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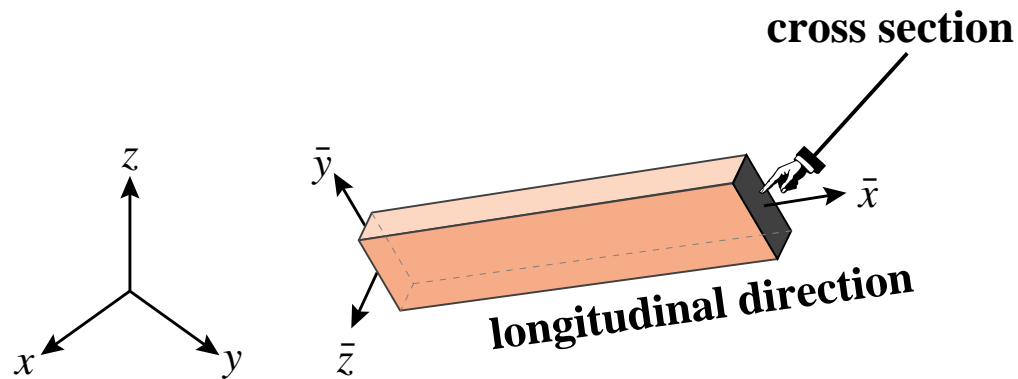
Constructing MoM Members

What Are MoM Members?

Skeletal structural members whose stiffness equations can be constructed by **Mechanics of Materials (MoM) methods**

Can be **locally** modeled as **1D elements**

MoM Members Tend to Look Alike ...

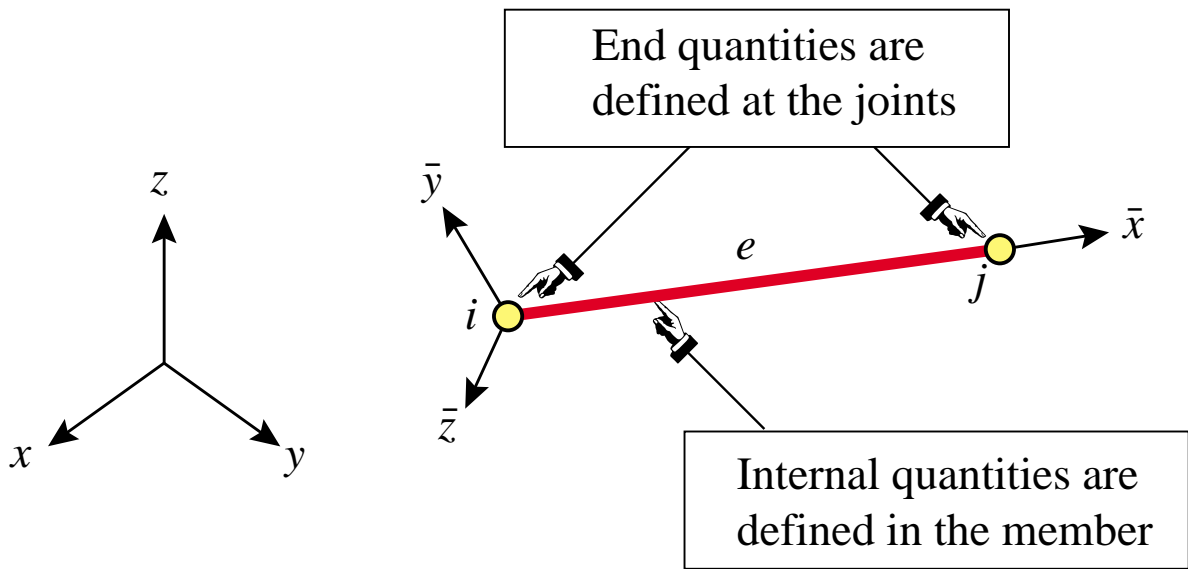


One dimension (**longitudinal**) much larger than the other two (**transverse**)

But Receive Different Names According to Structural Function

Bars:	transmit axial forces
Beams:	transmit bending
Shafts:	transmit torque
Spars (aka Webs):	transmit shear
Beam-columns:	transmit bending + axial force

