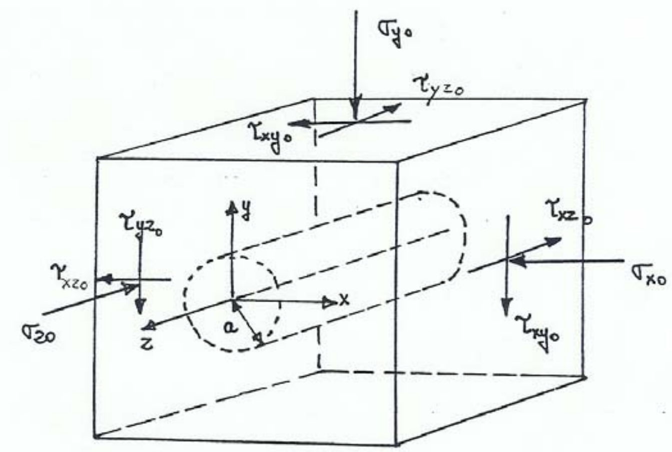
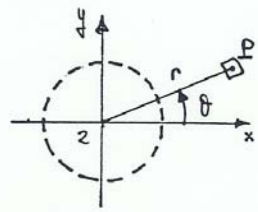
Stress, Strain and Displacement Distributions Around a Circular Hole of Radius, a, in a 3D Stress Field. Generalized Plane Strain Formulation (Summary).





1) Stresses:

- $$\sigma_r = \left(1-\frac{a^2}{r^2}\right)\left(\frac{\sigma_{xo}+\sigma_{yo}}{2}\right) + \left(1+\frac{3a^4}{r^4}-\frac{4a^2}{r^2}\right)\left(\frac{\sigma_{xo}-\sigma_{yo}}{2}\right)\cos 2\theta + \left(1+\frac{3a^4}{r^4}-\frac{4a^2}{r^2}\right)\tau_{xyo}\sin 2\theta$$

- $$\sigma_\theta = \left(1+\frac{a^2}{r^2}\right)\left(\frac{\sigma_{xo}+\sigma_{yo}}{2}\right) - \left(1+\frac{3a^4}{r^4}\right)\left(\frac{\sigma_{xo}-\sigma_{yo}}{2}\right)\cos 2\theta - \left(1+\frac{3a^4}{r^4}\right)\tau_{xyo}\sin 2\theta$$

- $$\sigma_z = \sigma_{zo} - 4\nu\frac{a^2}{r^2}\left(\frac{\sigma_{xo}-\sigma_{yo}}{2}\right)\cos 2\theta - 4\nu\frac{a^2}{r^2}\tau_{xyo}\sin 2\theta$$

- $$\tau_{r\theta} = -\left(1-\frac{3a^4}{r^4}+\frac{2a^2}{r^2}\right)\left(\frac{\sigma_{xo}-\sigma_{yo}}{2}\right)\sin 2\theta + \left(1-\frac{3a^4}{r^4}+\frac{2a^2}{r^2}\right)\tau_{xyo}\cos 2\theta$$

$$\bullet \quad \tau_{rz} = \left(1 - \frac{a^2}{r^2}\right)\left(\tau_{yz_0}\sin\theta + \tau_{xz_0}\cos\theta\right)$$

$$\bullet \quad \tau_{\theta z} = \left(1 + \frac{a^2}{r^2}\right)\left(\tau_{yz_0}\cos\theta - \tau_{xz_0}\sin\theta\right)$$

2) Strains

$$\varepsilon_r = \frac{1}{E}\left(\sigma_r - \nu\left(\sigma_\theta + \sigma_z\right)\right)$$
$$\varepsilon_\theta = \frac{1}{E}\left(\sigma_\theta - \nu\left(\sigma_r + \sigma_z\right)\right)$$
$$\varepsilon_z = \varepsilon_{z_0} = \frac{1}{E}\left(\sigma_{z_0} - \nu\left(\sigma_{x_0} + \sigma_{y_0}\right)\right)$$

$$\gamma_{r\theta} = \frac{1}{G}\tau_{r\theta} \quad ; \quad \gamma_{rz} = \frac{1}{G}\tau_{rz} \quad ; \quad \gamma_{\theta z} = \frac{1}{G}\tau_{\theta z}$$

3) Displacements

$$\bullet \quad u_r = -\frac{1}{E}\left[\left(\frac{\sigma_{x_0}+\sigma_{y_0}}{2}\right)\left(r + \frac{a^2}{r}\right) + \left(\frac{\sigma_{x_0}-\sigma_{y_0}}{2}\right)\left(r - \frac{a^4}{r^3} + 4\frac{a^2}{r}\right)\cos 2\theta\right]$$
$$+ \frac{\nu}{E}\left[\left(\frac{\sigma_{x_0}+\sigma_{y_0}}{2}\right)\left(r - \frac{a^2}{r}\right) - \left(\frac{\sigma_{x_0}-\sigma_{y_0}}{2}\right)\left(r - \frac{a^4}{r^3}\right)\cos 2\theta\right]$$
$$+ \frac{\nu}{E}\sigma_{z_0} r + 4\frac{\nu^2}{E}\frac{a^2}{r}\left(\frac{\sigma_{x_0}-\sigma_{y_0}}{2}\right)\cos 2\theta$$
$$- \frac{1}{E}\tau_{xy_0}\sin 2\theta\left(r - \frac{a^4}{r^3} + \frac{4a^2}{r}\right) - \frac{\nu}{E}\tau_{xy_0}\sin 2\theta\left(r - \frac{a^4}{r^3}\right)$$
$$+ 4\frac{\nu^2}{E}\frac{a^2}{r}\tau_{xy_0}\sin 2\theta$$

