

Homework Exercises for Chapter 14 - The Plane Stress Problem Solutions

EXERCISE 14.1 Here is the solution to go from plane strain to plane stress:

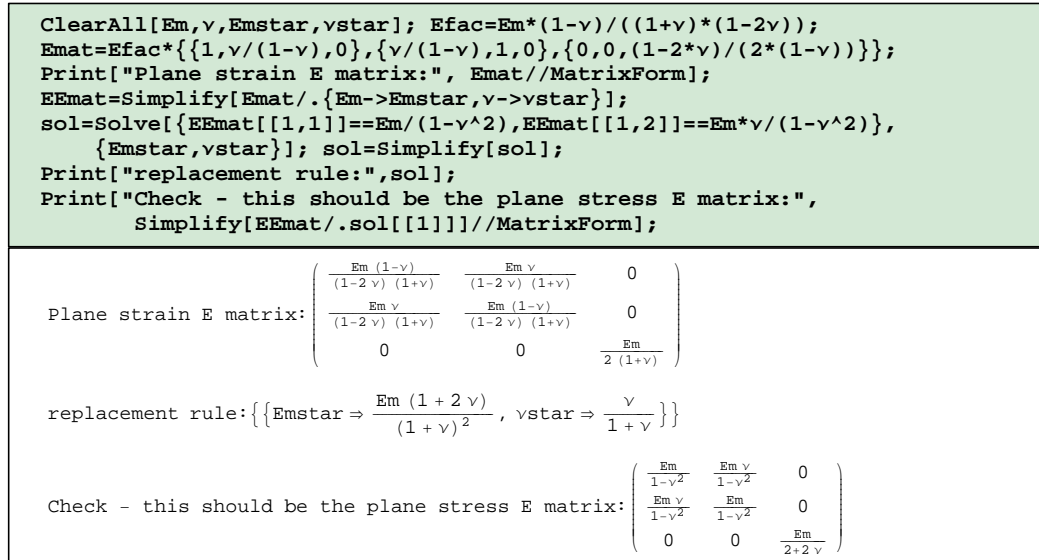


FIGURE E14.2. Solution for Exercise 14.1. Plane strain to plane stress.

It gives $E^* = E \frac{(1+2\nu)}{(1+\nu)^2}$ and $\nu^* = \frac{\nu}{1+\nu}$.

Here is to go from plane stress to plane strain:

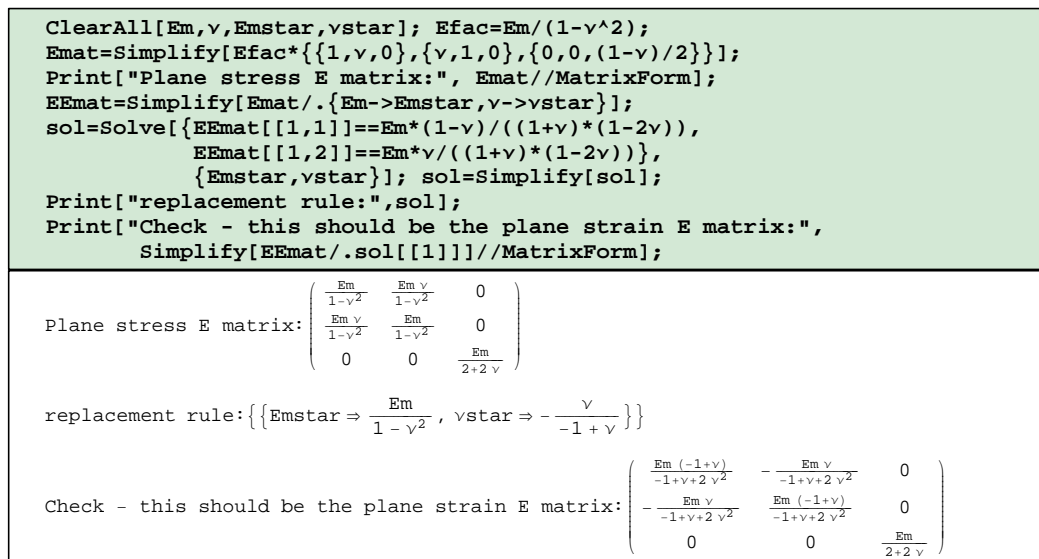


FIGURE E14.3. Solution for Exercise 14.1. Plane stress to plane strain.

