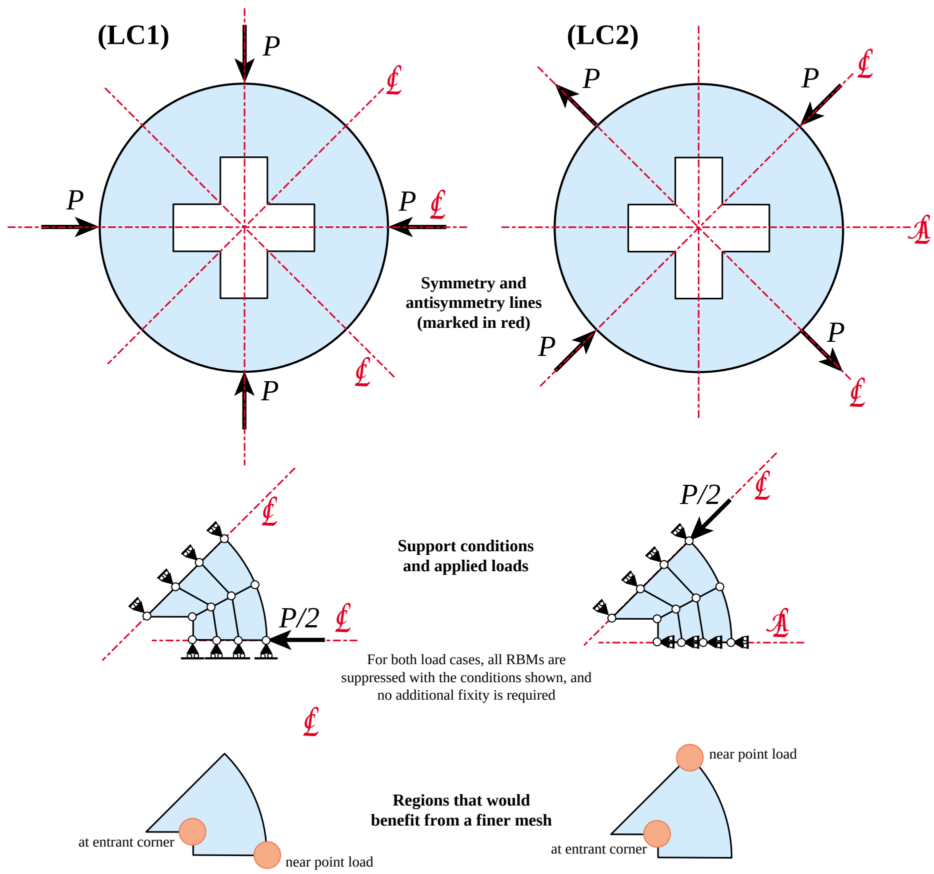
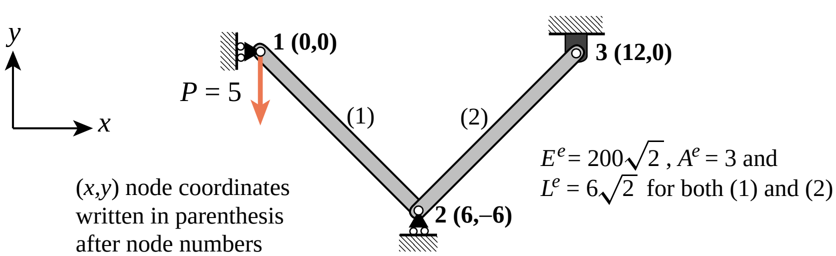
ASEN 5007 Fall 2009 First Midterm Exam - Solutions

QUESTION 1 30 pts: 15 (LC1) + 15 (LC2)



QUESTION 2. 40 pts: 10 + 10 + 10 + 10. Exam figure reproduced for convenience.