$$\bullet \quad v_\theta = \frac{1}{E}\left(\frac{\sigma_{x_0}-\sigma_{y_0}}{2}\right)\left(r + \frac{2a^2}{r} + \frac{a^4}{r^3}\right)\sin 2\theta + \frac{\nu}{E}\left(\frac{\sigma_{x_0}-\sigma_{y_0}}{2}\right)\left(r - \frac{2a^2}{r} + \frac{a^4}{r^3}\right)\sin 2\theta$$
$$- 4\frac{\nu^2}{E}\frac{a^2}{r}\left(\frac{\sigma_{x_0}-\sigma_{y_0}}{2}\right)\sin 2\theta - \frac{1}{E}\tau_{xy_0}\cos 2\theta\left(r + \frac{2a^2}{r} + \frac{a^4}{r^3}\right)$$
$$- \frac{\nu}{E}\tau_{xy_0}\cos 2\theta\left(r - 2\frac{a^2}{r} + \frac{a^4}{r^3}\right) + 4\frac{\nu^2}{E}\frac{a^2}{r}\tau_{xy_0}\cos 2\theta$$

$$\bullet \quad w = -\frac{z}{E}\left(\sigma_{z_0} - \nu\left(\sigma_{x_0} + \sigma_{y_0}\right)\right) - \frac{1}{G}\left(r + \frac{a^2}{r}\right)\left(\tau_{yz_0}\sin\theta + \tau_{xz_0}\cos\theta\right)$$

4) Along the contour of the hole $(r=a)$, expressions for ε_θ, ε_z, $\gamma_{\theta z}$, u_r/a, v_θ/a and w/a.

. $$E\varepsilon_\theta = \sigma_{x_0} + \sigma_{y_0} - 2(\sigma_{x_0} - \sigma_{y_0})\cos 2\theta - \nu\sigma_{z_0} + 2\nu^2(\sigma_{x_0} - \sigma_{y_0})\cos 2\theta - 4\tau_{xy_0}\sin 2\theta + 4\nu^2\tau_{xy_0}\sin 2\theta$$

$$\boxed{\varepsilon_\theta = f_{1\theta}\,\sigma_{x_0} + f_{2\theta}\,\sigma_{y_0} + f_{3\theta}\,\sigma_{z_0} + f_{6\theta}\,\tau_{xy_0}}$$

$$f_{1\theta} = \frac{1}{E}\left(1 + 2\cos 2\theta\,(\nu^2 - 1)\right)$$

$$f_{2\theta} = \frac{1}{E}\left(1 - 2\cos 2\theta\,(\nu^2 - 1)\right)$$

$$f_{3\theta} = -\frac{\nu}{E}$$

$$f_{6\theta} = \frac{4}{E}\sin 2\theta\,(\nu^2 - 1)$$

. $$E\varepsilon_z = \sigma_{z_0} - \nu(\sigma_{x_0} + \sigma_{y_0})$$

$$\boxed{\varepsilon_z = f_{1z}\,\sigma_{x_0} + f_{2z}\,\sigma_{y_0} + f_{3z}\,\sigma_{z_0} + f_{6z}\,\tau_{xy_0}}$$

$$f_{1z} = -\frac{\nu}{E}$$

$$f_{2z} = -\frac{\nu}{E}$$

$$f_{3z} = \frac{1}{E}$$

$$f_{6z} = 0$$

. $$G\gamma_{\theta z} = 2\tau_{yz_0}\cos\theta - 2\tau_{xz_0}\sin\theta$$

$$\boxed{\gamma_{\theta z} = f_{4\theta z}\,\tau_{yz_0} + f_{5\theta z}\,\tau_{xz_0}}$$

$$f_{4\theta z} = \frac{2}{G}\cos\theta \; ; \quad f_{5\theta z} = -\frac{2}{G}\sin\theta$$

$$\cdot\ E\frac{u_r}{a} = -\left(\sigma_{xo}+\sigma_{yo}\right) - 2\left(\sigma_{xo}-\sigma_{yo}\right)\cos 2\theta + \nu\,\sigma_{zo} + 2\nu^2\left(\sigma_{xo}-\sigma_{yo}\right)\cos 2\theta - 4\,\tau_{xyo}\sin 2\theta + 4\nu^2\tau_{xyo}\sin 2\theta$$

$$\boxed{\frac{u_r}{a} = f_{1u}\,\sigma_{xo} + f_{2u}\,\sigma_{yo} + f_{3u}\,\sigma_{zo} + f_{6u}\,\tau_{xyo}}$$

$$f_{1u} = \frac{1}{E}\left(2\cos 2\theta\left(\nu^2-1\right) - 1\right)$$

$$f_{2u} = \frac{1}{E}\left(2\cos 2\theta\left(1-\nu^2\right) - 1\right)$$

$$f_{3u} = \frac{\nu}{E}$$

$$f_{6u} = \frac{4}{E}\sin 2\theta\left(\nu^2-1\right)$$

$$\cdot\ E\frac{v_\theta}{a} = 2\left(\sigma_{xo}-\sigma_{yo}\right)\sin 2\theta - 2\nu^2\left(\sigma_{xo}-\sigma_{yo}\right)\sin 2\theta - 4\,\tau_{xyo}\cos 2\theta + 4\nu^2\tau_{xyo}\cos 2\theta$$

