

ASEN 5107 Nonlinear FEM Spring 2007 Quiz 3 (Last Exam)

Posted May 2, 2007. Due on or before 4PM Tu May 8, at AE 187 or instructors's mailbox at AE 194 or OT 613.

Work individually. Consult with instructor for clarifications, not with other students. Do two Questions: Pick one from 1-2, and one from 3-4. Bonus Questions provided. Begin answering each Question on a new page. Use of engineering paper is recommended. **Do not** attach any computer programs, just results such as plots.

Errors/typos found after May 2 will be posted on the web site "Final exam" link. Please check website periodically.

This test is largely a long homework on material covered in Chapters 29–30.

QUESTION 1 (40 pts) Choose one from Question 1 or Question 2.

(Analytical, may be speeded up by symbolic calculation). Consider a two-dimensional bar element as in Figure 29.6, submerged in a subsonic liquid flow running in the X direction. The current configuration \mathcal{C} tilts by θ , where $\theta = \text{angle}(\bar{x}, X)$ positive counterclockwise. Assume that the lateral aerodynamic force in the \bar{y} direction and the tangential drag force in the \bar{x} direction follow the laws

$$p_d = p_{d0} \sin^2 \theta, \quad p_t = p_{t0} \cos^2 \theta \quad (\text{Q3.1})$$

respectively, where p_d and p_t are forces per unit of current bar length, and p_{d0} and p_{t0} are known from experimental data.¹ Consider only 4 displacement degrees of freedom: $\mathbf{u}^T = [u_{X1}, u_{Y1}, u_{X2}, u_{Y2}]$. Determine \mathbf{f}_L and the load stiffness \mathbf{K}_L in terms of p_{d0} , p_{t0} , L and the nodal displacements. Use of trigonometric functions of θ to make expressions compact is recommended, cf. §29.6.

Give the numerical expression of \mathbf{K}_L if $L_0 = 5$, $u_{X1} = u_{Y1} = 0$, $u_{X2} = -2$, $u_{Y2} = 4$, $p_{d0} = 100$, $p_{t0} = 3$.

Grading based on: result, method, and analytic simplification level achieved.

QUESTION 2 (40 pts) Choose one from Question 1 or Question 2.

Exercise 30.2 but omit item (d), which deals with plots. Note that the applied loads were changed to $P_1 = P_2 = \frac{1}{2}P$, instead of $P_1 = P_2 = P$ on 5/2/07, then returned to $P_1 = P_2 = P$ on 5/3/07.

QUESTION 3 (60 pts) Choose one from Question 3 or Question 4.

Exercise 30.3(b). (Analytical+computer, tricky). This is the FEM analysis of Beck's column, which is sometimes used as a simplified model to illustrate stability of rockets (tilt effect). For this Question, do the two element case; four elements qualifies for Bonus Question 1. The solution of 30.3(a), which is the one element case, is given after that Exercise as a guide.

QUESTION 4 (60 pts) Choose one from Question 3 or Question 4.

(Analytical, computer may help but can be entirely done by hand). The simplest model to study supersonic flutter of an aircraft wing is shown in Figure Q3.1. It consists of a uniform, rigid panel (actually a flat plate) 1-2 of span b that has unit width in the z direction. It is elastically supported by a linear spring of stiffness k_S , which "lumps" the about- x bending wing stiffness, and a torsional spring of stiffness $k_T = \alpha k_S b^2$, where $\alpha \geq 0$ is a dimensionless parameter that characterizes the torsional wing stiffness.

The model has only two degrees of freedom: the transverse (y) edge displacements u_1 and u_2 , which are assumed to be *very small*.² The resisting forces on the panel are: the restoring transverse force $-k_S(u_1 + u_2)/2$, acting downward if $(u_1 + u_2)/2 > 0$; and the restoring torque $-k_T \tan \theta = k_T(u_1 - u_2)/b$, acting counterclockwise if $u_1 > u_2$.

The panel is immersed in a supersonic flow of speed $v \geq 0$ in the X direction. If the plate edges displace by u_1 and u_2 , the following lift force develops:

$$F_L = \frac{1}{2} C_L \rho_f v^2 (u_1 - u_2) \quad (\text{Q3.2})$$

¹ These laws are not the same as in Figure 29.6.