Common Features of MoM Finite Element Models



Governing Matrix Equations for Simplex MoM Element

From node displacements to internal deformations (strains)

$$\mathbf{v} = \mathbf{B} \bar{\mathbf{u}}$$

Kinematic

From deformations to internal forces

$$\mathbf{p} = \mathbf{S} \mathbf{v}$$

Constitutive

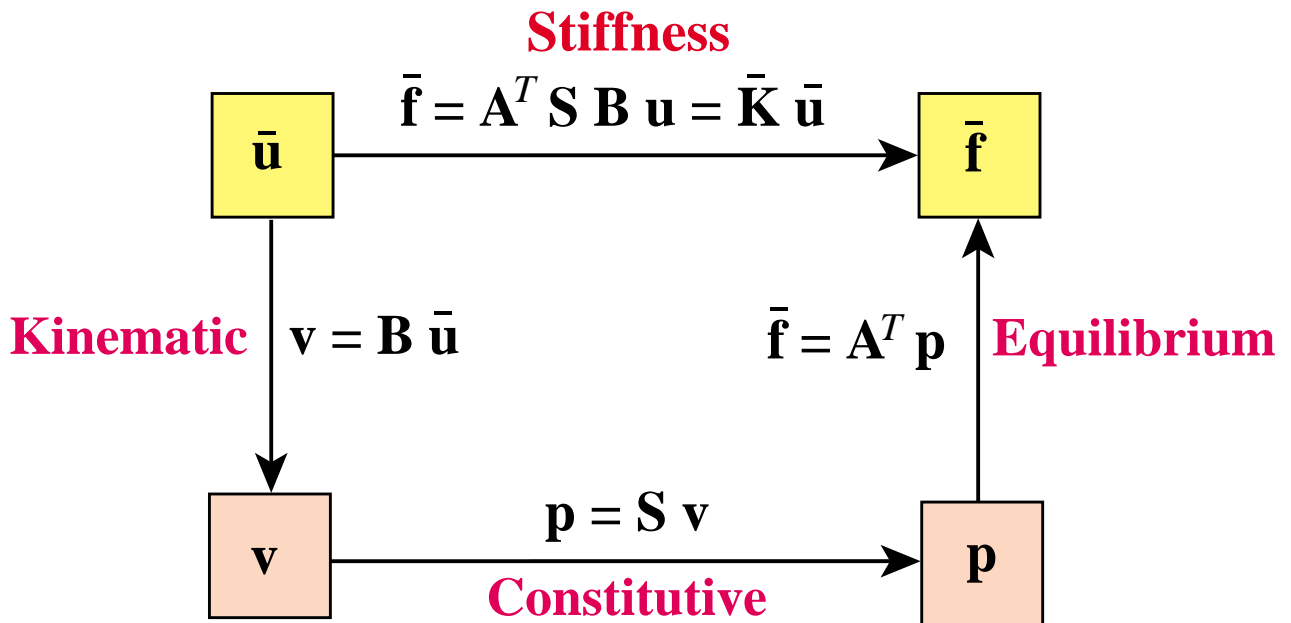
From internal forces to node forces

$$\bar{\mathbf{f}} = \mathbf{A} \mathbf{p}$$

Equilibrium

If $\bar{\mathbf{f}}$ and $\bar{\mathbf{u}}$ are PVW (Virtual Work) conjugate, $\mathbf{B} = \mathbf{A}$

Tonti Diagram of Governing Matrix Equations for Simplex MoM Element



Elimination of the Internal Quantities \mathbf{v} and \mathbf{p}
gives the Element Stiffness Equations
through Simple Matrix Multiplications

$$\bar{\mathbf{f}} = \mathbf{A}^T \mathbf{S} \mathbf{B} \bar{\mathbf{u}} = \bar{\mathbf{K}} \bar{\mathbf{u}}$$