$$\boxed{\frac{v_\theta}{a} = f_{1v}\,\sigma_{xo} + f_{2v}\,\sigma_{yo} + f_{3v}\,\sigma_{zo} + f_{6v}\,\tau_{xyo}}$$

$$f_{1v} = \frac{2}{E}\sin 2\theta\left(1-\nu^2\right)$$

$$f_{2v} = \frac{2}{E}\sin 2\theta\left(\nu^2-1\right)$$

$$f_{3v} = 0$$

$$f_{6v} = \frac{4}{E}\cos 2\theta\left(\nu^2-1\right)$$

$$\cdot\ E\frac{w}{a} = -\frac{z}{a}\left(\sigma_{zo} - \nu\left(\sigma_{xo}+\sigma_{yo}\right)\right) - \frac{2E}{G}\left(\tau_{yzo}\sin\theta + \tau_{xzo}\cos\theta\right)$$

$$\boxed{\frac{w}{a} = f_{1w}\,\sigma_{xo} + f_{2w}\,\sigma_{yo} + f_{3w}\,\sigma_{zo} + f_{4w}\,\tau_{yzo} + f_{5w}\,\tau_{xzo} + f_{6w}\,\tau_{xyo}}$$

$$f_{1w} = f_{2w} = \frac{\nu}{E}\frac{z}{a}\quad;\quad f_{3w} = -\frac{z}{a}\frac{1}{E}$$

$$f_{4w} = -\frac{2}{G}\sin\theta\quad;\quad f_{5w} = -\frac{2}{G}\cos\theta\quad;\quad f_{6w} = 0$$

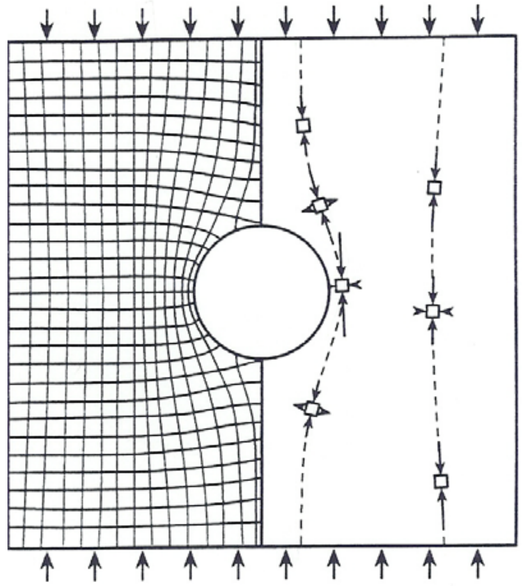


Fig. 1

9/95G2a'

