ASEN 5007 Fall 2009 - Midterm Exam #1 - Open Notes and HWs

On-campus: Th October 8, 2009, 3:30–4:45 pm. CAETE students: see instructions on cover sheet

Begin answering **each Question on a new page**; *Question items may continue on the same page. Do not write on the exam sheets. Write your name or initials on each page. Use this sheet as cover.*

QUESTION 1. 30 pts: 15 (LC1) + 15 (LC2)

The structure of Figure Q1.1 is a flat plate of constant thickness and uniform material with a hole of the shape shown. All edges are free (there are no supports). Two load cases are considered and identified as (LC1) and (LC2) in the figure. All loads have the same magnitude P. For each load case:

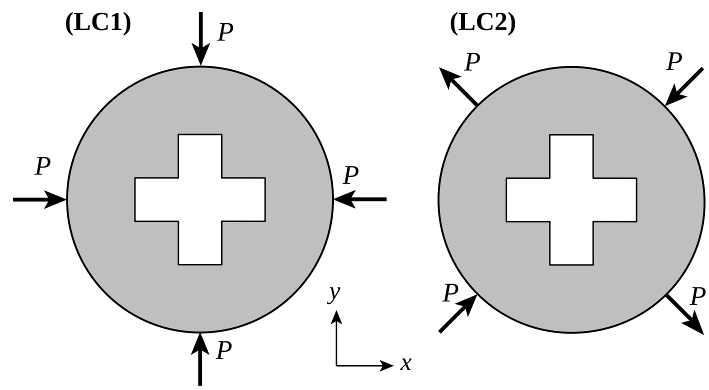

Figure Q1.1. Plane structure for Question 1. In (LC2) (right figure) the loads act at ±45° from x.

(a) Try to reduce the portion to be discretized as much as possible by identifying symmetry and/or antisymmetry lines. Sketch a diagram of the *whole plate* and identify such lines.

(b) Sketch a *coarse* finite element mesh over the smaller region you picked in (a). *Draw this separately from the sketch in (a)*. Make sure that there are several nodes over the cutout and exterior edge. Identify (for example with arrows) regions over which a finer mesh may be desirable (but *do not* draw the finer mesh). Show the loads that are to be applied on the smaller region.

(c) Show how you would apply the displacement boundary conditions on the nodes of the mesh picked in (b), so as to enforce symmetry and/or antisymmetry conditions.

Important: Draw on your own paper, not on the exam sheet. Do diagrams for LC1 and LC2 separately, indicating which is which; do not mix them.

QUESTION 2. 40 pts: 10 + 10 + 10 + 10

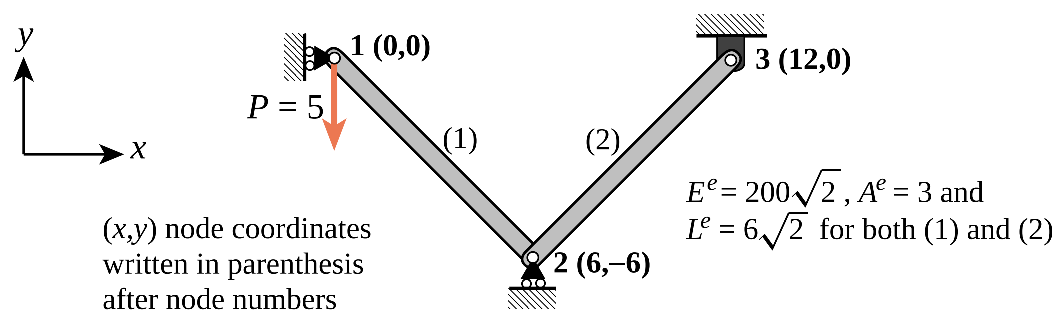

Figure Q1.2. Pin-jointed, plane truss structure for Question 2.

The plane truss problem defined in Figure Q1.2 has two elements and three nodes. Node 3 is fixed whereas 1 and 2 move over rollers as shown. The only nonzero applied load acts downward on node 1.

Solve this problem by the Direct Stiffness Method. Start from the element stiffness equations given in (Q1.1) below. These are listed so you do not need to refer to the Notes, and *already incorporate the* $E^e A^e / L^e$ *factor* in the stiffness matrices.