$$\bar{\mathbf{K}} = \mathbf{A}^T \mathbf{S} \mathbf{B}$$

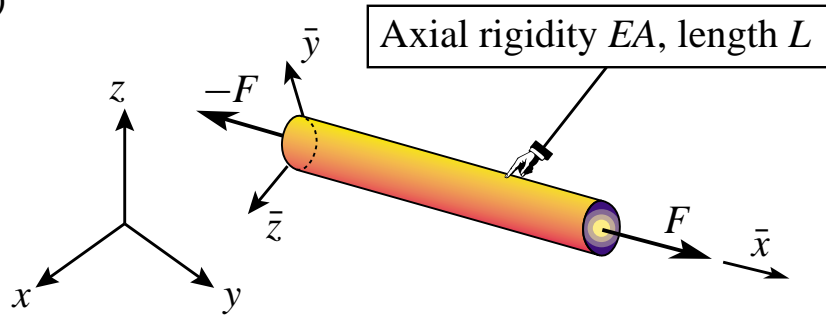
If $\mathbf{B} = \mathbf{A}$ ↓

$$\bar{\mathbf{K}} = \mathbf{B}^T \mathbf{S} \mathbf{B}$$

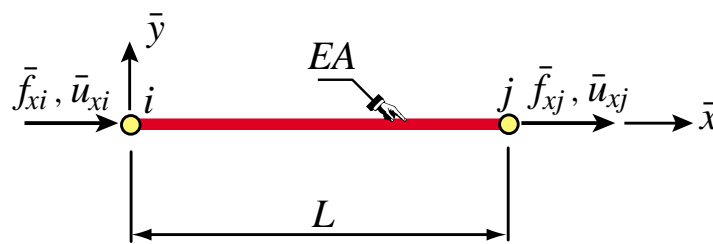
symmetric if \mathbf{S} is

The Bar Element Revisited

(a)



(b)



The Bar Element Revisited (cont'd)

$$d = \begin{bmatrix} -1 & 1 \end{bmatrix} \begin{bmatrix} \bar{u}_{xi} \\ \bar{u}_{xj} \end{bmatrix} = \mathbf{B}\bar{\mathbf{u}}$$

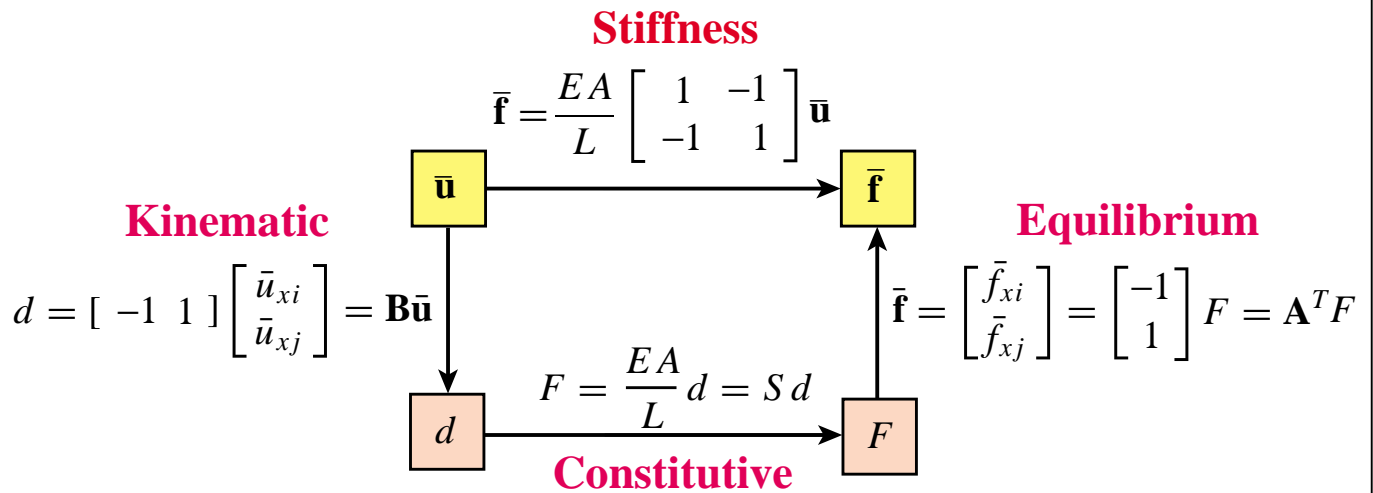
$$F = \frac{EA}{L}d = Sd,$$

$$\bar{\mathbf{f}} = \begin{bmatrix} \bar{f}_{xi} \\ \bar{f}_{xj} \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \end{bmatrix} F = \mathbf{A}^T F$$