It gives $E^* = \frac{E}{1-\nu^2}$ and $\nu^* = \frac{\nu}{1-\nu}$. Credit is given for doing it either way.

EXERCISE 14.2 For plane strain:

$$\begin{aligned} \mathbf{E} &= \begin{bmatrix} \lambda + 2\mu & \lambda & 0 \\ \lambda & \lambda + 2\mu & 0 \\ 0 & 0 & \mu \end{bmatrix} = \mu \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} + \lambda \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \\ &= \frac{E}{2(1+\nu)} \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} + \frac{E\nu}{(1+\nu)(1-2\nu)} \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \mathbf{E}_\mu + \mathbf{E}_\lambda. \end{aligned} \quad (\text{E14.10})$$

For plane stress:

$$\begin{aligned} \mathbf{E} &= \begin{bmatrix} \frac{4\mu(\lambda + \mu)}{\lambda + 2\mu} & \frac{2\lambda\mu}{\lambda + 2\mu} & 0 \\ \frac{2\lambda\mu}{\lambda + 2\mu} & \frac{4\mu(\lambda + \mu)}{\lambda + 2\mu} & 0 \\ 0 & 0 & \mu \end{bmatrix} = \begin{bmatrix} 2\mu - \bar{\lambda} & 0 & 0 \\ 0 & 2\mu - \bar{\lambda} & 0 \\ 0 & 0 & \mu \end{bmatrix} + \bar{\lambda} \begin{bmatrix} 2 & 1 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix} \\ &= \mu \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} + \bar{\lambda} \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \frac{E}{2(1+\nu)} \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} + \frac{E\nu}{1-\nu^2} \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \mathbf{E}_\mu + \mathbf{E}_\lambda. \end{aligned} \quad (\text{E14.11})$$

(The second step above is actually unnecessary, one could go directly to the third expression.) Here $\bar{\lambda} = 2\lambda\mu/(\lambda + 2\mu) = E\nu/(1 - \nu^2)$ is a modified Lamé constant; the inverse relation being $\lambda = 2\bar{\lambda}\mu/(2\mu - \bar{\lambda})$.

EXERCISE 14.3 A solution using *Mathematica* is shown in Figure E14.4. In this script `gxy` stands for $2e_{xy}$.

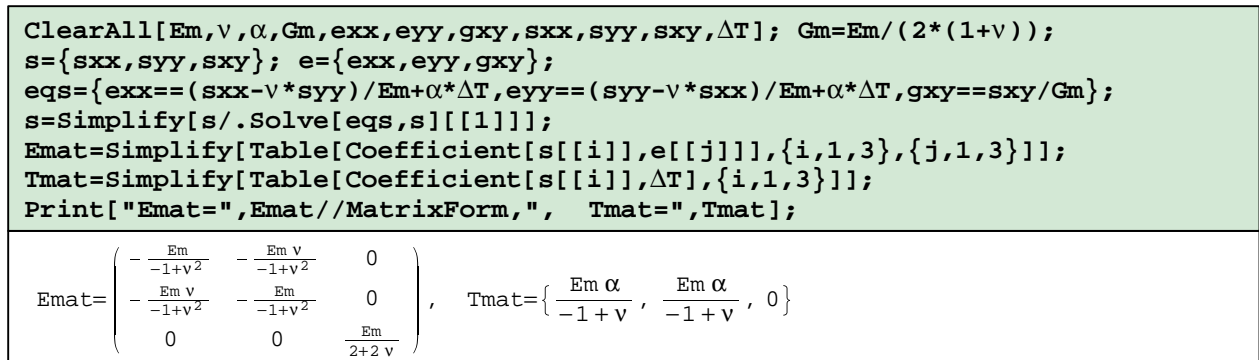


FIGURE E14.4. Solution for Exercise 14.3.

