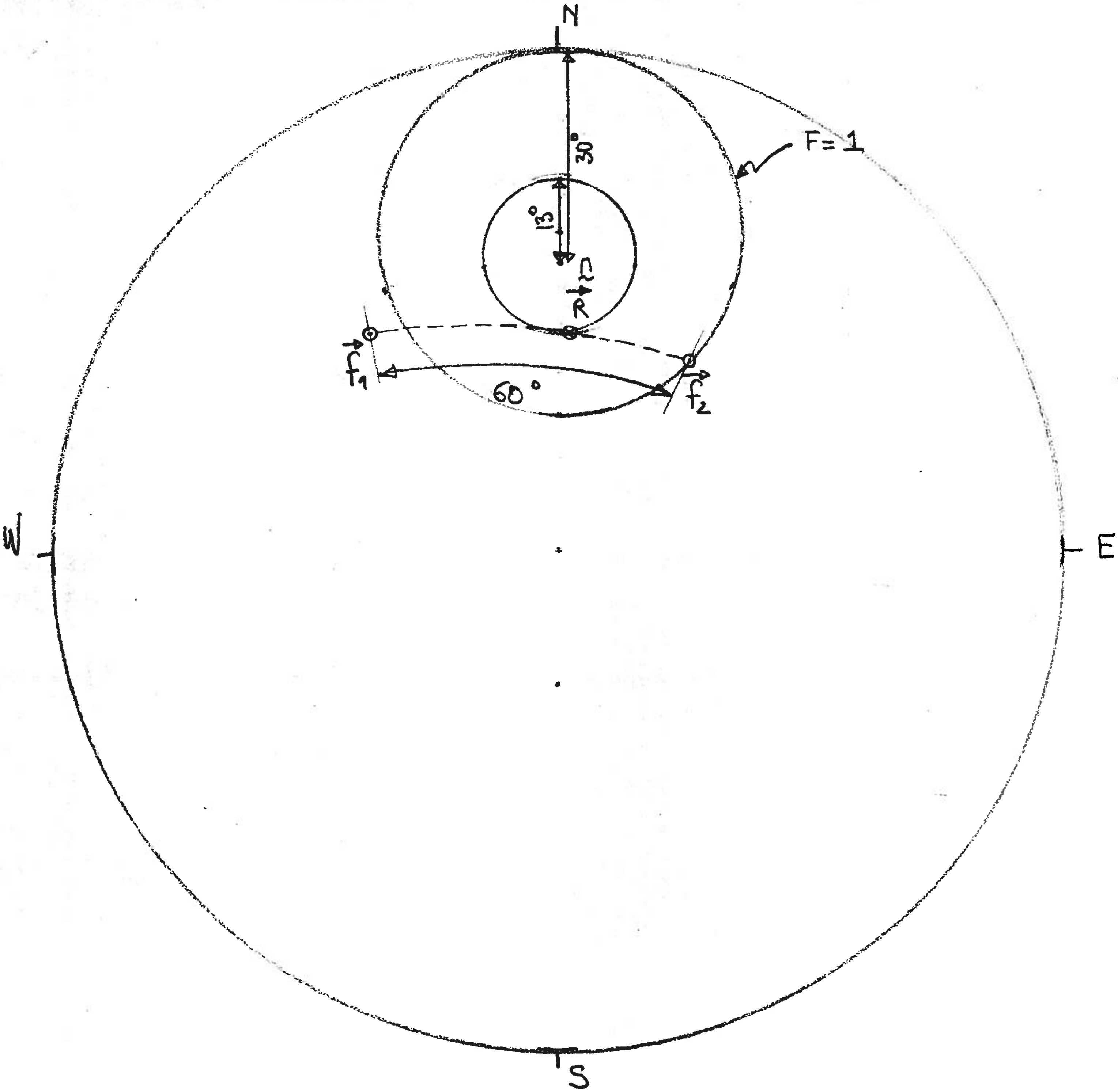
$F_1 = 20$ MN; and $\vec{f}_1$ plunges 30° to the N 40° W
$F_2 = 30$ MN; and $\vec{f}_2$ plunges 40° to the N 35° E

Let $\vec{R}$ be the resultant force acting on a block resting on a plane with orientation N 90° E 60° S (60/180). The friction angle between the block and the plane is equal to 30 degrees. Is the block safe under the effect of $\vec{R}$ ?




$$\frac{R}{\sin 120} = \frac{F_1}{\sin 24} = \frac{F_2}{\sin 36}$$

Thus, the magnitude, R, of the resultant vector $\vec{R}$ is equal to 42.6 MN.




Safety factor $F = \frac{\tan 30}{\tan 13} = 2.5$

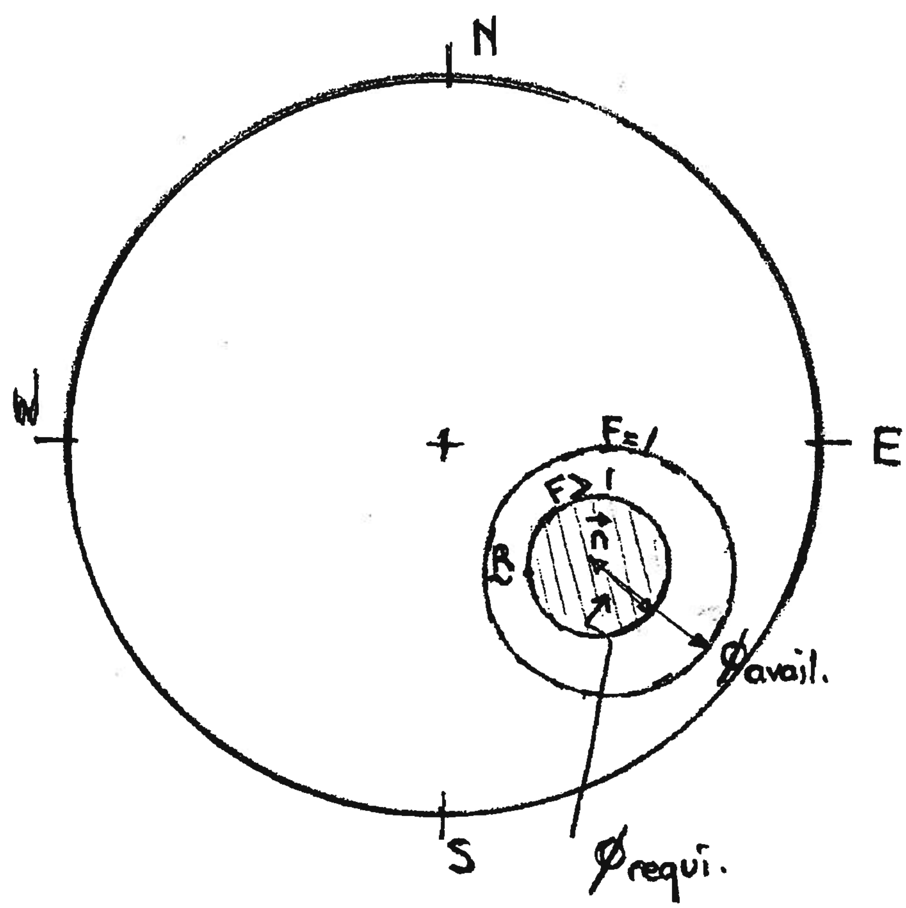
Let $F$ be the safety factor against sliding such that

$$F = \frac{\tan \phi_{avail}}{\tan \phi_{req.}}$$

where $\phi_{avail}$ is the friction angle available for design (equal to $\phi$), and
$\phi_{req}$ is the friction angle required for design and to reach a fixed value of the safety factor.

