Homework Exercises for Chapter 7. - Solutions

FEM Modeling: Mesh, Loads and BCs

EXERCISE 7.1 Trouble spots from recipe are: B, F, J, M (entrant corners), N, D, I (concentrated forces). See Figure E7.7 for a physical justification.

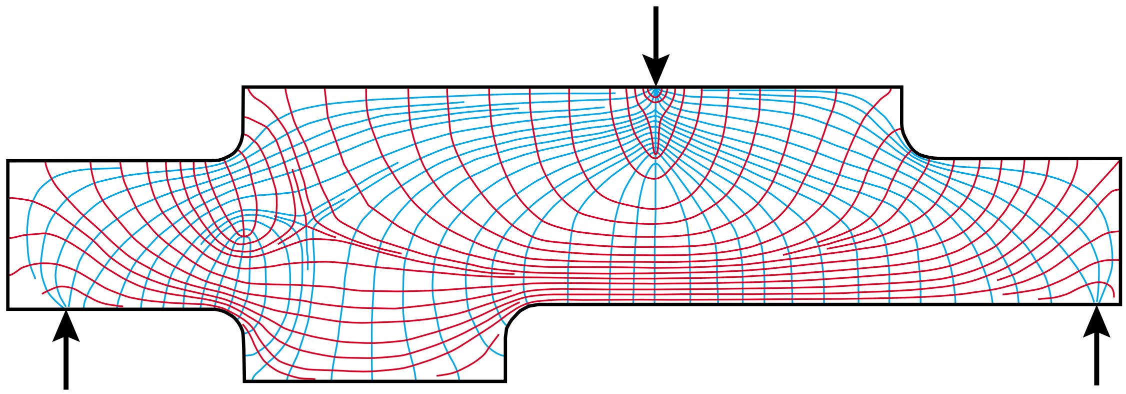

FIGURE E7.7. Figure shows principal stress trajectories or "isostatics." They are drawn in red for tension and blue for compression. (Determined from experimental data gathered in 1964 photoelasticity project at UC Berkeley.) Trajectories "bunch up" in regions of high stress gradients near loads and entrant corners.

EXERCISE 7.2 What perhaps are the two simplest solutions for the transition zone are pictured in Figure E7.8.

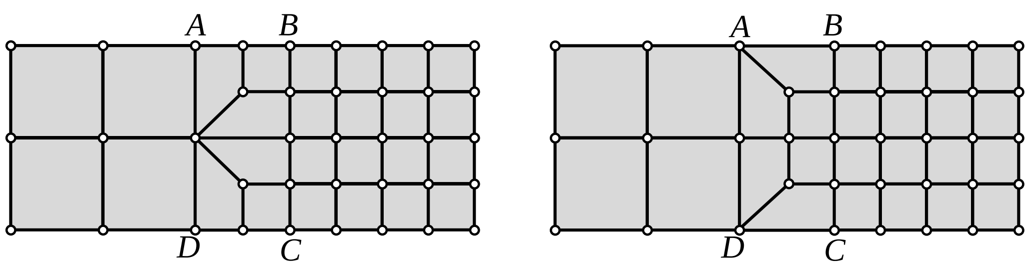

FIGURE E7.8. Two simple transition-mesh solutions for Exercise 7.2.

Several quadrilateral-based "transition meshes" are illustrated in Figure E7.9 for completeness; redrawn from Irons and Ahmad [147]. The solutions of Figure E7.8 are essentially variations of (e) and (f). Solution (d) is interesting in that it produces somewhat better looking element shapes.

EXERCISE 7.3

(a) Counting "tributary squares" of size $a \times a$ rapidly gives the answer: $f_{y1} = f_{y5} = -2a^2\,\gamma\,h$, $f_{y2} = f_{y3} = f_{y4} = -4a^2\,\gamma\,h$, $f_{y6} = f_{y10} = -3a^2\,\gamma\,h$, $f_{y7} = f_{y8} = f_{y9} = -6a^2\,\gamma\,h$, $f_{y11} = f_{y15} = -a^2\,\gamma\,h$, $f_{y12} = f_{y13} = f_{y14} = -2a^2\,\gamma\,h$. Check: $W = \sum_{n=1}^{n=15} f_{yn} = -(2\times 2 + 3\times 4 + 2\times 3 + 3\times 6 + 2\times 1 + 3\times 2)\,a^2\,\gamma\,h = -48\,a^2\,\gamma\,h$.

(b) The results are identical to NbN. When doing it by hand, EbE is slower because NbN can be done by "grid eyeballing."