(a) Assembled master stiffness equations, obtained with any method:

$$\begin{bmatrix} 50 & -50 & -50 & 50 & 0 & 0 \\ -50 & 50 & 50 & -50 & 0 & 0 \\ -50 & 50 & 100 & 0 & -50 & -50 \\ -50 & 50 & 0 & 100 & -50 & -50 \\ 0 & 0 & -50 & -50 & 50 & 50 \\ 0 & 0 & -50 & -50 & 50 & 50 \end{bmatrix} \begin{bmatrix} u_{x1} \\ u_{y1} \\ u_{x2} \\ u_{y2} \\ u_{x3} \\ u_{y3} \end{bmatrix} = \begin{bmatrix} f_{x1} \\ f_{y1} \\ f_{x2} \\ f_{y2} \\ f_{x3} \\ f_{y3} \end{bmatrix} \qquad \text{(Q1.3)}$$

(b) BCs: $u_{x1} = u_{y2} = u_{x3} = u_{y3} = 0,\ f_{y1} = -5,\ f_{x2} = 0$. Crossing out rows and columns 1, 4, 5 and 6 gives the reduced stiffness equation:

$$\boxed{\begin{bmatrix} 50 & 50 \\ 50 & 100 \end{bmatrix} \begin{bmatrix} u_{y1} \\ u_{x2} \end{bmatrix} = \begin{bmatrix} -5 \\ 0 \end{bmatrix}} \tag{Q1.4}$$

(c) Solving:

$$u_{x2} = 1/10 = 0.10, \quad u_{y1} = -1/5 = -0.20 \tag{Q1.5}$$

Complete displacement solution:

$$\boxed{\mathbf{u} = [\,0 \quad -0.20 \quad 0.10 \quad 0 \quad 0 \quad 0\,]^T} \tag{Q1.6}$$

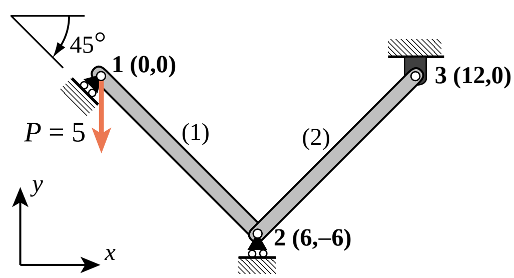
(d) Compute the internal bar force $F^{(2)}$ in element (2), which goes from node 2 to node 3. The orientation angle from x to 2→3 (+CCW) is 45°. We have $c = \cos 45^\circ = 1/\sqrt{2}$ and $s = \sin 45^\circ = 1/\sqrt{2}$. The local displacements are recovered from the displacement transformation

$$\begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 0 & 0 \\ -1/\sqrt{2} & 1/\sqrt{2} & 0 & 0 \\ 0 & 0 & 1/\sqrt{2} & 1/\sqrt{2} \\ 0 & 0 & -1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix} \begin{bmatrix} u_{x2} = 1/10 \\ u_{y2} = 0 \\ u_{x3} = 0 \\ u_{y3} = 0 \end{bmatrix} = \begin{bmatrix} \bar{u}_{x2} = 1/(10\sqrt{2}) \\ * \\ \bar{u}_{x3} = 0 \\ * \end{bmatrix} \tag{Q1.7}$$

where * are values of no interest for this computation. The member elongation is $d^{(2)} = \bar{u}_{x3} - \bar{u}_{x2} = 0 - (-1/(10\sqrt{2})) = -1/(10\sqrt{2})$, whence

$$\boxed{F^{(2)} = \frac{E^{(2)} A^{(2)}}{L^{(2)}} d^{(2)} = \frac{600\sqrt{2}}{6\sqrt{2}} \times -\frac{1}{10\sqrt{2}} = -5\sqrt{2} = -7.07 \ \text{(C)}} \tag{Q1.8}$$

QUESTION 3 30 pts = 5 + 10 + 10 + 5. Exam figure reproduced for convenience.