As an example, if $\phi_{avail} = 30°$ and $\phi_{req} = 15°$, the safety factor would be equal to 2.5.

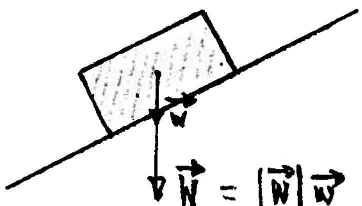
If we decide to design a slope with a fixed safety factor $F$, the resultant force would have to be located on a friction circle of radius $\phi_{req}$.



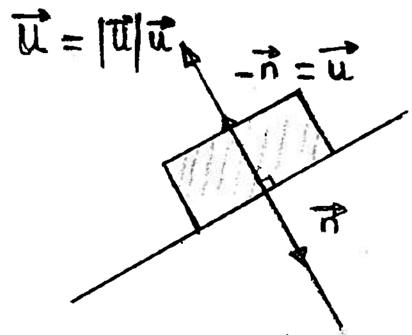
Forces that can enter into rock slope stability calculations include:

- rock block weight, $\vec{W}$
- loads transmitted by adjacent blocks
- water forces, $\vec{U}$
- earthquake forces
- support forces.

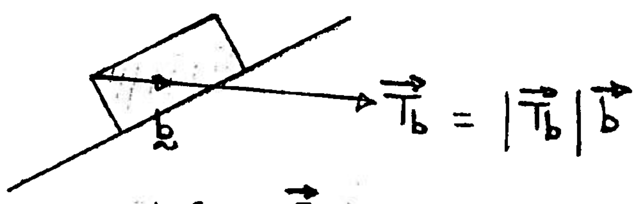
Convention: use the lower hemisphere (LH) stereographic projection. $\vec{n}$ is the normal pointed out of the block, into the support, and the forces plotted are those acting on the block.




Block weight $\vec{W}$.




Water force $\vec{U}$ normal to contact plane in the $-\vec{n}$ direction.




support force $\vec{T}_b$

The earthquake force is treated as a "pseudostatic" force with constant acceleration $\vec{a} = g\vec{K}$. The inertlal force is then equal to

$\vec{F} = W\vec{K}$

The direction of $\vec{K}$ is opposed to the earthquake acceleration.

## 2.4 Example # 1

A block weighing 100 MN rests on a plane whose orientation is N 40° W, 30° SW (30/230) .The available friction angle is believed to be 45 degrees.

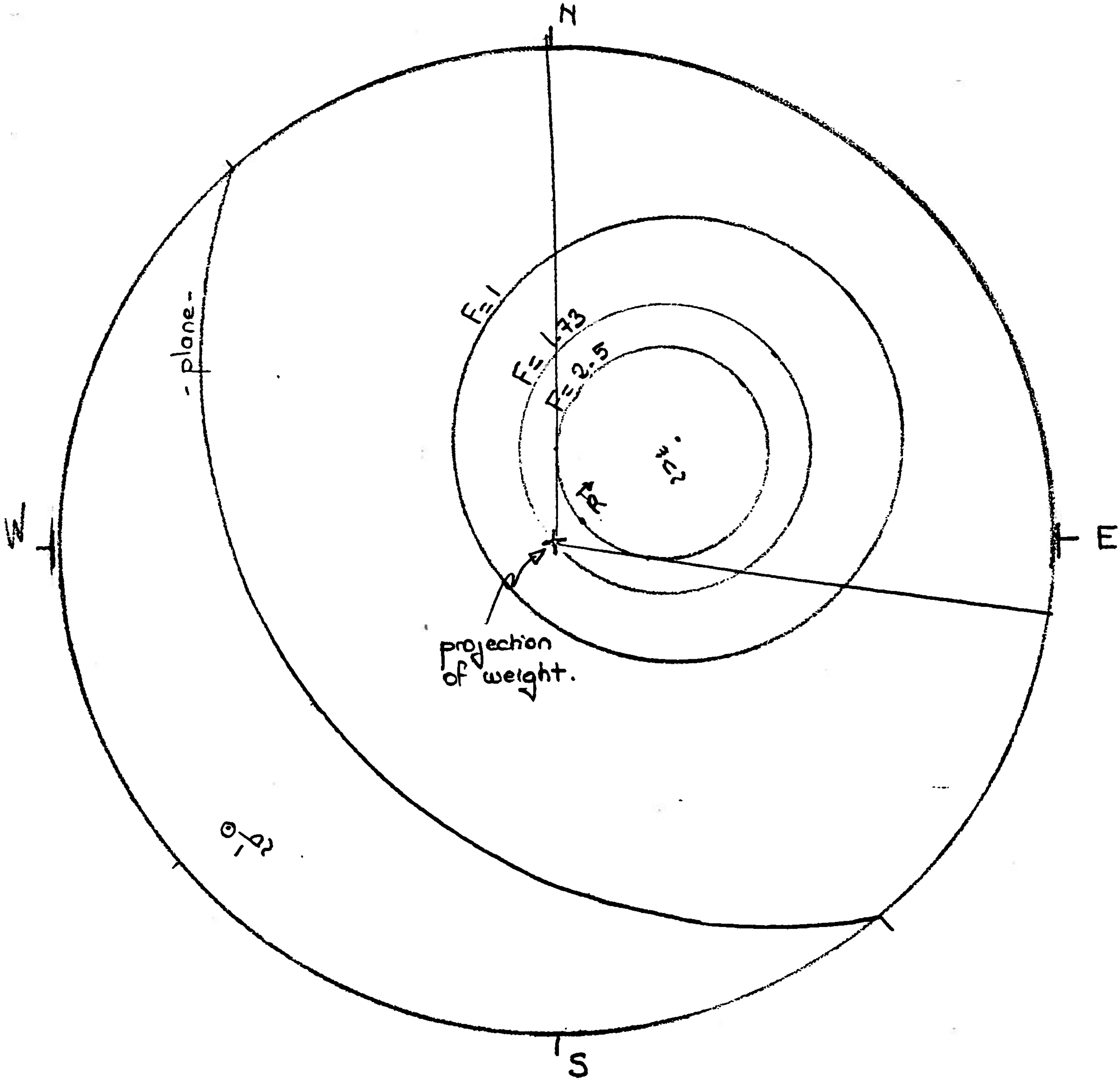
a) Is the block stable under its own weight?

b) What is the safety factor of the block under its own weight?

c) We want to raise the safety factor to 2.5 by using rock bolts. What is the minimum bolt force that has to be applied to reach that safety factor?

d) What is the bolt force that has to be applied to reach that safety factor if the bolts are installed 60 degrees below horizontal (minimum bolt length criterion)?