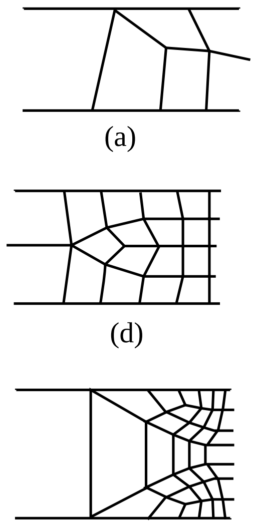

FIGURE E7.9. Quad-only transition meshes for various situations.

EXERCISE 7.4

This Exercise was solved with the *Mathematica* scripts listed in Figure E7.10. Note that the logic of the EbE script is simpler, with less "ifs and buts" than NbN. This is generally true of their computer implementations. Results of running these scripts are collected in the Table 7.1.

Table 7.1. Results for Exercise 7.4 (forces in lbs)

Node	Node by Node (NbN)		Element by Element (EbE)	
	x-force	y-force	x-force	y-force
1	15100.8	0	20134.4	0
2	137779.2	0	139776.0	0
3	296400.0	0	295360.0	0
4	369532.8	0	366912.0	0
5	192067.2	−393120.0	188697.6	−393120.0
6	0	−1179360.0	0	−1179360.0
7	0	−1572480.0	0	−1572480.0
8	0	−1572480.0	0	−1572480.0
9	0	−786240.0	0	−786240.0
Sum	1010880.0	−5503680.0	1010880.0	−5503680.0

The sum of x-forces and y-forces checks out with the total hydrostatic forces $\frac{1}{2}180^2 \times 62.4 = 1010880.0$ lbs and $-490 \times 180 \times 62.4 = -5503680.0$ lbs, respectively.

Exercise 7.4 - Hydrostatic load lumping by NbN method

```
(* Assume face AB is vertical to simplify formulas *)
ClearAll[γ,whead];  γ=62.4; whead=180;
xn={  0,  0, 0, 0,0,-70,-210,-350,-490};
yn={180,136,84,36,0,  0,   0,   0,   0};
fxn=fyn=Table[0,{9}];
For [j=1,j<=9,j++,
     {xi,yi}={xj,yj}={xk,yk}={xn[[j]],yn[[j]]};
     If [j>1, {xi,yi}={xn[[j-1]],yn[[j-1]]} ];
     If [j<9, {xk,yk}={xn[[j+1]],yn[[j+1]]} ];
     {xd1,yd1,xd2,yd2}={xj-xi,yi-yj,xk-xj,yj-yk}/2;
     {di,dj,dk}=whead-{yi,yj,yk};
     If [j<=5, dm=(di+2*dj+dk)/4, dm=dj]; dmx=dmy=dm;
     If [j==5, dmx=(di+3*dj)/4; dmy=dj];
     fxn[[j]]=(yd1+yd2)*dmx*γ; fyn[[j]]=(xd1+xd2)*dmy*γ;
     ];
Print["NbN hydro force lumping results:"];
Print[Table[{i,fxn[[i]],fyn[[i]]},{i,1,9}]//InputForm];
sx=Sum[fxn[[i]],{i,1,9}];  sy=Sum[fyn[[i]],{i,1,9}];
Print["x equil check: ", sx," vs ",whead^2*γ/2];
Print["y equil check: ", sy," vs ",whead*(xn[[9]]-xn[[5]])*γ];
```

Exercise 7.4 - Hydrostatic load lumping by EbE method

```
(* Assume face AB is vertical to simplify hand computations *)
ClearAll[γ,whead];  γ=62.4; whead=180;
xn={  0,  0, 0, 0,0,-70,-210,-350,-490};
yn={180,136,84,36,0,  0,   0,   0,   0}; dn=(whead-yn)*γ;
fxn=fyn=Table[0,{9}];
For [e=1,e<=8,e++, i=e; j=e+1;
     {xi,xj}={xn[[i]],xn[[j]]}; {yi,yj}={yn[[i]],yn[[j]]};
     {di,dj}={dn[[i]],dn[[j]]}; xe=xi-xj; ye=yi-yj;
     fexi= (2*di+dj)*ye/6; fexj= (di+2*dj)*ye/6;
     feyi=-(2*di+dj)*xe/6; feyj=-(di+2*dj)*xe/6;
     fxn[[i]]+=fexi; fxn[[j]]+=fexj; fyn[[i]]+=feyi; fyn[[j]]+=feyj;
     ];
Print["EbE hydro force lumping results:"];
Print[Table[{i,fxn[[i]],fyn[[i]]},{i,1,9}]//TableForm];
sx=Sum[fxn[[i]],{i,1,9}];  sy=Sum[fyn[[i]],{i,1,9}];
Print["x equil check: ", sx," vs ",whead^2*γ/2];
Print["y equil check: ", sy," vs ",whead*(xn[[9]]-xn[[5]])*γ];
```

FIGURE E7.10. Solution of Exercise 7.4 with *Mathematica*.