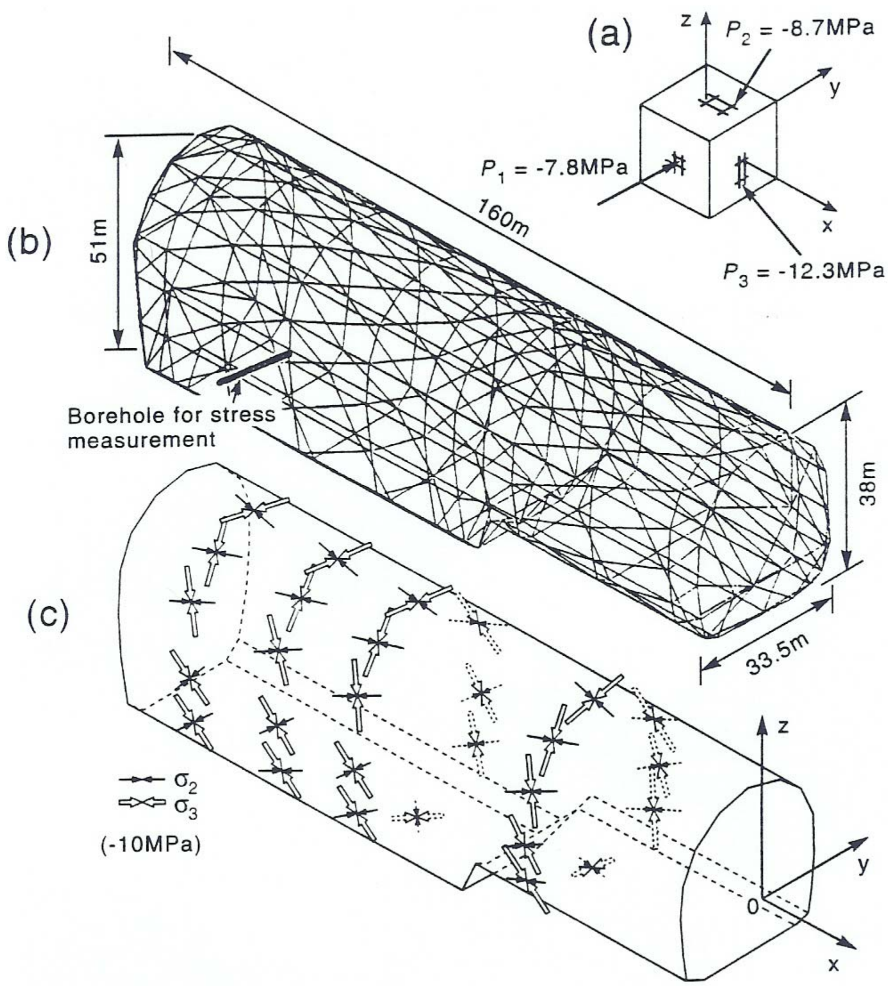
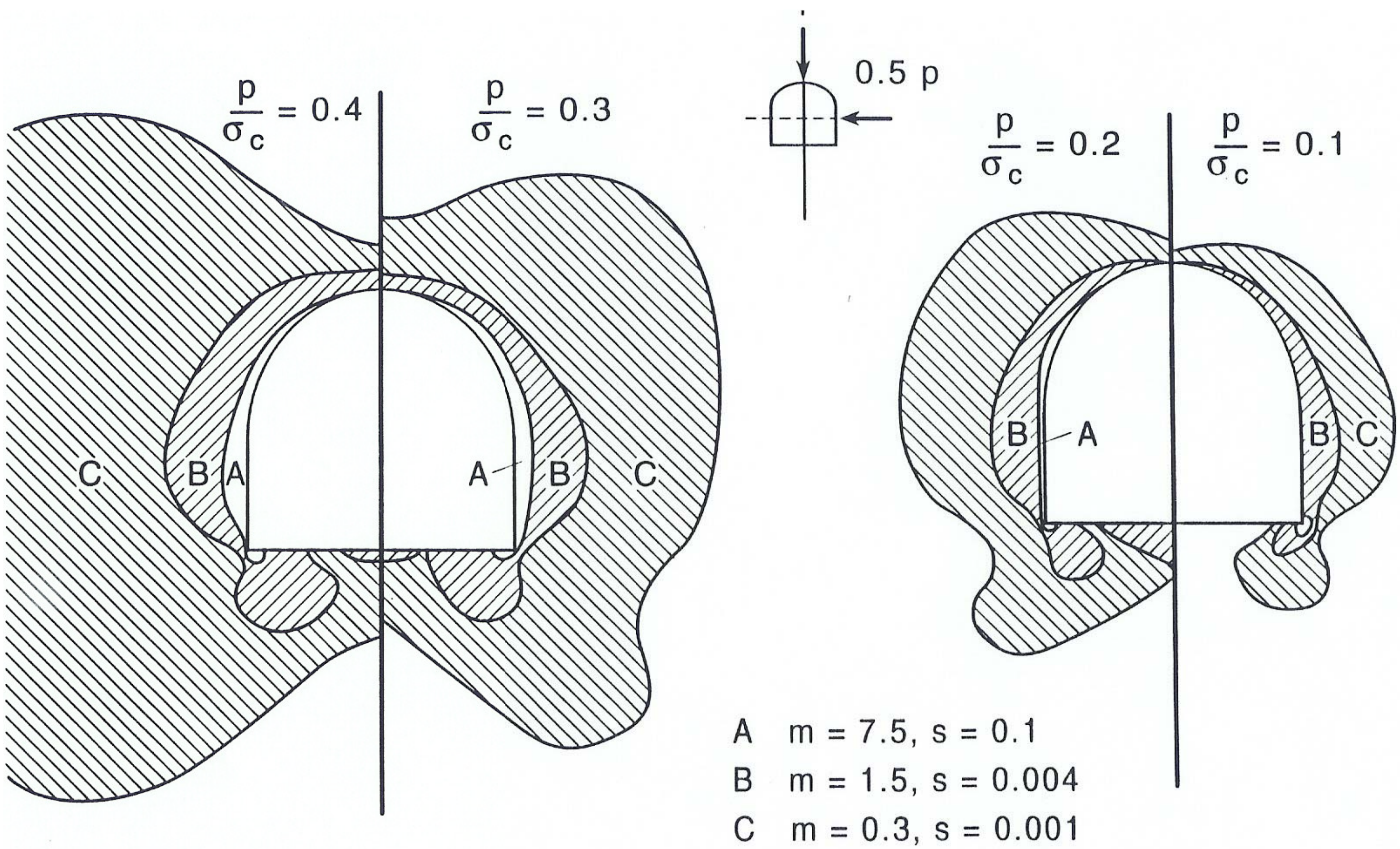

(a)
z
y
x
P_2 = -8.7MPa
P_1 = -7.8MPa
P_3 = -12.3MPa
(b)
51m
160m
38m
33.5m
Borehole for stress
measurement
(c)
z
y
x
0
σ2
σ3
(-10MPa)

Fig. 2



p/σc = 0.4
p/σc = 0.3
0.5 p
p/σc = 0.2
p/σc = 0.1
C
B
A
A
B
C
B
A
B
C
A m = 7.5, s = 0.1
B m = 1.5, s = 0.004
C m = 0.3, s = 0.001