Example # 1



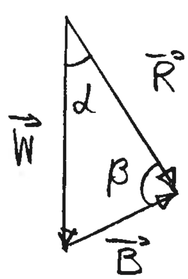
a) Block safe under its own weight

b) Corresponding safety factor

$$F = \frac{\tan 45}{\tan 30} = \underline{1.73}$$

c) For a safety factor of 2.5, $\phi_{req.}$ must be such that $2.5 = \frac{\tan 45}{\tan \phi_{req.}} \Rightarrow \phi_{req} = \underline{21.8^\circ} \ (\simeq 22^\circ)$

Consider a vertical plane common to the weight $\vec{W}$ and the bolt force $\vec{B}$. The resultant force of $\vec{W}$ and $\vec{B}$ must be located on the friction circle corresponding to F = 2.5. $\vec{B}$ can have any bearing between north and N 99 E

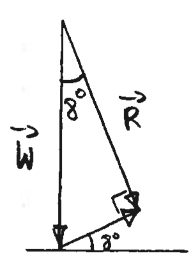



$$\frac{|\vec{B}|}{\sin\alpha} = \frac{|\vec{W}|}{\sin\beta}.$$

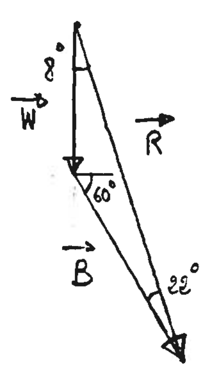
The minimum bolt force is reached when β is maximum and α is minimum.

For this example β = 90° and α = 8° ⇒ $\boxed{|\vec{B}| = 13.9 \text{ MN}}$
Bolt direction N 50 E

d)




Minimum bolt force condition




$$|\vec{B}| = \frac{\sin 8}{\sin 22} |\vec{W}|$$

$$|\vec{B}| = \underline{37.2 \text{ MN}}$$

Minimum bolt length condition.

**2.5 Example # 2**

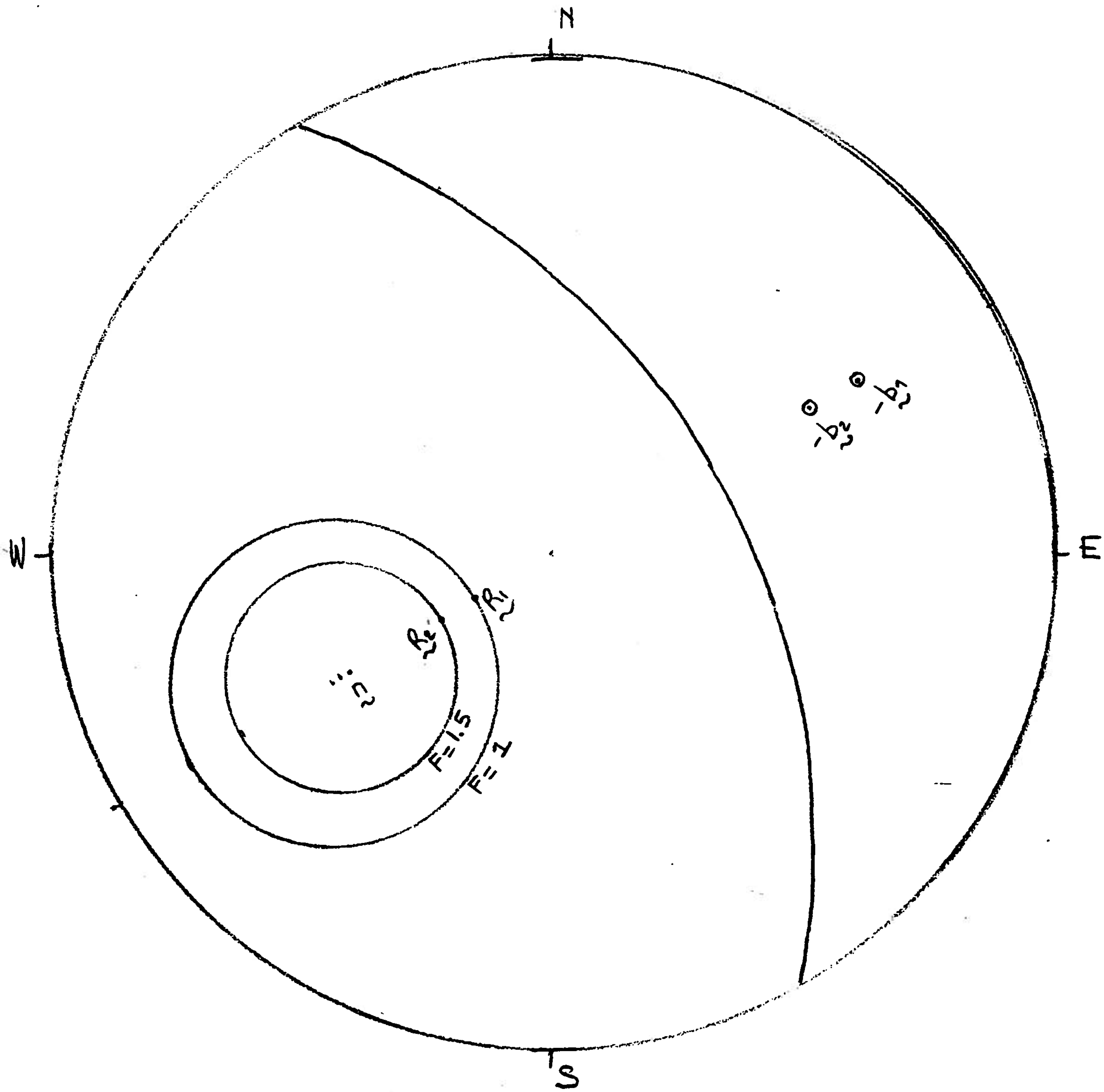
We are given a plane P daylighting into a cut and having attitude as follows: N 30° W, 50° NE (50/060) .The weight of a potentially sliding mass resting on plane P is 400 tons (metric tons) on an area of 200 m .The friction angle is believed to be 30 degrees.

a) Find the direction and magnitude of the minimum rock bolt force to achieve a factor of safety of 1.0, and a factor of safety of 1.5.

b) What water pressure acting on plane P could cause failure after rock bolts are installed for a safety factor of 1.5?

