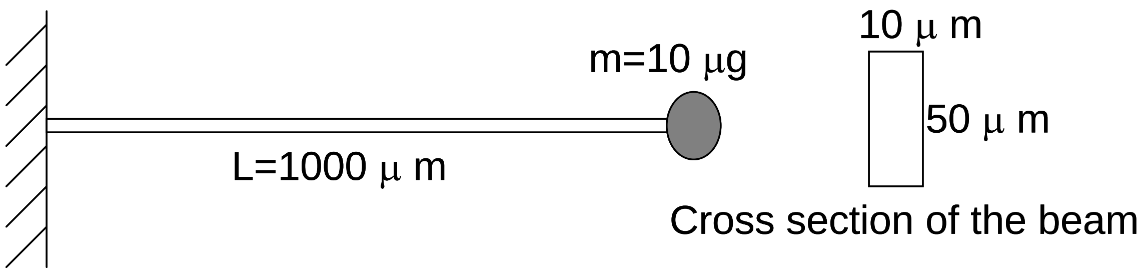
Homework Micro-scale Engineering #4, Due Date September 30, 2008

A cantilever beam element is illustrated below. The beam is made of silicon with a Young's modulus of 190,000 MPa.



Its spring constant and the natural frequency can be calculated as follows:

I = width X thickness^3 / 12 = (10 x 10^{-6})(50 x 10 $^{-6}$)^3/12 = 0.1042 x 10^{-18} m^4

Spring constant k = 3EI/L^3 = 3 x (190,000 x 10^6) (0.1042 x 10^{-18})/(1000 x 10^{-6})^3 = 59.39 N/m.

Natural frequency ω = SQRT (k/m) = SQRT (59.39/(10 x 10-6)) = 2437 rad/s = 776 Hz.

We would like to make a nano-scaled beam by reducing all the dimensions by 1000X (L=1000 nm, width = 10 nm and the thickness = 50 nm). If we would like to reach the nature frequency around 1000 Hz (1 kHz), what would be the mass attached to the nano-scaled beam? (10 points)

width – W , thickness – t

$I = \frac{Wt^3}{12}$

$k = \frac{3EI}{L^3} = \frac{1}{4}\frac{EWt^3}{L^3}$

$w = \sqrt{K/m} = \frac{1}{2}\sqrt{\frac{EWt^3}{L^3 \cdot m}}$

$W_0 = \frac{1}{2}\sqrt{\frac{E W_0 t_0^3}{L_0^3 m_0}} = 776\ Hz$ (1)

$W_1 = \frac{1}{2}\sqrt{\frac{E W_1 t_1^3}{L_1 m_1}} = 1000\ HZ$ (2)

$W_1/W_0 = L_1/L_0 = t_1/t_0 = \cancel{\ } 10^{-3}$

use (2) divide (1) we get

$$\sqrt{\frac{(W_1/W_0)(t_1/t_0)^3}{(L_1/L_0)^3(m_1/m_0)}} = \frac{1000}{776}$$

$$\sqrt{\frac{10^{-3} m_0}{m_1}} = \frac{1000}{776}$$

$$m_1 = \frac{776^2}{1000^2} \times 10^{-3} m_0$$

$= 6.02.176\ m_0 \times 10^{-6} = \cancel{6.02176 \times 10^{-9}}$ 6.02176 $\times 10^{-9}$ g