² Lateral displacements are grossly exaggerated in the figure for visibility

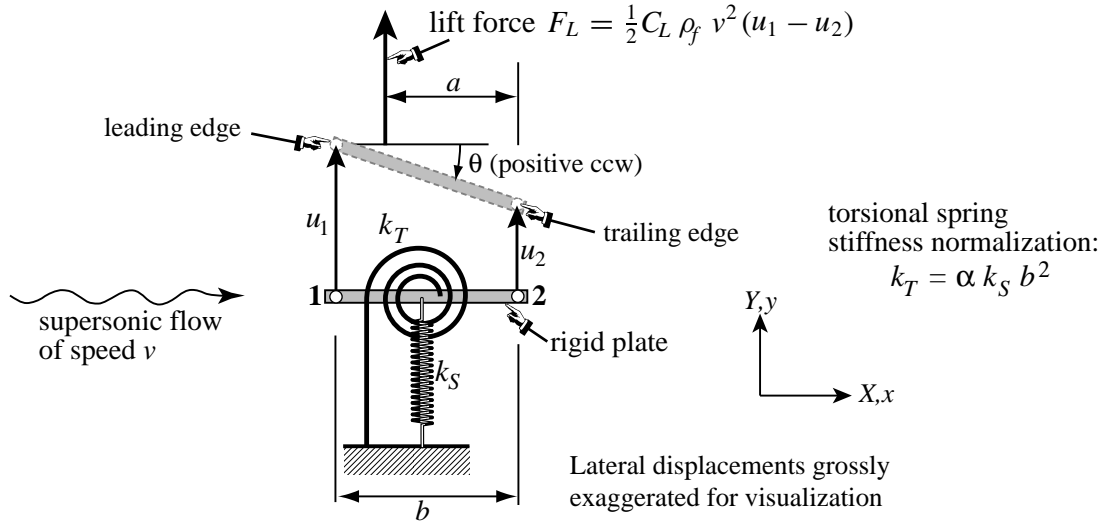


Figure Q3.1. A highly simplified model to study supersonic panel flutter.

in which ρ_f is the fluid mass density and $C_L > 0$ is a lift coefficient. This lift acts at a distance a from the trailing edge, as shown in the figure. C_L , ρ_f and a are kept fixed through the analysis whereas v is increased from zero until instability occurs.³

For the dynamic stability analysis take $\mathbf{K} = \mathbf{K}_M + \mathbf{K}_L$, where the material stiffness \mathbf{K}_M is the *linear* stiffness due to the springs in the reference state, and \mathbf{K}_L must be obtained from the lift force “lumping” to the nodes as a nonconservative force vector \mathbf{f}_L . Begin by proving that

$$\mathbf{K}_M = \frac{1}{4}k_S \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} + \alpha k_S \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}, \quad \mathbf{f}_L = F_L \begin{bmatrix} a/b \\ 1 - a/b \end{bmatrix} \quad (\text{Q3.3})$$

As consistent mass matrix take (as a given)

$$\mathbf{M} = \frac{1}{6}mb \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \quad (\text{Q3.4})$$

where m is the given panel mass per unit of x length. Find the expressions for the divergence speed v_d and the flutter speed v_f in terms of the data (k_S , α , etc) assuming $a = 3b/4$ (“quarter point wing”) and discuss which one is critical, *i.e.* happens first when the flow speed is increased. (Always take the + root for v).

Plot v_f and v_d against $0.0834 \leq \alpha \leq 1$ by taking $b = 1$, $k_S = 1$, $\rho_f = 1$, $C_L = 1$ and $a = 3b/4$ (m drops out).

Note: only one Bonus Question counts. If you do more than one, mark the one you want graded.

BONUS QUESTION 1 (up to 5 points, total may exceed 100)

Do Beck’s column with 4 equal elements.

BONUS QUESTION 2 (up to 10 points, total may exceed 100)

Exercise 30.4. This is PhD level.

BONUS QUESTION 3 (up to 5 points, total may not exceed 100)

Prove that flutter cannot occur if the system has only one degree of freedom (“it takes two to flutter”)

³ The fluid speed v may be viewed here as the control parameter λ . A common alternative is to take $v = \lambda c$, where c is the sound speed; if so λ is called the Mach number and is generally denoted as M (not to be confused with mass or moment).