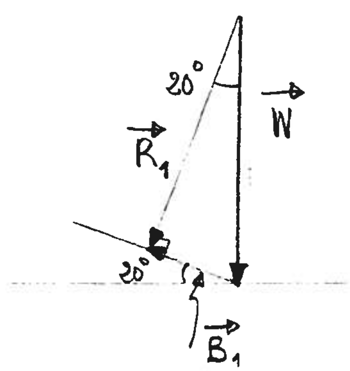
c) Consider the sliding rock mass without the rock bolts. What is the cohesion needed to increase the frictional resistance along plane P such that a safety factor of 1.0 is achieved when the sliding mass is under its own weight?

Example #2



a) Sliding mass is not stable under its own weight. Therefore, there is a need for support.

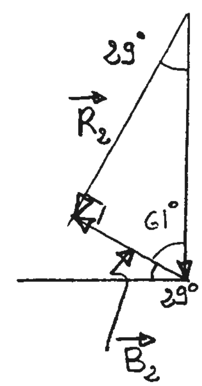
| F = 1 $\quad \phi_{req} = \phi_{av.} = 30^\circ$ | F = 1.5 $\quad \phi_{req} = \dfrac{\tan 30}{1.5} = 21^\circ$ |
|---|---|




$|\vec{B_1}| = |\vec{W}| \sin 20$

$|\vec{B_1}| = 137$ tons

Direction S 60 W




$|\vec{B_2}| = |\vec{W}| \sin 29$

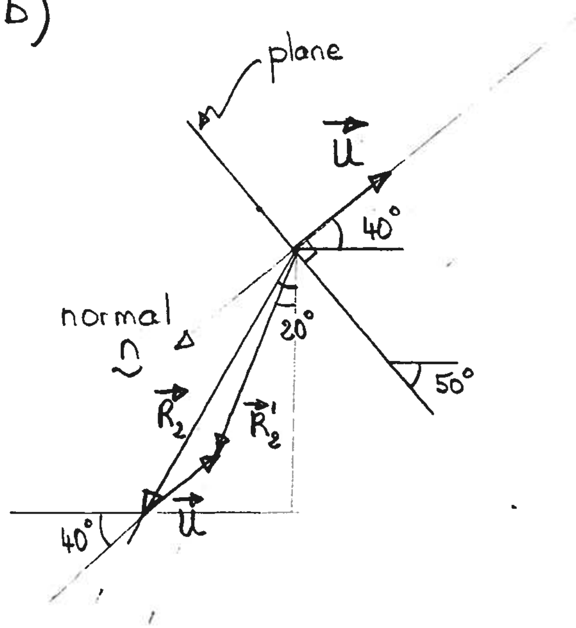
$|\vec{B_2}| = 194$ tons

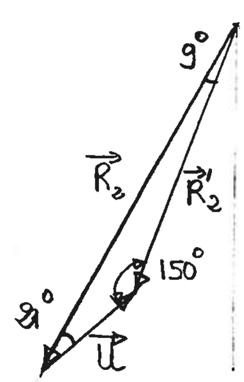
$|\vec{R_2}| = \sin 61 \, |\vec{W}|$

$|\vec{R_2}| = 350$ tons

Direction S 60 W

b)



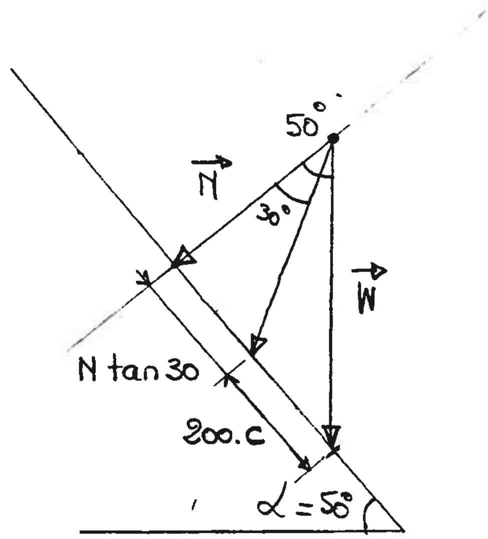

$$\frac{|\vec{U}|}{\sin 9} = \frac{|\vec{R_2}|}{\sin 150}$$

$\Rightarrow |\vec{U}| = 110$ tons

For a uniform water pressure along the plane of contact

$$u = \frac{1100}{200} = 5.5 \ kN/m^2$$

c)




$$N = |\vec{w}| \cos\alpha = |\vec{w}| \cos 50 = 257.1 \text{ tons}$$

For a safety factor of 1, there must be a cohesion c such that.

$$|\vec{w}| \sin 50 = 200\, c + |\vec{w}| \cos 50 \,.\, \tan 30$$

$$\Rightarrow c = \frac{|\vec{w}|}{200} \left[\sin 50 - \cos 50 \,.\, \tan 30\right]$$