Transcribing the answer:

$$\begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{bmatrix} = \frac{E}{1-\nu^2} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix} \begin{bmatrix} e_{xx} \\ e_{yy} \\ 2e_{xy} \end{bmatrix} - \frac{E\alpha}{1-\nu} \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \Delta T, \quad (\text{E14.12})$$

The only difference with respect to the first of (E14.1) is the thermal stress vector, which vanishes if $\Delta T = 0$. This form may be found in any book on elasticity.

EXERCISE 14.4

Equilibrium along x and y give $t_x ds = \sigma_{xx} dx + \sigma_{yx} dy$ and $t_y ds = \sigma_{xy} dx + \sigma_{yy} dy$, respectively. Dividing through by ds and setting $dx/ds = n_x$, $dy/ds = n_y$, $\sigma_{yx} = \sigma_{xy}$, yields (14.10).

EXERCISE 14.5

The verification of (a) and (b) is immediate on expanding the quadratic forms. Item (c) is more difficult. A brute force solution using *Mathematica* for an arbitrary material matrix \mathbf{E} is shown in Figure E14.5.

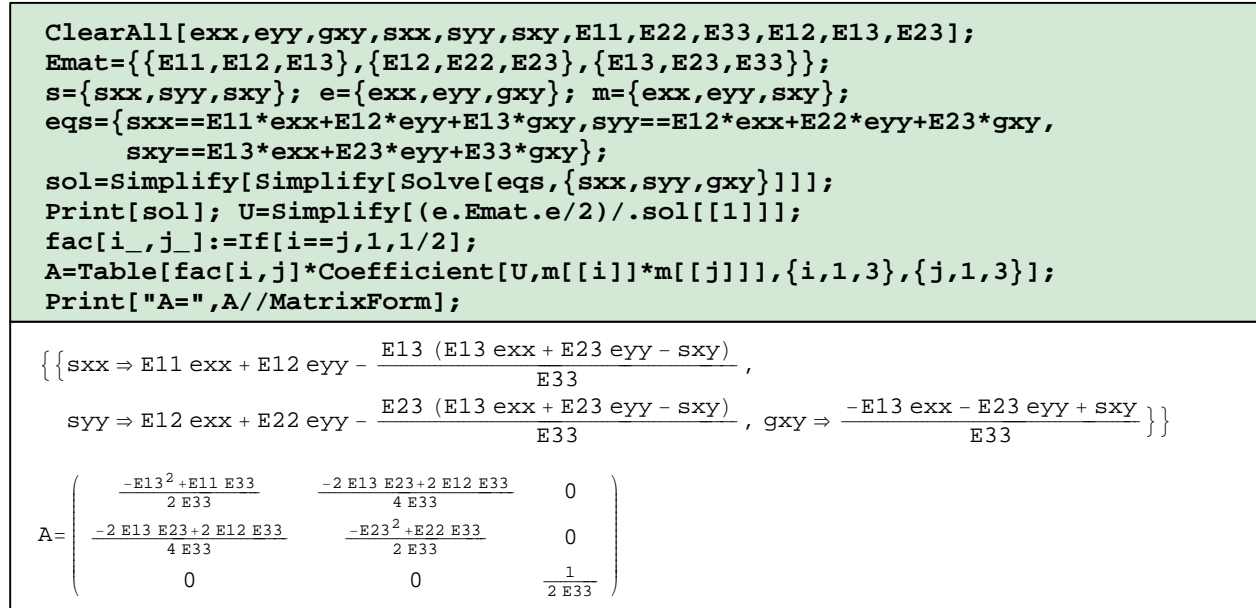


FIGURE E14.5. Solution for Exercise 14.5.

Transcribing the result, the matrix for item (c) is

$$A = \begin{bmatrix} \frac{E_{11} E_{33} - E_{13}^2}{2 E_{33}} & \frac{E_{12} E_{33} - 2 E_{13} E_{23}}{4 E_{33}} & 0 \\ & \frac{E_{22} E_{33} - E_{23}^2}{2 E_{33}} & 0 \\ \text{symm} & & \frac{1}{2 E_{33}} \end{bmatrix} \quad (\text{E14.13})$$