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The element stiffness equations *in global coordinates* are

$$\begin{bmatrix} 50 & -50 & -50 & 50 \\ -50 & 50 & 50 & -50 \\ -50 & 50 & 50 & -50 \\ 50 & -50 & -50 & 50 \end{bmatrix} \begin{bmatrix} u_{x1}^{(1)} \\ u_{y1}^{(1)} \\ u_{x2}^{(1)} \\ u_{y2}^{(1)} \end{bmatrix} = \begin{bmatrix} f_{x1}^{(1)} \\ f_{y1}^{(1)} \\ f_{x2}^{(1)} \\ f_{y2}^{(1)} \end{bmatrix}, \quad \begin{bmatrix} 50 & 50 & -50 & -50 \\ 50 & 50 & -50 & -50 \\ -50 & -50 & 50 & 50 \\ -50 & -50 & 50 & 50 \end{bmatrix} \begin{bmatrix} u_{x2}^{(2)} \\ u_{y2}^{(2)} \\ u_{x3}^{(2)} \\ u_{y3}^{(2)} \end{bmatrix} = \begin{bmatrix} f_{x2}^{(2)} \\ f_{y2}^{(2)} \\ f_{x3}^{(2)} \\ f_{y3}^{(2)} \end{bmatrix} \tag{Q1.1}$$

(a) Assemble the master stiffness equations. (This result is reused in Question 3 below).

(b) Apply the given force and displacement BCs to get a reduced system of 2 equations and show it.

(c) Solve the reduced stiffness system for the unknown displacements and show the complete node displacement vector. *Skip recovery of node forces and reactions.*

(d) Recover the axial force $F^{(2)}$ in element (2) using the displacements you got in (c), noting sign.

QUESTION 3. 30 pts = 5 + 10 + 10 + 5

This Question reuses the truss structure solved in Question 2. Everything remains the same except that the support at 1 is changed to a skew rollers at 45° with respect to x, as pictured in Figure Q1.3. This support condition becomes a multifreedom constraint (MFCs). The departing point for the Question is the master stiffness system obtained upon deleting u_{y2}, u_{x3} and u_{y3} but *before* applying the MFCs:

$$\begin{bmatrix} * & * & * \\ * & * & * \\ * & * & * \end{bmatrix} \begin{bmatrix} u_{x1} \\ u_{y1} \\ u_{x2} \end{bmatrix} = \begin{bmatrix} 0 \\ * \\ 0 \end{bmatrix} \tag{Q1.2}$$

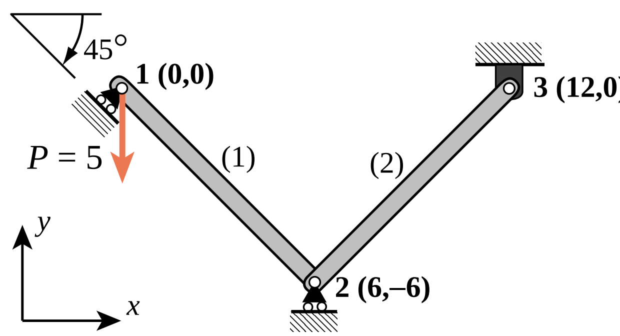

Figure Q1.3. Structure for Question 3.

Asterisks are numeric entries you got in item (a) of Question 2, after removing rows and columns 4, 5 and 6. (For this Question, do not remove u_{x1}).

(a) Write down the MFC and put it in canonical form.

(b) Apply the MFC by the master-slave (M/S) method by picking a slave. Write down the M/S transformation matrix but do not proceed further.

(c) Apply the MFC by the penalty function method. Write down the stiffness equations of the penalty element and show how it will modify (Q1.2) but do not proceed further.

(d) Apply the MFCs by Lagrange multipliers. Show the resulting system of equations, but do not solve.

BONUS QUESTION. Up to 5 pts, total may not exceed 100.

The plate of Question 1 has thickness h and is now loaded only by uniform pressure p, which is expressed as force per unit area, inside the hole. A very coarse mesh, shown in Figure Q1.4, is established over the whole plate. (Note that this mesh is too coarse to be useful — it is used only to make this Question specific.)

Using the NbN load-lumping method, determine forces at node 1 in terms of p, h and a, in which a is the distance defined in the figure.

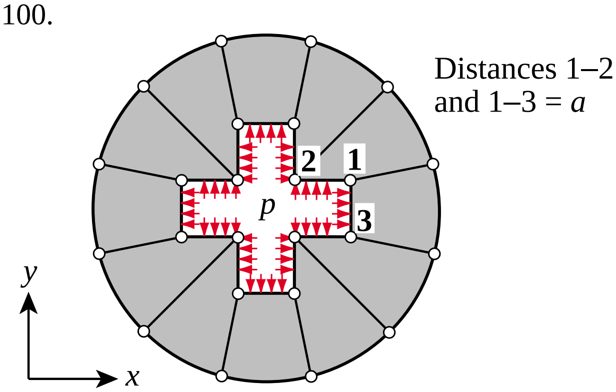

Figure Q1.4. Structure for Bonus Question.