$$\underline{c = 0.79 \; t/m^2.}$$

**2.6 Example # 3**

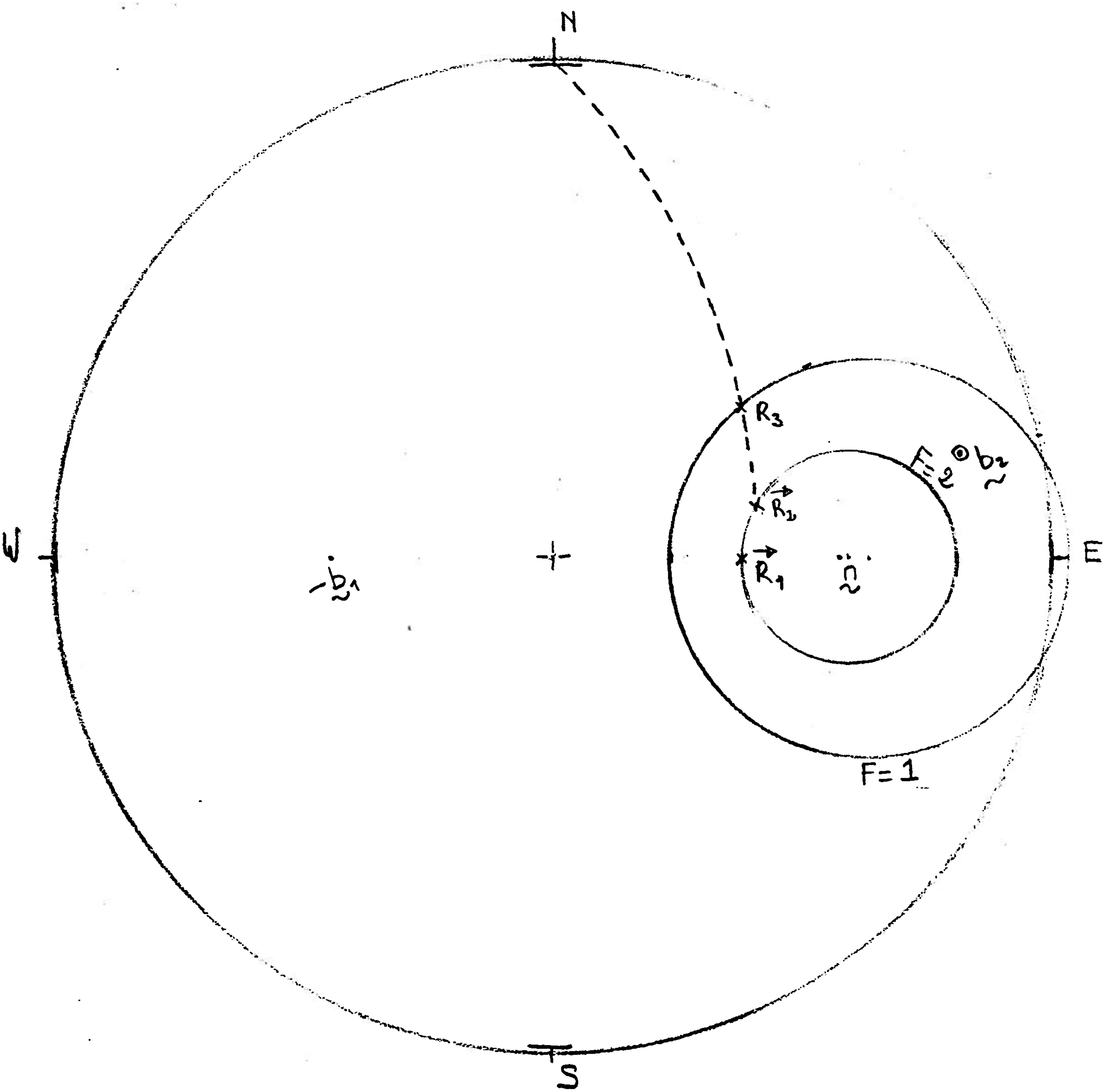
A block weighing 200 MN rests on a plane striking North and dipping 60° W (60/270) .The available friction angle is believed to be 33 degrees.

a) Find the minimum force for stabilizing the block with a safety factor of 2 using rock bolts.

b) Find the force for stabilizing the block with a safety factor of 2 if the bolts are installed 10 degrees below horizontal to the N 76 E.

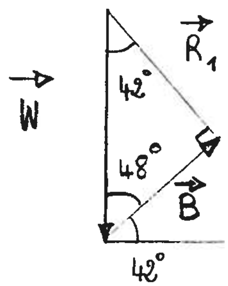
c) What inertia force associated with ground motion initiates slip if the inertia force acts horizontally to the North? The bolts are installed as in case b) before the earthquake.

Example #3



a) Safety factor F = 2

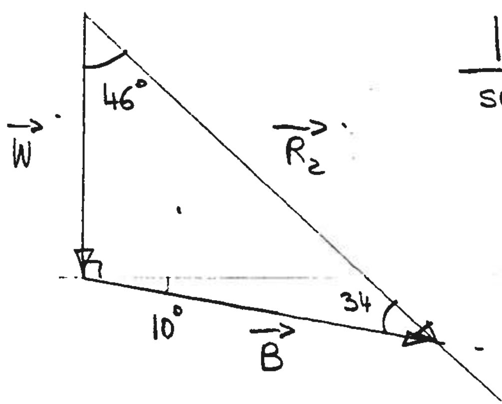
$$\tan \phi_{req} = \frac{\tan 33}{2} \Rightarrow \phi_{req} = 18^\circ$$




$|\vec{B}| = |\vec{W}| \sin 42 = 134$ MN.

Bolt direction. N 90 E

b)

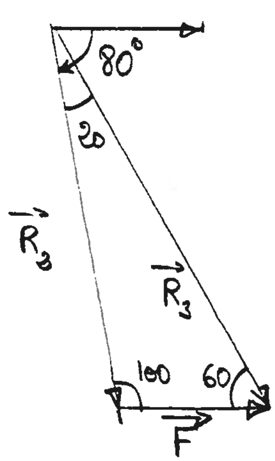



$$\frac{|\vec{B}|}{\sin 46} = \frac{|\vec{W}|}{\sin 34} = \frac{|\vec{R}_2|}{\sin 100}$$

$\Rightarrow |\vec{B}| = 257$ MN

$|\vec{R}_2| = 352$ MN

c)




$$\frac{|\vec{F}|}{\sin 20} = \frac{|\vec{R}_2|}{\sin 60}$$

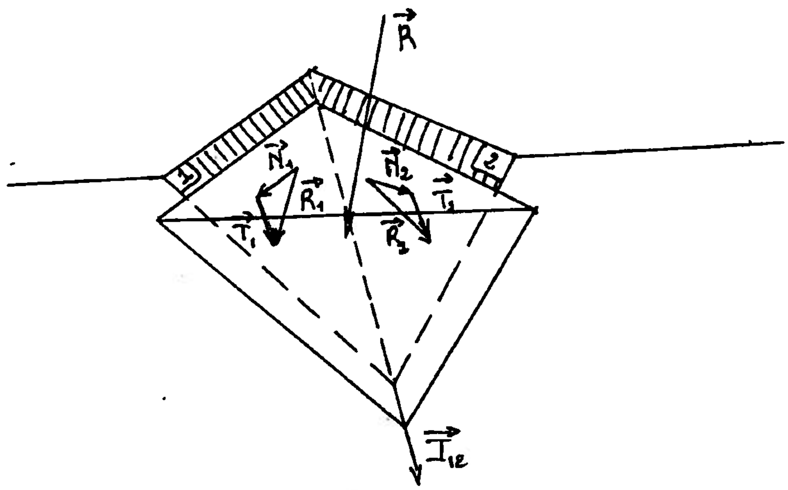
$\Rightarrow |\vec{F}| = 139$ MN

$|\vec{K}| = 0.7$

## 3. WEDGE SLIDING WITH TWO FREE SURFACES

### 3.1 Fundamentals

Intersecting discontinuity surfaces may liberate tetrahedral wedges. Consider only wedges with two of their four surfaces being free surfaces.