Fig. 3

9/95(

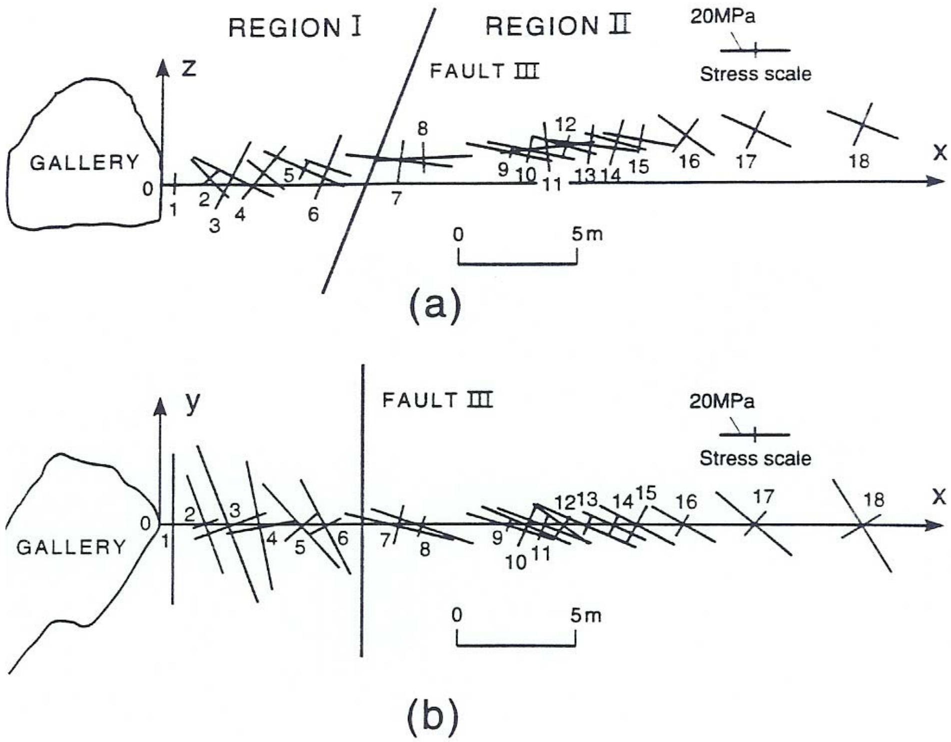
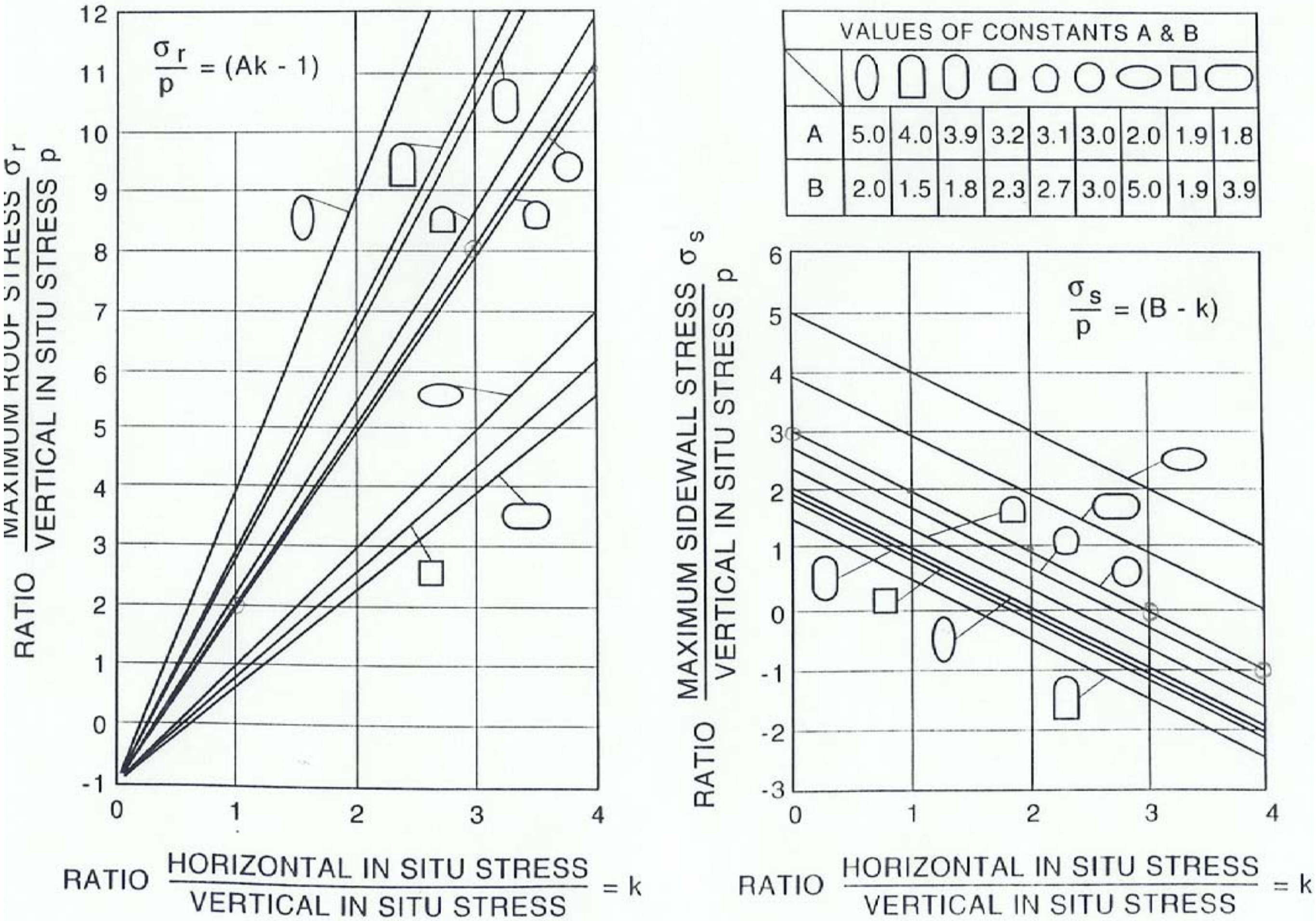

REGION I
REGION II
FAULT III
20MPa
Stress scale
GALLERY
z
x
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
0
5m
(a)
y
FAULT III
20MPa
Stress scale
GALLERY
x
0
5m
(b)

Fig. 4



VALUES OF CONSTANTS A & B

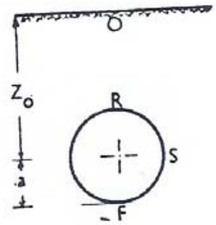
A	5.0	4.0	3.9	3.2	3.1	3.0	2.0	1.9	1.8
B	2.0	1.5	1.8	2.3	2.7	3.0	5.0	1.9	3.9

Fig. 5.