EXERCISE 7.5 Exercise never assigned.

EXERCISE 7.6 Exercise never assigned.

EXERCISE 7.7 Symmetry and antisymmetry lines are identified on Figure E7.11. Problem domains may be reduced to the darker regions. Appropriate supports to realize these symmetry and antisymmetry conditions as well as actual supports (if given) are depicted in Figure E7.11.

EXERCISE 7.8 (a) From the beak to ground one "worm-bar" element is enough. Smaller elements inside the ground, specially near G, to capture high force gradient. May increase in size as end E is approached.

(b) Pull force as axial load at B. Friction forces, assumed known may be represented by tangentially lumping on worm nodes. Lateral worm displacement precluded by rollers over GE.

(c) A 2D or 3D continuum-element solid model representing skin and insides.

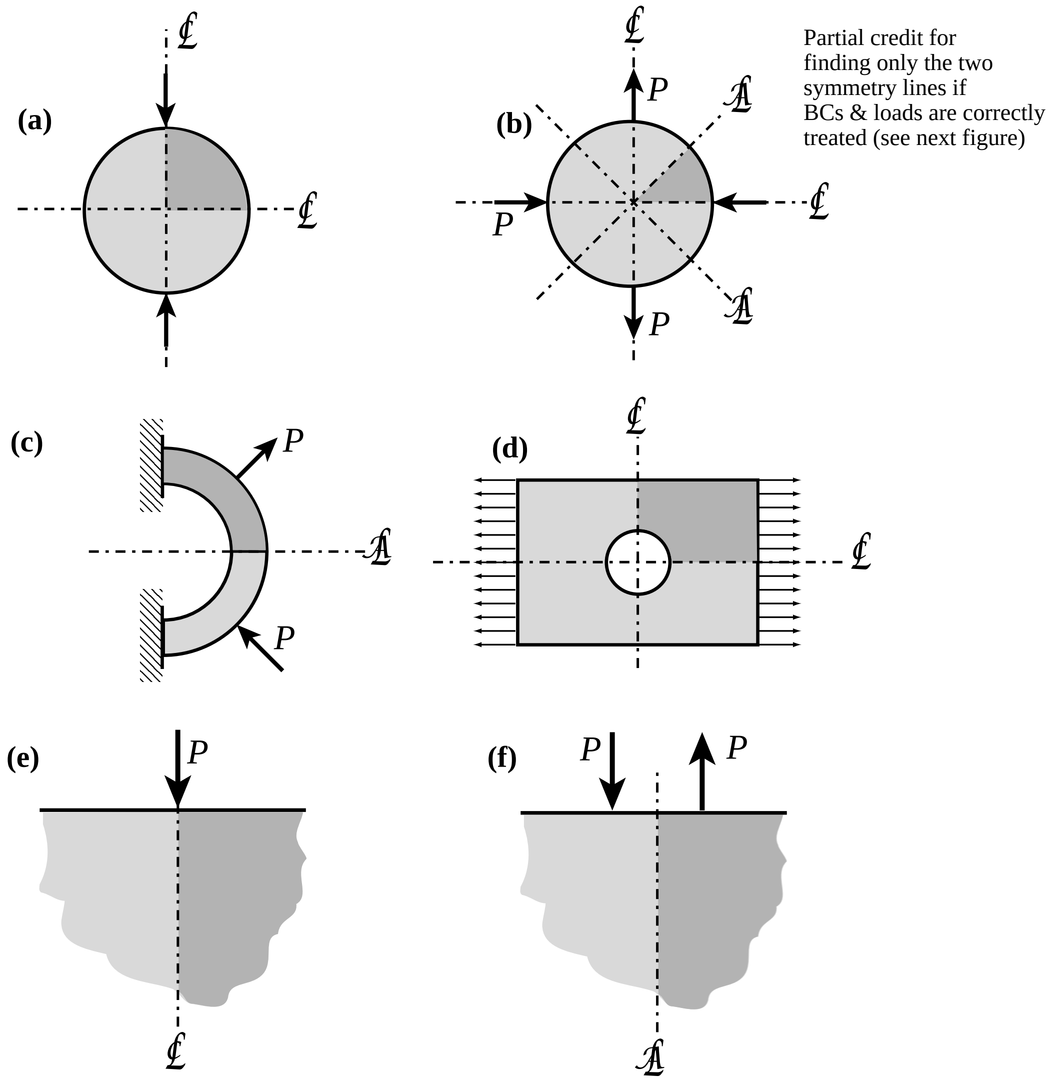

FIGURE E7.11. Symmetry and antisymmetry lines in problems of Exercise 7.4.