Consider two planes 1 and 2 with friction angles $\phi_1$ and $\phi_2$, respectively. Let $\vec{n}_1$ and $\vec{n}_2$ be unit vectors normal to planes 1 and 2 respectively. The unit vectors are pointing into the corresponding supporting planes. Let $\vec{I}_{12}$ be a unit vector parallel to the line of intersection of planes 1 and 2. Vector $\vec{I}_{12}$ points into the free space.

The wedge can experience three possible modes of sliding: (1) Sliding on plane 1 alone, (2) Sliding on plane 2 alone, and (3) sliding on planes 1 and 2 parallel to the line of intersection $\vec{I}_{12}$. Wedge failure requires that the wedge is free to move kinematically and that friction is mobilized along one or two planes depending on the failure mode.

Let $\vec{R}$ be the total applied force on the wedge with $\vec{R} = \vec{R}_1 + \vec{R}_2$ where $\vec{R}_1$ is the fraction of $\vec{R}$ that is acting on plane 1 and $\vec{R}_2$ the fraction of $\vec{R}$ that is acting on plane 2.

In addition,

$$\vec{R}_1 = \vec{N}_1 + \vec{T}_1 = \mathrm{N}_1\, \vec{n}_1 + \mathrm{T}_1\, \vec{I}_{12}$$

$$\vec{R}_2 = \vec{N}_2 + \vec{T}_2 = \mathrm{N}_2\, \vec{n}_2 + \mathrm{T}_2\, \vec{I}_{12}$$

The mechanics of sliding on plane l or plane 2 alone is identical to the one discussed in Section 2. However, there is an additional constraint: sliding on a single plane is possible only within a restricted set of directions in the plane. There are sliding directions on plane 1 that will be associated with closing on plane 2 and vice versa. When sliding takes place on one plane only, the total force $\vec{R}$ is carried by that plane.

The great circle ($\vec{I}_{12}$, $\vec{n}_1$) bounds the region of kinematically possible slip on plane 1. Likewise, the great circle ($\vec{I}_{12}$, $\vec{n}_2$) bounds the region of kinematically possible slip on plane 2. Between these two great circles, there is a region in which slip is possible along the line of intersection $\vec{I}_{12}$. At limiting equilibrium in the intersection failure mode, full friction has been mobilized on planes land 2 simultaneously. The force $\vec{R}_1$ consists of a normal force and a shear force parallel to $\vec{I}_{12}$ and is $\phi_1$ degrees from $\vec{n}_1$. Likewise, the force$\vec{R}_2$ consists of a normal force and a shear force parallel to $\vec{I}_{12}$ and is $\phi_2$ degrees from $\vec{n}_2$.

Given the friction angles $\phi_1$ and $\phi_2$, it is possible to construct a generalized safe zone including all possible modes of movement of the wedge. When the safe zone has been constructed, one examines the orientation of the resultant $\vec{R}$ with respect to the safe zone. The orientation tells whether the wedge is stable or unstable and indicates the mode of potential slip.

For a wedge, two safety factors are introduced

$$F_1 = \frac{\tan \phi_{1\ avail}}{\tan \phi_{1\ req.}} \quad ; \quad F_2 = \frac{\tan \phi_{2\ avail}}{\tan \phi_{2\ req.}}$$

Note that for a given position of $\vec{R}$ in the safe zone of the wedge, there are several combinations for $F_1$ and $F_2$.

### 3.2 Example

Consider a rock wedge with weight equal to 500 MN.

Plane 1 : N 59° W 60° W $\phi_1 = 30°$

Plane 2 : N 0° E 45° $\phi_2 = 40°$

a) Is the block safe under its own weight?

b) Find the magnitude of bolting force for a safety factor of 2 on each plane. Bolts are installed 10 degrees below horizontal to the North.

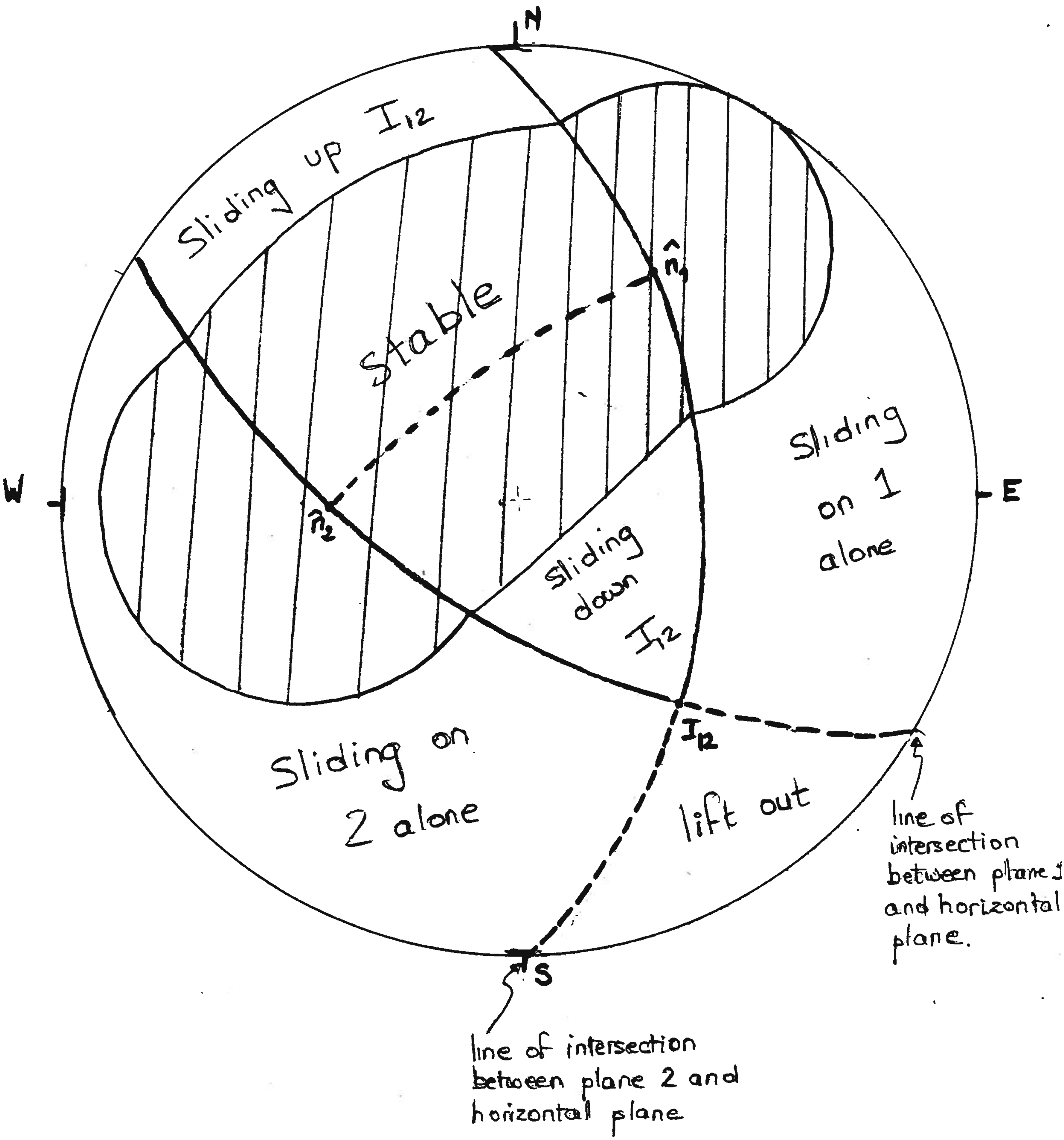
c) Assume the bolts are installed as in question b) What should be the water pressure on planes 1 and 2 to create sliding?

