Homework Exercises for Chapter 8. - Solutions

MultiFreedom Constraints I

EXERCISE 8.1 Here are the results of running the *Mathematica* program under version 4.2:

$$\text{Stiffness K}=\begin{pmatrix} 100 & -100 & 0 & 0 & 0 & 0 & 0 \\ -100 & 200 & -100 & 0 & 0 & 0 & 0 \\ 0 & -100 & 200 & -100 & 0 & 0 & 0 \\ 0 & 0 & -100 & 200 & -100 & 0 & 0 \\ 0 & 0 & 0 & -100 & 200 & -100 & 0 \\ 0 & 0 & 0 & 0 & -100 & 200 & -100 \\ 0 & 0 & 0 & 0 & 0 & -100 & 100 \end{pmatrix}$$

$$\text{Applied forces}=\{1, 2, 3, 4, 5, 6, 7\}$$

$$\text{Transformation matrix T}=\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\text{Constraint gap vector g}=\left\{0, 0, 0, 0, 0, -\frac{1}{5}, 0\right\}$$

$$\text{Modified Stiffness upon fixing node 1:}\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 400 & -100 & 0 & -100 & -100 \\ 0 & -100 & 200 & -100 & 0 & 0 \\ 0 & 0 & -100 & 200 & -100 & 0 \\ 0 & -100 & 0 & -100 & 200 & 0 \\ 0 & -100 & 0 & 0 & 0 & 100 \end{pmatrix}$$

$$\text{Modified RHS upon fixing node 1:}\{0, 48, 3, 4, -15, -13\}$$

$$\text{Computed umod (lacks slave u6)}=\left\{0, \frac{27}{100}, \frac{11}{40}, \frac{1}{4}, \frac{37}{200}, \frac{7}{50}\right\}$$

$$\text{Complete solution u}=\left\{0, \frac{27}{100}, \frac{11}{40}, \frac{1}{4}, \frac{37}{200}, \frac{7}{100}, \frac{7}{50}\right\}$$

$$\text{Numerical u}=\{0., 0.27, 0.275, 0.25, 0.185, 0.07, 0.14\}$$

$$\text{Recovered forces K.u with reactions}=\left\{-27, \frac{53}{2}, 3, 4, 5, -\frac{37}{2}, 7\right\}$$

$$\text{Numerical K.u}=\{-27., 26.5, 3., 4., 5., -18.5, 7.\}$$

EXERCISE 8.2

The following *Mathematica* script solves this Exercise:

```
(* Exercise 8.2 - Master-Slave Method *)
(* MFCs: u2-u6=1/5, u3+2u4=-2/3, 2u3-u4+u5=0 - slaves: u4,u5,u6 *)
K=MasterStiffnessOfSixElementBar[100];
Print["Stiffness K=",K//MatrixForm];
f={1,2,3,4,5,6,7}; Print["Applied forces=",f];
T={{1, 0, 0, 0}, {0, 1, 0, 0}, {0, 0, 1, 0}, {0, 0, -1/2, 0},
   {0, 0, -5/2, 0}, {0, 1, 0, 0}, {0, 0, 0, 1}};
Print["Transf matrix T (transposed)=",Transpose[T]//MatrixForm];
g={0, 0, 0, -1/3, -1/3, -1/5, 0};
Print["Constraint gap vector g=",g];
Khat=Simplify[Transpose[T].K.T]; fhat=Simplify[Transpose[T].(f-K.g)];
{Kmod,fmod}=FixLeftEndOfSixElementBar[Khat,fhat]; (* fix left end *)
Print["Modified Stiffness upon fixing node 1:",Kmod//MatrixForm];
Print["Modified RHS upon fixing node 1:",fmod];
umod=LinearSolve[Kmod,fmod];
Print["Computed umod (lacks slaves)=",umod];
u=T.umod+g; Print["Complete solution u=",u];
Print["Numerical u=",SetPrecision[N[u],5]];
fu=K.u; Print["Recovered forces K.u with reactions=",fu];
Print["Numerical K.u=",SetPrecision[N[fu],5]];
```

Modules `MasterStiffnessOfSixElementBar` and `FixLeftEndOfSixElementBar` are listed in the statement of Exercise 8.1. `T` and `g` were actually computed by the commands shown in the statement of the Exercise, printed in InputForm and fed into the script. But they could have also been computed by hand. Running this code gives

$$\text{Stiffness K=}\begin{pmatrix} 100 & -100 & 0 & 0 & 0 & 0 & 0 \\ -100 & 200 & -100 & 0 & 0 & 0 & 0 \\ 0 & -100 & 200 & -100 & 0 & 0 & 0 \\ 0 & 0 & -100 & 200 & -100 & 0 & 0 \\ 0 & 0 & 0 & -100 & 200 & -100 & 0 \\ 0 & 0 & 0 & 0 & -100 & 200 & -100 \\ 0 & 0 & 0 & 0 & 0 & -100 & 100 \end{pmatrix}$$

$$\text{Applied forces=}\{1, 2, 3, 4, 5, 6, 7\}$$

$$\text{Transf matrix T (transposed)=}\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & -\frac{1}{2} & -\frac{5}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\text{Constraint gap vector g=}\left\{0, 0, 0, -\frac{1}{3}, -\frac{1}{3}, -\frac{1}{5}, 0\right\}$$

$$\text{Modified Stiffness upon fixing node 1:}\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 400 & 150 & -100 \\ 0 & 150 & 1350 & 0 \\ 0 & -100 & 0 & 100 \end{pmatrix}$$

$$\text{Modified RHS upon fixing node 1:}\left\{0, \frac{44}{3}, -\frac{569}{6}, -13\right\}$$

$$\text{Computed umod (lacks slaves)=}\left\{0, \frac{659}{15300}, -\frac{287}{3825}, -\frac{133}{1530}\right\}$$

$$\text{Complete solution u=}\left\{0, \frac{659}{15300}, -\frac{287}{3825}, -\frac{2263}{7650}, -\frac{223}{1530}, -\frac{2401}{15300}, -\frac{133}{1530}\right\}$$

$$\text{Numerical u=}\{0, 0.043072, -0.075033, -0.29582, -0.14575, -0.15693, -0.086928\}$$

$$\text{Recovered forces K.u with reactions=}\left\{-\frac{659}{153}, \frac{274}{17}, \frac{1571}{153}, -\frac{5674}{153}, \frac{2467}{153}, -\frac{138}{17}, 7\right\}$$

$$\text{Numerical K.u=}\{-4.3072, 16.118, 10.268, -37.085, 16.124, -8.1176, 7.0000\}$$

EXERCISE 8.3 Never assigned.

EXERCISE 8.4 Never assigned.

EXERCISE 8.5 Never assigned.

EXERCISE 8.6 Never assigned.

EXERCISE 8.7 If you can solve this one as a hobby, apply for a job with MSC Software.