The reduced stiffness equations (Q1.2) result on applying three BCs: $u_{y2} = u_{x3} = u_{y3} = 0$ on the master stiffness equations obtained in item (a) of Question 1. Removing rows and columns 4, 5 and 6 we get

$$\begin{bmatrix} 50 & -50 & -50 \\ -50 & 50 & 50 \\ -50 & 50 & 100 \end{bmatrix} \begin{bmatrix} u_{x1} \\ u_{y1} \\ u_{x2} \end{bmatrix} = \begin{bmatrix} 0 \\ -5 \\ 0 \end{bmatrix} \tag{Q1.9}$$

Note that the RHS contains the values for the applied forces. It is an error to put f_{x1}, f_{y1} or f_{x2} there.

(a) Multifreedom constraint, by inspection:

$$\boxed{u_{x1} = -u_{y1}, \quad \text{or} \quad u_{x1} + u_{y1} = 0.} \tag{Q1.10}$$

(b) One slave freedom must be selected, since there is one MTC. Picking u_{y1} as slave, the M/S transformation equation is

$$\begin{bmatrix} u_{x1} \\ u_{y1} \\ u_{x2} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} u_{x1} \\ u_{x2} \end{bmatrix} \quad \text{or} \quad \mathbf{u} = \mathbf{T}\,\hat{\mathbf{u}}. \tag{Q1.11}$$

The remaining operations of the M/S method are not required in the test.

(b) To applied the penalty function method the MFC is placed in canonical form: $u_{x1} + u_{y1} = 0$. Rewrite in matrix form:

$$[\,1 \quad 1\,] \begin{bmatrix} u_{x1} \\ u_{y1} \end{bmatrix} = 0. \tag{Q1.12}$$

Premultiply both sides by $[\,1 \;\; 1\,]^T$ to get

$$\begin{bmatrix} 1 \\ 1 \end{bmatrix} [\,1 \quad 1\,] \begin{bmatrix} u_{x1} \\ u_{y1} \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} u_{x1} \\ u_{y1} \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} 0, \tag{Q1.13}$$

and finally scale by the weight w to get the penalty element

$$\boxed{w \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} u_{x1} \\ u_{y1} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix},} \tag{Q1.14}$$

Note that the RHS is a column 2-vector. Putting the scalar 0 as RHS is a considered an error because if so the matrix equations (Q1.14) have inconsistent dimensions. Merging into (Q1.9) gives

$$\begin{bmatrix} 50+w & -50+w & -50 \\ -50+w & 50+w & 50 \\ -50 & 50 & 100 \end{bmatrix} \begin{bmatrix} u_{x1} \\ u_{y1} \\ u_{x2} \end{bmatrix} = \begin{bmatrix} 0 \\ -5 \\ 0 \end{bmatrix} \tag{Q1.15}$$

The remaining steps of the penalty function method are not required in the test.

(c) To apply the Lagrange multiplier method one multiplier is required because there is only one MFC. Again one uses the canonical form: $u_{x1} + u_{y1} = 0$. On augmenting the coefficient matrix of (Q1.9) with one row and one column we get

$$\boxed{\begin{bmatrix} 50 & -50 & -50 & 1 \\ -50 & 50 & 50 & 1 \\ -50 & 50 & 100 & 0 \\ 1 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} u_{x1} \\ u_{y1} \\ u_{x2} \\ \lambda \end{bmatrix} = \begin{bmatrix} 0 \\ -5 \\ 0 \\ 0 \end{bmatrix}} \tag{Q1.16}$$

Switching signs in the last row and column is also considered correct. The solution of this system is not required in the test.

BONUS QUESTION Up to 5 pts, total may not exceed 100.

Tributary areas of node 1 extend for $a/2$ to its left along x and $a/2$ down along y. The contribution of each tributary area is $f_{x1} = p\,(a/2)\,h = \frac{1}{2}pah$ along $+x$ and $f_{y1} = p\,(a/2)\,h = \frac{1}{2}pah$ along $+y$. Consequently the NbN lumped force components at node 1 are

$$f_{x1} = f_{y1} = \tfrac{1}{2}\,p\,a\,h$$

Adding these two values is considered an error because they do not act in the same direction.