Eissa (1980)

effect of the ground surface and is only applicable when $z_o/a > 5$.

TABLE 3.3.2 STRESS CONCENTRATION ($\sigma_\theta/\gamma z_o$) AROUND THE BOUNDARY OF A SHALLOW CIRCULAR TUNNEL



z_o/a	FLOOR (F)			SIDEWALL (S)			ROOF (R)		
	K=0.0	K=0.333	K=1.0	K=0.0	K=0.333	K=1.0	K=0.0	K=0.333	K=1.0
1.081	-2.158	-0.445	3.152	2.98	2.57	1.64	-3.974	-2.09	6.06
1.185	-2.00	-0.375	2.938	3.126	2.69	1.84	-2.375	-1.03	1.84
1.337	-1.856	-0.332	2.8	3.24	2.77	1.911	-2.05	-0.914	1.496
1.543	-1.656	-0.264	2.69	3.29	2.83	1.968	-1.8	-0.744	1.44
1.811	-1.575	-0.2045	2.577	3.27	2.843	1.984	-1.55	-0.573	1.452
2.151	-1.446	-0.155	2.48	3.25	2.841	1.997	-1.36	-0.396	1.532
2.577	-1.322	-0.115	2.4	3.19	2.8	2.012	-1.18	-0.273	1.595
3.107	-1.24	-0.083	2.336	3.15	2.78	2.003	-1.085	-0.179	1.657
3.762	-1.18	-0.06	2.284	3.11	2.747	2.008	-1.024	-0.118	1.723
5	-1.126	-0.037	2.207	3.067	2.72	2.007	-0.978	-0.067	1.792
6	-1.20	-0.025	2.20	3.05	2.70	2.006	-0.951	-0.055	1.778
7.5	-1.072	-0.015	2.138	3.03	2.69	2.00	-0.963	-0.03	1.862
10	-1.055	-0.007	2.104	3.018	2.678	2.00	-0.974	-0.015	1.896
15	-1.035	-0.0025	2.069	3.007	2.672	2.00	-0.977	-0.0074	1.93
25	-1.019	-0.0015	2.04	3.003	2.668	2.00	-0.983	-0.003	1.958
37.5	-1.012	-0.001	2.028	3.0015	2.667	2.00	-0.9886	-0.002	1.972
50	-1.009	-0.0007	2.02	3.00	2.667	2.00	-0.991	-0.0015	1.979
∞	-1.00	0.0	2.0	3.00	2.667	2.0	-1.0	0.0	2.0

Eissa (1980)

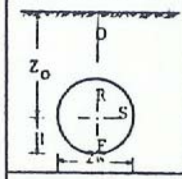
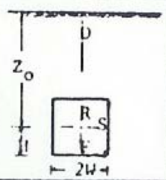
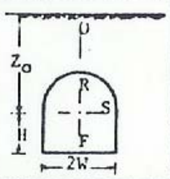
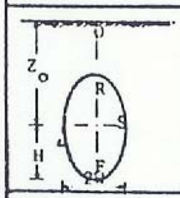
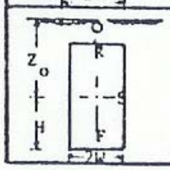
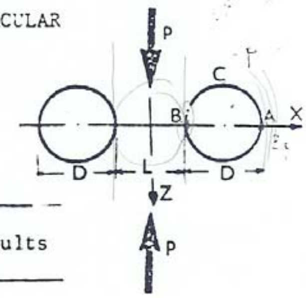
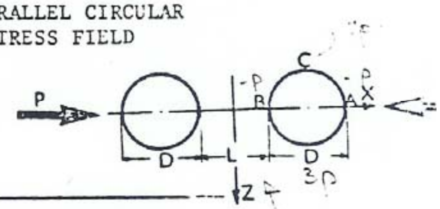
		K = 0.0				K = 0.5				K = 1.0			
	Z/H_c	σ_θ (F)	σ_θ (S)	σ_θ (R)	σ_θ (O)	σ_θ (F)	σ_θ (S)	σ_θ (R)	σ_θ (O)	σ_θ (F)	σ_θ (S)	σ_θ (R)	σ_θ (O)
	1.5	-1.8025	3.26	-1.753	0.988	0.3704	3.161	-0.691	0.247	2.765	2.1235	0.938	0.444
	3.0	-1.333	3.16	-1.062	0.383	0.555	2.593	0.247	0.309	2.42	2.0123	1.543	0.1605
	6.0	-1.142	3.05	-0.951	0.111	0.537	2.525	0.414	0.0617	2.216	2.006	1.778	0.049
	∞	-1.0	3.0	-1.0	-	0.5	2.5	0.5	-	2.0	2.0	2.0	-
	1.5	-1.679	2.568	-2.64	-0.222	-0.2963	2.074	-1.654	0.222	1.111	1.555	-0.6913	0.691
	3.0	-1.148	1.85	-0.889	-0.136	-0.0247	1.395	-0.1235	0.2346	1.0864	0.938	0.642	0.284
	6.0	-0.994	1.796	-0.8086	0.0926	0.0	1.352	-0.006	0.0926	0.994	0.9074	0.7901	0.0864
	∞	-1.0	1.85	-1.0	-	-0.075	1.35	-0.075	-	0.9	0.9	0.9	-
	1.5	-1.654	3.111	-2.123	0.07407	-0.1975	2.222	-0.346	0.321	1.26	1.358	1.407	0.568
	3.0	-1.173	2.469	-0.9506	0.1605	-0.062	1.914	0.395	0.185	1.05	1.407	1.753	0.1975
	6.0	-1.025	2.343	-0.876	0.0864	-0.043	1.8796	0.537	0.074	0.938	1.404	1.944	0.068
	∞	-0.95	2.3	-1.0	-	-0.075	1.8	0.6	-	0.80	1.30	2.25	-
	1.5	-1.778	2.321	-1.284	-0.025	1.9506	1.469	0.815	0.0124	5.679	0.63	2.9	0.272
	3.0	-1.272	2.02	-0.9012	0.0123	1.753	1.494	1.1975	0.037	4.778	0.963	3.31	0.0741
	6.0	-1.093	2.006	-0.895	0.0185	1.636	1.5	1.358	0.0123	4.364	0.994	3.61	0.0123
	∞	-1.0	2	-1.0	-	1.5	1.5	1.5	-	4.0	1.0	4.0	-
	1.5	-1.506	1.83	-1.06	0.049	0.222	1.21	0.02	0.037	1.951	0.568	1.086	0.667
	3.0	-1.1	1.47	-0.716	0.037	0.284	1.00	0.235	0.074	1.667	0.531	1.17	0.124
	6.0	-0.96	1.45	-0.747	0.025	0.278	0.994	0.259	0.025	1.52	0.543	1.265	0.025
	∞	-1.0	1.4	-1.0	-	0.2	0.9	0.2	-	1.4	0.4	1.4	-