d) Assume the bolts are installed as in question b) .What should be the water pressure on plane 2 alone to create sliding on both planes?

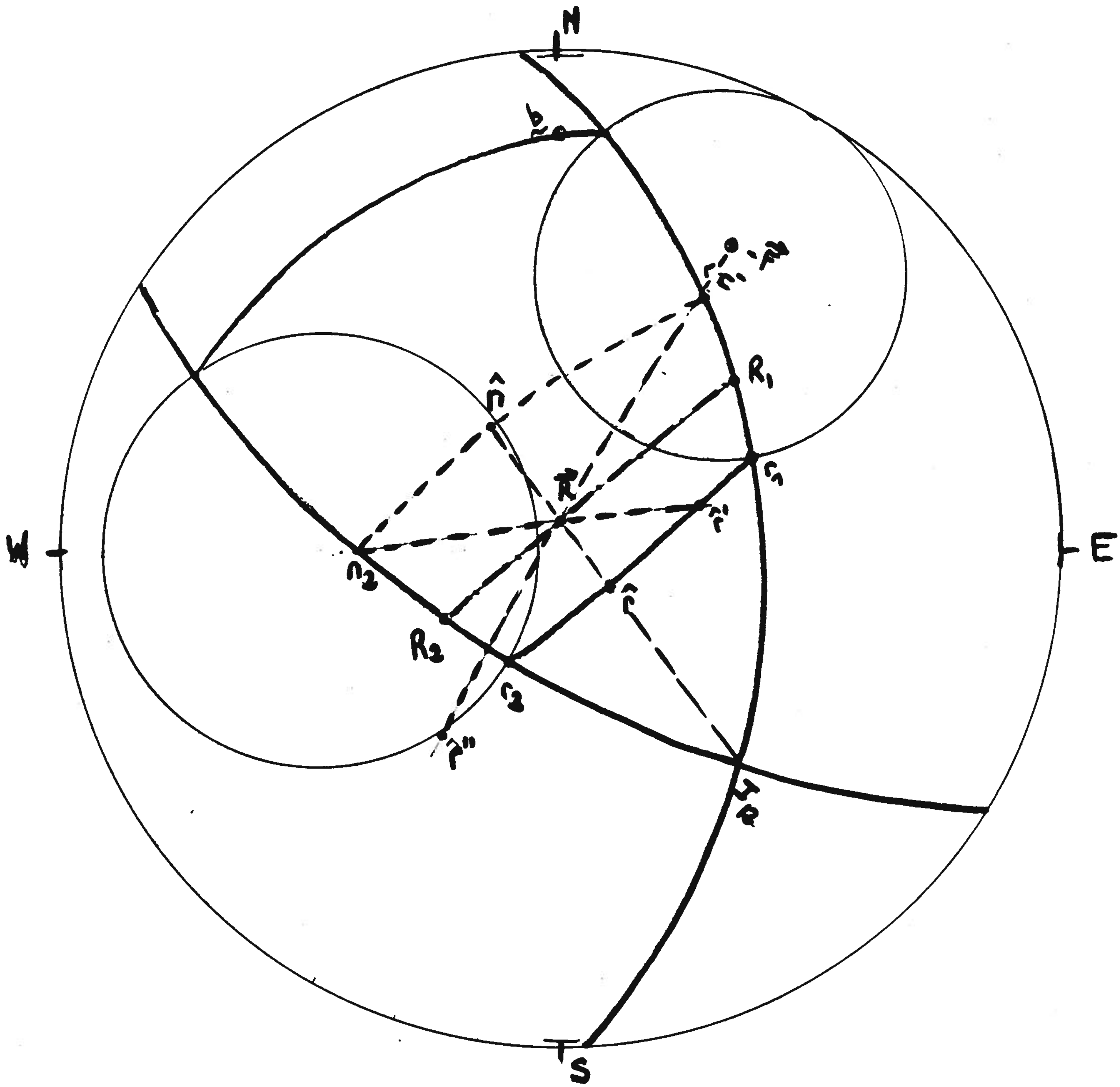
e) What inertia force associated with ground motion initiates slip if the inertia force acts 20 degrees above horizontal to the S 30° W?

Note: The rock wedge line of intersection $\vec{I}_{12}$ is assumed to daylight. The upper free surface of the wedge is assumed to be horizontal for all practical purposes.

Example


N
W
E
S
Sliding up $I_{12}$
Stable
$\hat{n}_1$
$\hat{n}_2$
Sliding on 1 alone
sliding down $I_{12}$
$I_{12}$
Sliding on 2 alone
lift out
line of intersection between plane 1 and horizontal plane.
line of intersection between plane 2 and horizontal plane

Example



$\phi_1 = 30°$

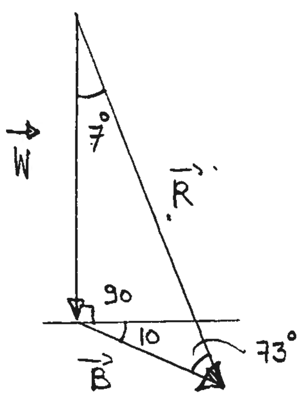
$\phi_2 = 40°$

$W = 500$ MN

a) Block safe under its own weight

b) $F_1 = F_2 = 2 \Rightarrow \tan \phi_{r_1} = \dfrac{\tan 30}{2} \Rightarrow \phi_{r_1} = 16.1°$

