Remaining ASEN 5007 Schedule

Today **Solving FEM Equations (descriptive only, no HW)**

12/7 **Complete plane stress program (Ch 27) - FCQs**

12/12 **Demos of Ch 27 program. Stress recovery (Ch 28)**

12/13 **Take-home exam posted on web, due in six days.**
Based on plane stress program of Ch 27

12/14 **Discussion of posted exam**
Last HW (#11) due 12/14 for on-campus students.

12/19 **Take-home exam due**

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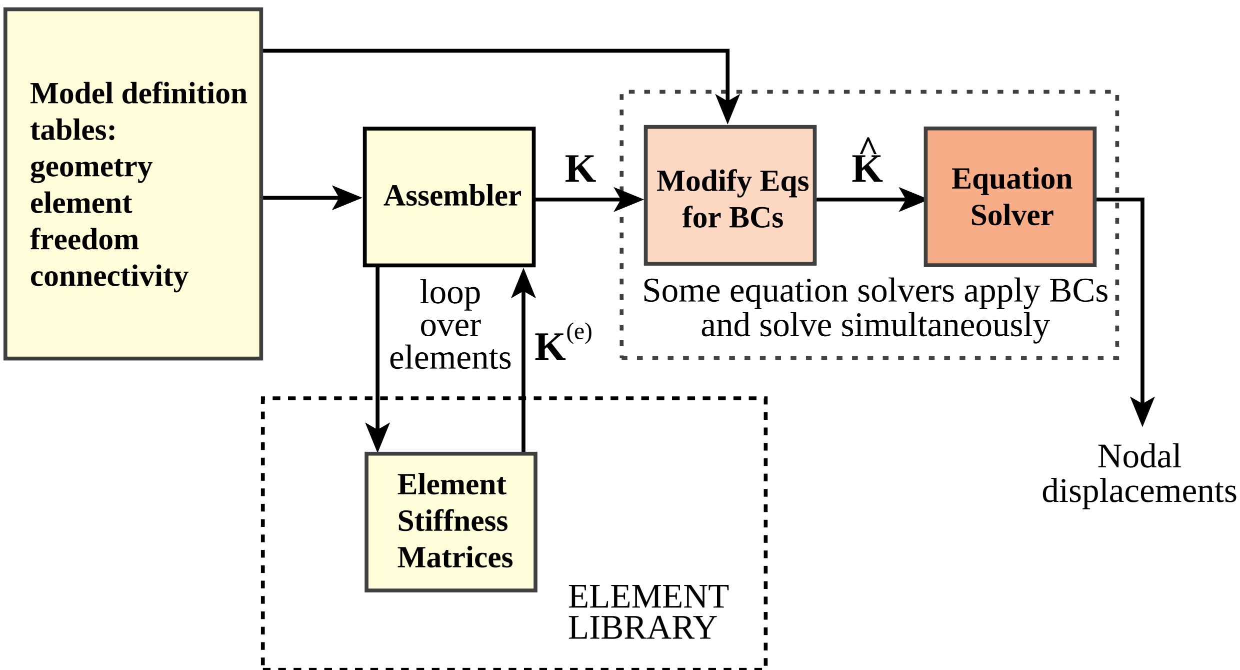
Solving FEM Equations