TABLE 2.3.5 BOUNDARY STRESSES FOR TWO PARALLEL CIRCULAR TUNNELS IN UNIT VERTICAL STRESS FIELD



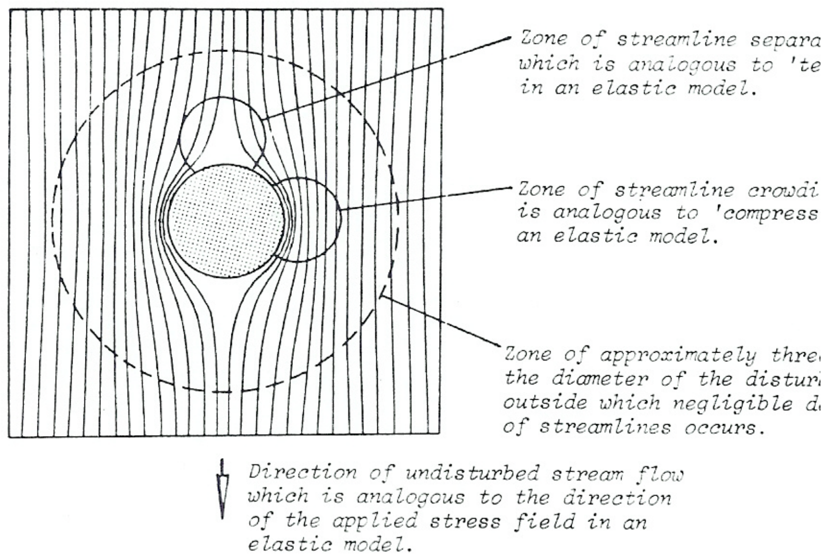
L	Boundary Element Results			Analytical Results		
	$\sigma_{(A)}$	$\sigma_{(B)}$	$\sigma_{(C)}$	$\sigma_{(A)}$	$\sigma_{(B)}$	$\sigma_{(C)}$
0.0	3.8521	21.8056	-1.0858	3.868	∞	-
D/4	3.2737	4.0621	-0.9067	-	-	-
D/2	3.1567	3.2931	-0.8753	3.151	3.264	-
D	3.0673	3.0326	-0.8739	3.066	3.020	-
2D	3.0199	2.9948	-0.9157	3.02	2.992	-
4D	3.005	2.9980	-0.962	3.004	2.997	-
7D	3.001	2.9997	-0.984	3.001	2.999	-
∞	3.00	3.00	-0.9997	3.00	3.00	-

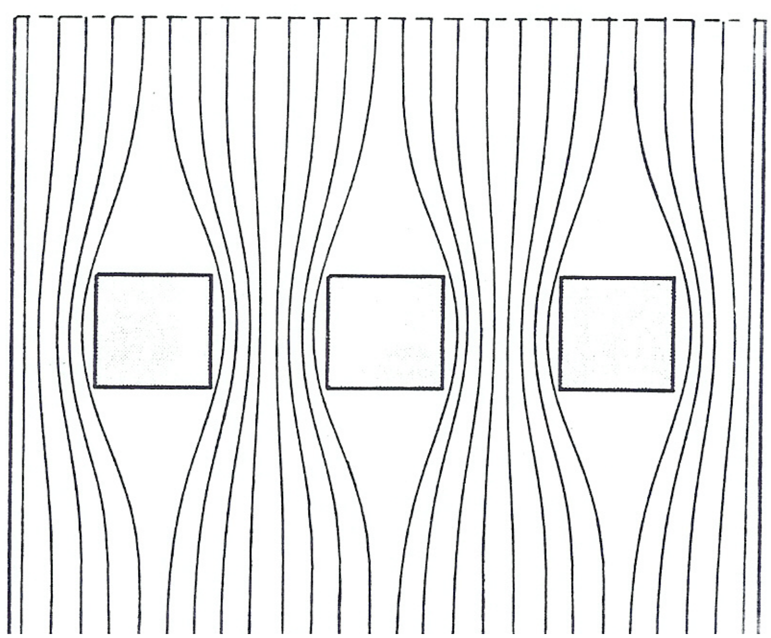
TABLE 2.3.6 BOUNDARY STRESSES FOR TWO PARALLEL CIRCULAR TUNNELS IN UNIT HORIZONTAL STRESS FIELD