$\tan \phi_{r_2} = \dfrac{\tan 40}{2} \Rightarrow \phi_{r_2} = 22.7°$

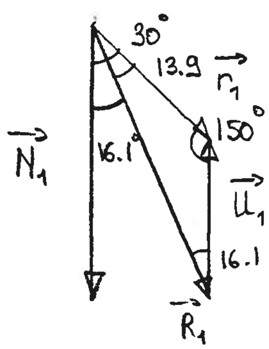



$$\Rightarrow \frac{|\vec{B}|}{\sin 7} = \frac{|\vec{W}|}{\sin 73} = \frac{|\vec{R}|}{\sin 100}$$

$\Rightarrow |\vec{B}| = 64$ MN

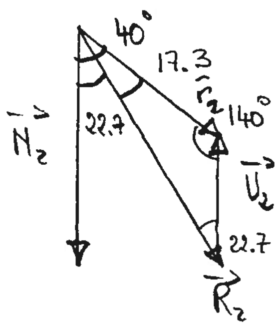
$|\vec{R}| = 515$ MN.

c)



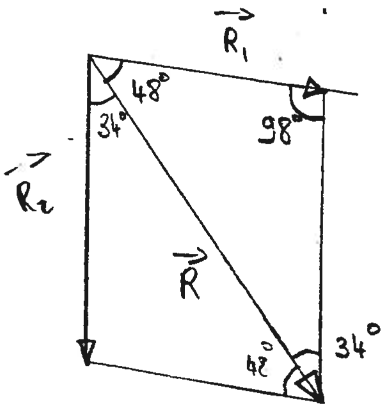

$$\frac{|\vec{U_1}|}{\sin 13.9} = \frac{|\vec{R_1}|}{\sin 150} = \frac{|\vec{r_1}|}{\sin 16.1}$$

$$\Rightarrow \boxed{\begin{aligned} |\vec{U_1}| &= 0.48\,|\vec{R_1}| \\ |\vec{r_1}| &= 0.554\,|\vec{R_1}| \end{aligned}}$$




$$\frac{|\vec{U_2}|}{\sin 17.3} = \frac{|\vec{R_2}|}{\sin 140} = \frac{|\vec{r_2}|}{\sin 22.7}$$

$$\boxed{|\vec{U_2}| = 0.46\,|\vec{R_2}| \;;\; |\vec{r_2}| = 0.60\,|\vec{R_2}|}$$



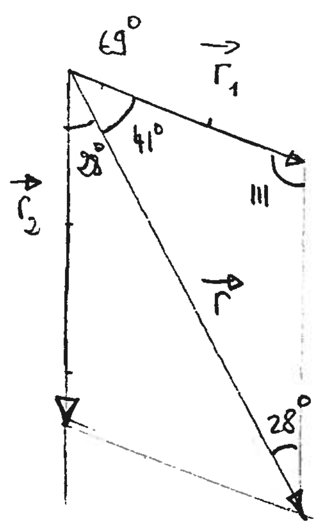

$$\frac{|\vec{R_1}|}{\sin 34} = \frac{|\vec{R}|}{\sin 98°} = \frac{|\vec{R_2}|}{\sin 48°}$$

$$|\vec{R_1}| = 291 \text{ MN}$$

$$|\vec{R_2}| = 386 \text{ MN}$$

$\Rightarrow$ $|\vec{U_1}| = 140$ MN $\qquad |\vec{r_1}| = 161$ MN

$|\vec{U_2}| = 177$ MN. $\qquad |\vec{r_2}| = 232$ MN.

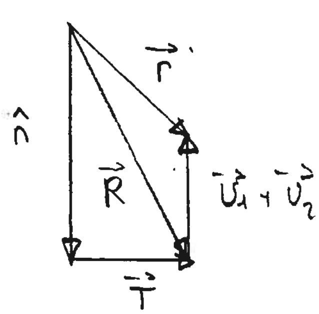



$$\vec{r} = \vec{r_1} + \vec{r_3}$$

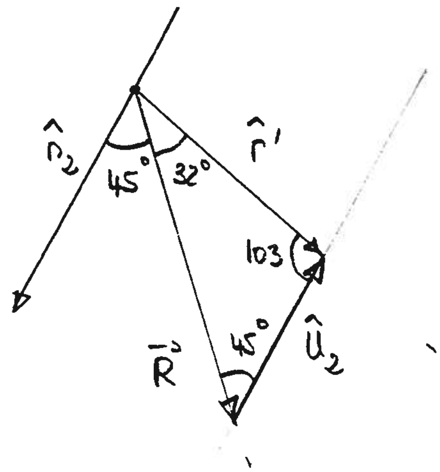
$$\frac{|\vec{r}|}{\sin 111} = \frac{|\vec{r_1}|}{\sin 28}$$

$$\Rightarrow \underline{|\vec{r}| = 320 \text{ MN}}$$

$$\hat{r} = \hat{r}_1 + \hat{r}_2$$
$$= \vec{R_1} + \vec{U_1} + \vec{R_2} + \vec{U_2}$$
$$= \vec{N_1} + \vec{T_1} + \vec{N_2} + \vec{T_2} + (\vec{U_1} + \vec{U_2})$$
$$= (\vec{N_1} + \vec{N_2}) + (\vec{T_1} + \vec{T_2}) + (\vec{U_1} + \vec{U_2}).$$
$$= \underbrace{\hat{n} + \vec{T}}_{\vec{R}} + (\vec{U_1} + \vec{U_2}).$$



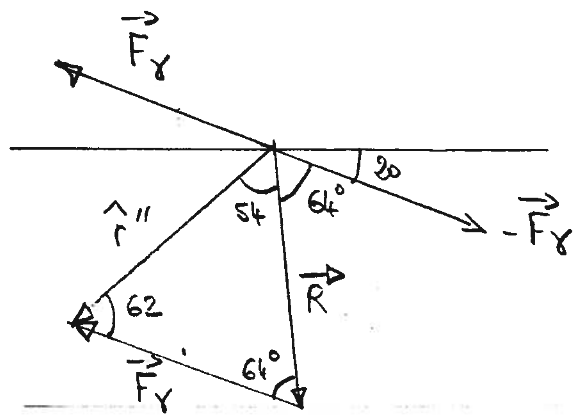
d).




$$\frac{|\vec{U}_2|}{\sin 32} = \frac{|\vec{R}|}{\sin 103}$$

$$|\vec{U}_2| = 280 \text{ MN}$$

e)




$$\frac{|\vec{F}_\gamma|}{\sin 54} = \frac{|\vec{R}|}{\sin 62} \Rightarrow |\vec{F}_\gamma| = 478 \text{ MN}$$

$$|\vec{K}| = 0.94$$