

ASEN 5107: Nonlinear FEM First Midterm Quiz Spring 2007

Begin answering each Question on a new page. Attach this exam as cover

Important: Work individually. No consultation with others is permitted

Take home Tuesday February 27, 2001. Due on or before Thursday March 1 at class time. Attach this exam as cover sheet with your name on it.

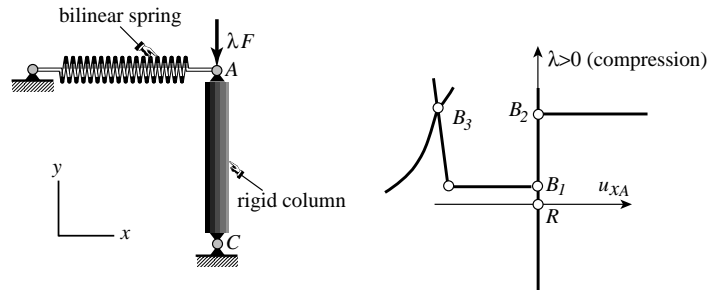


Figure Q1.1. Structure for Question 1(a).

QUESTION 1 Conceptual questions, 30 pts, 5 pts/item.

- (a) The structure shown in Figure Q1.1 is an axially loaded rigid column pinned at 2 and propped at 1 by a *bilinear spring* that is very stiff in tension but less so in compression. Briefly explain the response: λ ($\lambda > 0$ means compression) versus lateral deflection u_{xA} sketched on the right of that Figure. Note: the secondary paths go on further; they are truncated for convenience.
- (b) The structure is in Question 1(a), but is now subject to a *horizontal* load λF where $\lambda > 0$ means the load acts toward the left, compressing the spring. (No vertical load is applied). Sketch the response for $\lambda > 0$ and $\lambda < 0$. Any critical points expected?
- (c) You crush an empty styrofoam coffee cup by pushing down on the lid. What kind of nonlinearities do you think are involved in the process? Explain.
- (d) In preparation to mail a letter you tear down a stamp from an old-fashioned stamp book. What kind of nonlinearities are involved in the tearing process?
- (e) A submarine goes too deep. Its hull collapses by buckling and then breaks. What kind of nonlinearities are involved? Ignore what happens after collapse starts.

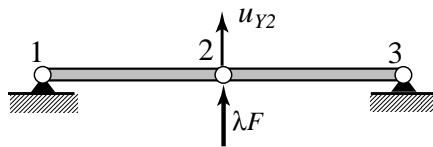


Figure Q1.2. Structure for Question 1(f).

- (f) The 2-bar shallow arch of §8.3 of Notes has initially a *zero* height H when unloaded. The bars are not prestressed. An upwards vertical load λF is applied at the center node 2 as sketched in Figure Q1.2. Sketch the response of λ versus vertical deflection u_{Y2} , with emphasis in the vicinity of the reference configuration. (Hint: the tangent stiffness is initially zero).

QUESTION 2 (35 points, 15-20 pts/item).

The approximate potential energy of a rigid column of length L , loaded by an axial compressive load λF , hinged at one end and propped by a linear spring of stiffness k at the other (similar to Figure Q1.1, but the spring is now linear all the way) is

$$\Pi(u, \lambda) = U(u) - V(u, \lambda) = \frac{1}{2}ku^2 - \frac{1}{2}FL\lambda \left(\frac{u^2}{L^2} + \frac{u^4}{2L^4} \right) \quad (1)$$

where k , F and L are constant, and the only degree-of-freedom u is the horizontal displacement under the load. In the reference configuration $\lambda = 0$ and $u = 0$. Note that (1) is not separable.

- Find the residual force equation $r = 0$ and the first-order rate equation $\dot{r} = 0$. From the former verify that $u = 0$ is the fundamental equilibrium path.
- Find the λ_{cr} (in terms of k , L_0 , and F) at which the structure encounters a critical point on the fundamental path. Find whether it is a limit or bifurcation point.

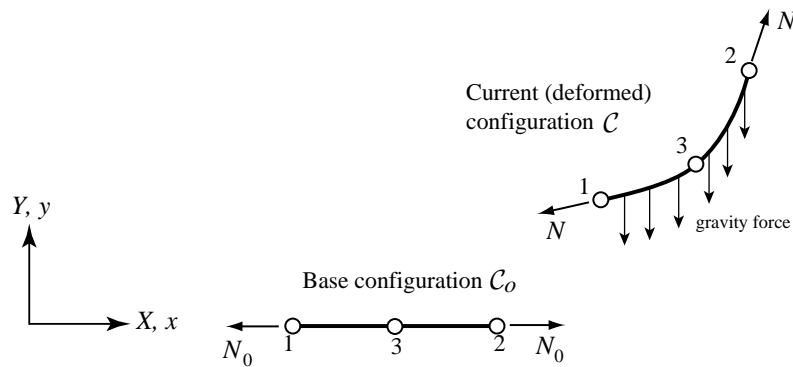


Figure Q1.3. 3-node cable element for Question 3.

QUESTION 3 (35 points, 10-15-10 pts/item)

You go to work for a software company that sells a nonlinear FEM code. Your first project is to develop a small-strain, two-dimensional, 3-node, plane, slack cable element with parabolic shape to be used in suspension bridges and guyed towers. See Figure Q1.3. The element nodal DOFs are the two displacements at each node. The element has cross section area A , density ρ and is pulled down by gravity g . The only internal force is the axial force N .

- How would you pick a parabola that goes through 3 points in the plane $\{X, Y\}$? Is that always possible? (explain without equations).
- Explain briefly, *minimizing equations*, how the TL bar element of §8 could be modified to become an elastic cable element. Note: accounting for prestress in the reference configuration is essential since if $N \leq 0$ the cable has *zero axial stiffness* (try to push a rope).
- Would there be any substantial gain in modeling cables with beam elements as opposed to bar elements as in (b)? If you were to do that, would a Euler-Bernoulli or Timoshenko model be preferable, or would it make no difference?

BONUS QUESTION Up to 5 points, total may not exceed 100.

How would you handle the situation when $N \leq 0$ in the cable element of Figure Q1.3? Is static analysis possible or a different type of analysis might be required?