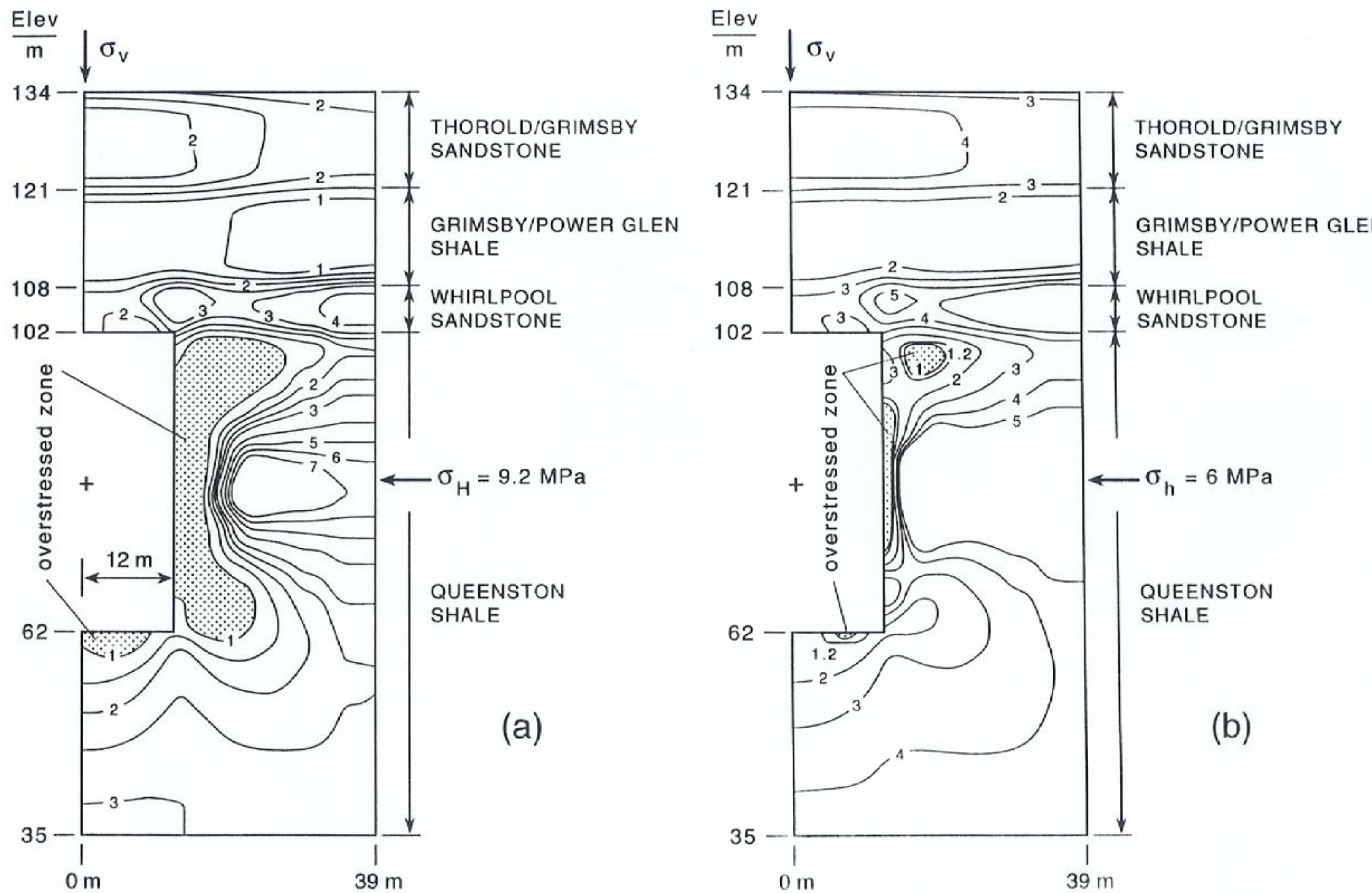


L	Boundary Element Results			Analytical Results		
	$\sigma_{(A)}$	$\sigma_{(B)}$	$\sigma_{(C)}$	$\sigma_{(A)}$	$\sigma_{(B)}$	$\sigma_{(C)}$
0.0	-0.9632	2.6558	2.5355	-1.074	-	2.569
D/4	-0.8923	-0.4088	2.5584	-	-	-
D/2	-0.8875	-0.4067	2.5999	-0.896	-0.377	2.623
D	-0.9016	-0.6203	2.6871	-0.908	-0.609	2.703
2D	-0.9358	-0.8381	2.8174	-0.940	-0.837	2.825
4D	-0.969	-0.947	2.922	-0.971	-0.948	2.927

Hoek and Brown (1980)






Elev
m
σ_v
134
121
108
102
62
35
0 m
39 m
12 m
overstressed zone
THOROLD/GRIMSBY
SANDSTONE
GRIMSBY/POWER GLEN
SHALE
WHIRLPOOL
SANDSTONE
QUEENSTON
SHALE
σ_H = 9.2 MPa
σ_h = 6 MPa
(a)
(b)

Fig