(d) A 2D or 3D continuum-element model for soil. No infinite elements are needed because the problem is highly localized (a worm is not a dam). Friction may be represented by special nonlinear elements on the worm surface, but that is an advanced topic.

EXERCISE 7.9 Exercise never assigned.

EXERCISE 7.10 Exercise never assigned.

EXERCISE 7.11 Exercise never assigned.

EXERCISE 7.12 Exercise never assigned.

EXERCISE 7.13 Exercise never assigned.

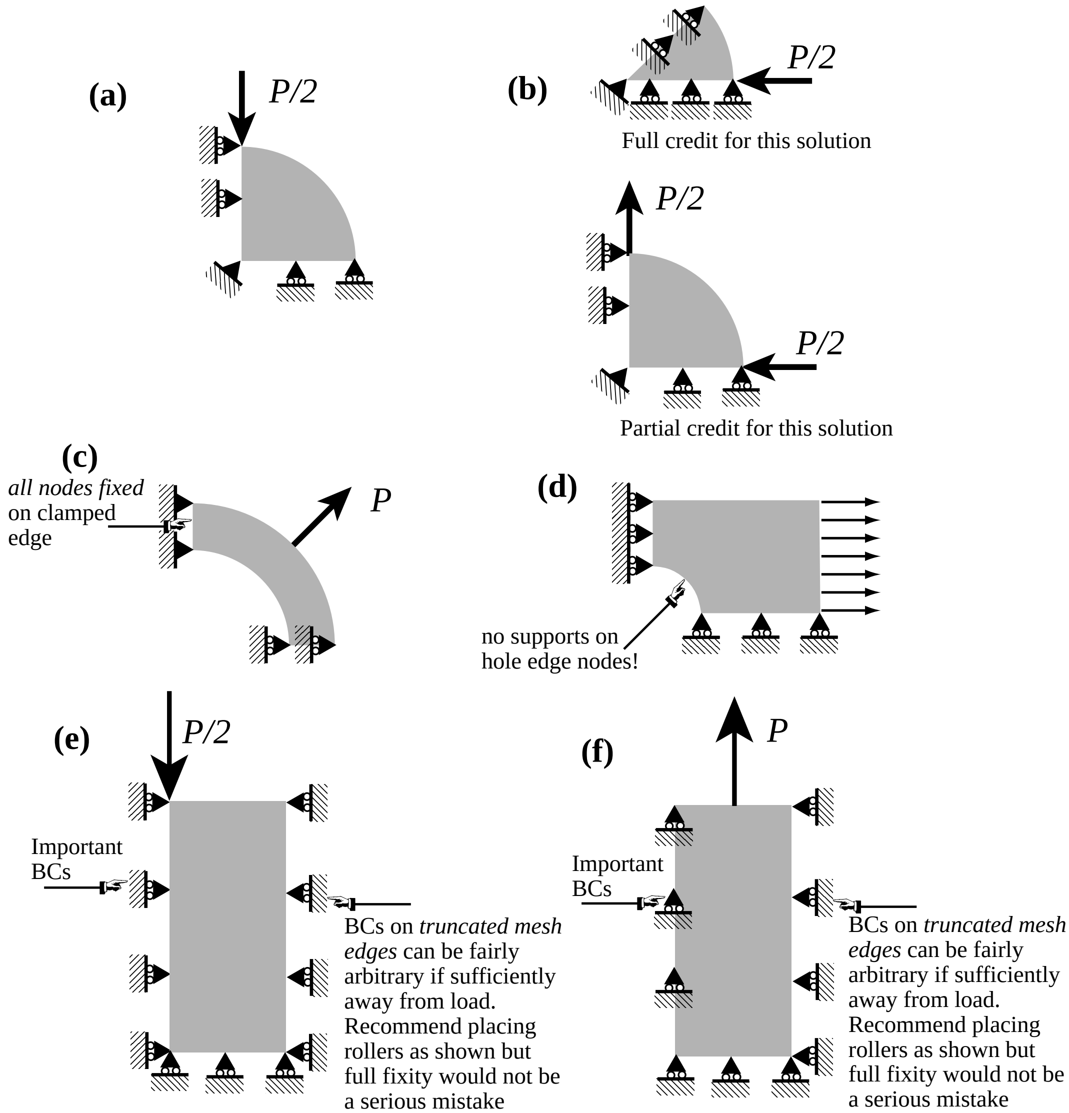

FIGURE E7.12. Representation of the kinematic boundary conditions on the reduced regions (darker areas) of the previous figure. FEM meshes not explicitly shown since the only important answers is the treatment of boundary conditions and loads.

EXERCISE 7.14 Exercise never assigned.

EXERCISE 7.15 Exercise never assigned.

EXERCISE 7.16 $n(n+1)/2$.