Role of the Solver in a FEM Code



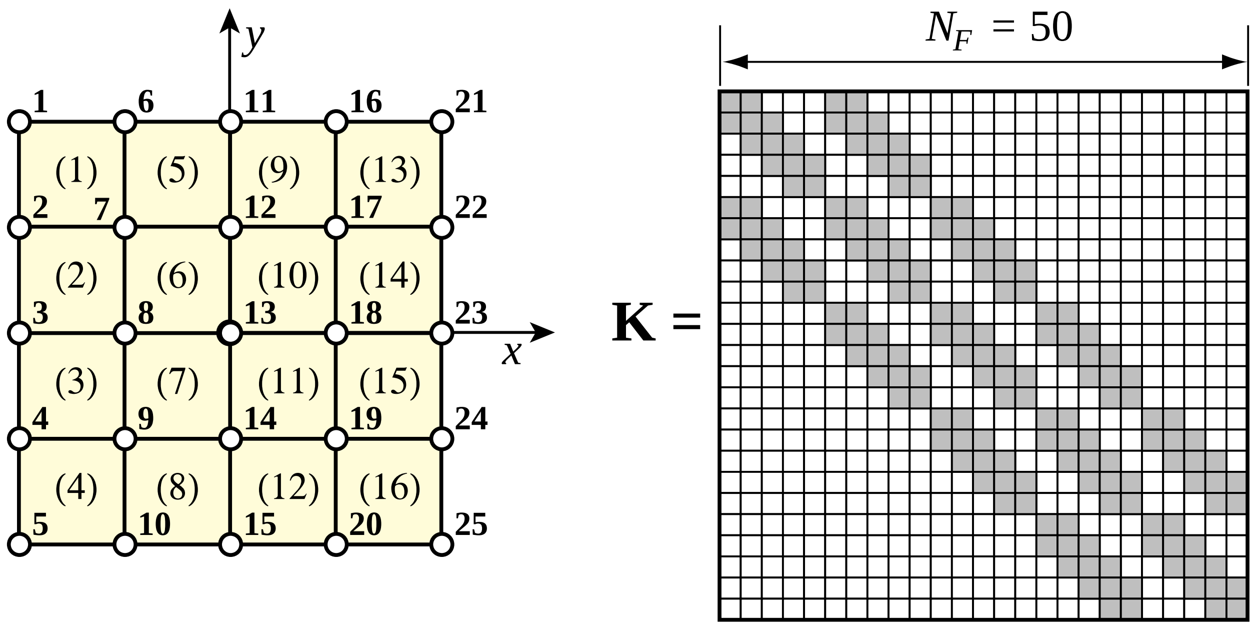
Computer Resources Req'd by FEM Solver

Storage and Solution Times for a Fully Stored Stiffness Matrix

Matrix order *N*	Storage (double prec)	Factor op. units	Factor time workstation/PC	Factor time supercomputer
10^4	800 MB	$10^{12}/6$	3 hrs	2 min
10^5	80 GB	$10^{15}/6$	4 mos	30 hrs
10^6	8 TB	$10^{18}/6$	300 yrs	3 yrs

time numbers last adjusted in 1998
to get current times divide by 10-20

Typical Stiffness Matrix Sparsity Pattern



Computer Resources Req'd by FEM Solver

Storage and Solution Times for a Skyline Stiffness Matrix
Assuming $B = \sqrt{N}$

Matrix order N	Storage (double prec)	Factor op. units	Factor time workstation/PC	Factor time supercomputer
10^4	8 MB	$10^8/2$	5 sec	0.05 sec
10^5	240 MB	$10^{10}/2$	8 min	5 sec
10^6	8 GB	$10^{12}/2$	15 hrs	8 min

time numbers last adjusted in 1998
to get current times divide by 10-20

What We Will Cover Today

How the Master Stiffness Equations are stored in a commonly used "skyline" sparse format

How to Mark BC on the Master Stiffness Eqs (if you write your own solver)

The Basic Solution Steps

Note: Implementation Details will be Skipped since Built-in Mathematica Solver will be used for Demo Programs

Skyline Storage
(aka Profile or Variable-Band Storage)

$$\mathbf{K} = \begin{bmatrix} 11 & & 13 & & & 16 \\ & 22 & 0 & 24 & & 0 \\ & & 33 & 34 & & 0 \\ & & & 44 & & 46 \\ & & & & 55 & 56 \\ & \textit{symm} & & & & 66 \end{bmatrix}$$

Skyline "template"

Mathematica **representation**

p = { 0, 1, 2, 5, 8, 15};
s = {11, 22, 13, 0, 33, 24, 34, 44, 55, 16, 0, 0, 46, 66};
S = { p, s};

Availability of Sparse Solvers in High Level languages

Matlab

Excellent sparse solvers, well tested over two decades

Limitations:

- **RAM bound, virtual memory paging can kill runs for large systems**
- **no parallel solvers**

Mathematica

Sparse solver part of system since version 5.0
Claim to take advantage of multiple processors on a chip

Limited experienc so far, needs "shaking"

Marking Displacement BCs

Equations for which the displacement component is known or prescribed are identified by a ***negative*** diagonal location value. For example, if u_3 and u_5 are prescribed displacement components in the example system,

$$\texttt{p}: \quad [\,0,\ 1,\ 2,\ -5,\ 8,\ -9,\ 15\,]$$

Solution Steps for K u = f

Factorization

$$\mathbf{K} = \mathbf{L}\,\mathbf{D}\,\mathbf{U} = \mathbf{L}\,\mathbf{D}\,\mathbf{L}^{T}$$

Solution

Forward reduction:	***solve***	$\mathbf{L}\,\mathbf{z} = \mathbf{f}$	***for***	$\mathbf{z}$
Diagonal Scaling:	***solve***	$\mathbf{D}\,\mathbf{y} = \mathbf{z}$	***for***	$\mathbf{y}$
Back substitution:	***solve***	$\mathbf{U}\,\mathbf{u} = \mathbf{y}$	***for***	$\mathbf{u}$

SkySolver Implementation (sketched only, Notes give details)

```
S = { p, s }
```

$$\mathbf{K} = \begin{bmatrix} 11 & & 13 & & & 16 \\ & 22 & 0 & 24 & & 0 \\ & & 33 & 34 & & 0 \\ & & & 44 & & 46 \\ & & & & 55 & 56 \\ symm & & & & & 66 \end{bmatrix}$$

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