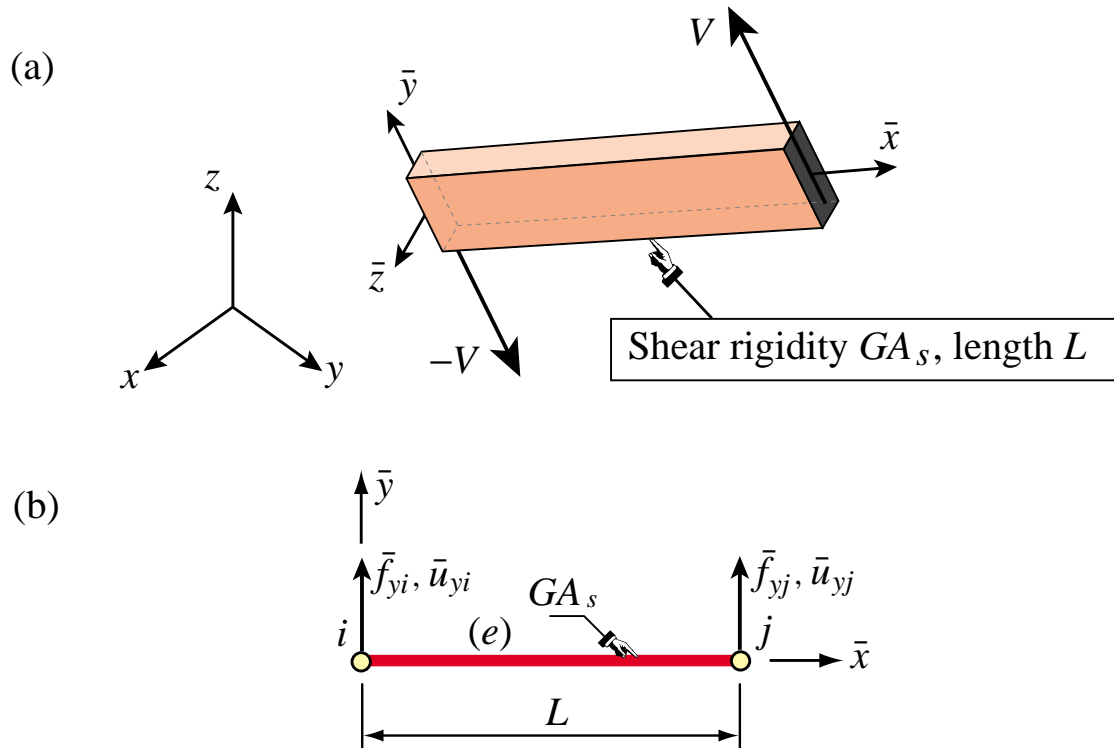
$$\bar{\mathbf{K}} = \mathbf{A}^T S \mathbf{B} = S \mathbf{B}^T \mathbf{B} = \frac{EA}{L} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

Can be expanded to the 4 x 4 of Chapter 2 by adding two zero rows and columns to accommodate \bar{u}_{yi} and \bar{u}_{yj}

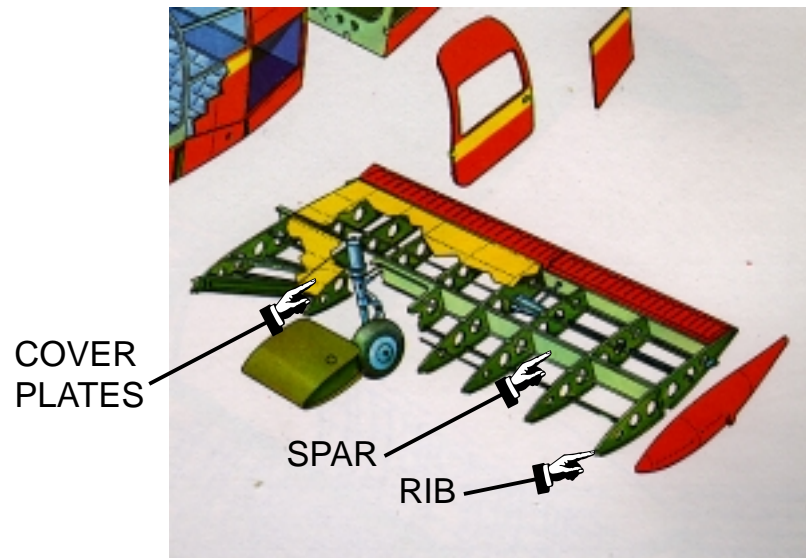
Discrete Tonti Diagram for Bar Element



The Spar (a.k.a. Shear-Web) Element



Spars used in Wing Structure (Piper Cherokee)



The Spar Element (cont'd)

$$\gamma = \frac{1}{L} \begin{bmatrix} -1 & 1 \end{bmatrix} \begin{bmatrix} \bar{u}_{yi} \\ \bar{u}_{yj} \end{bmatrix} = \mathbf{B}\bar{\mathbf{u}}$$

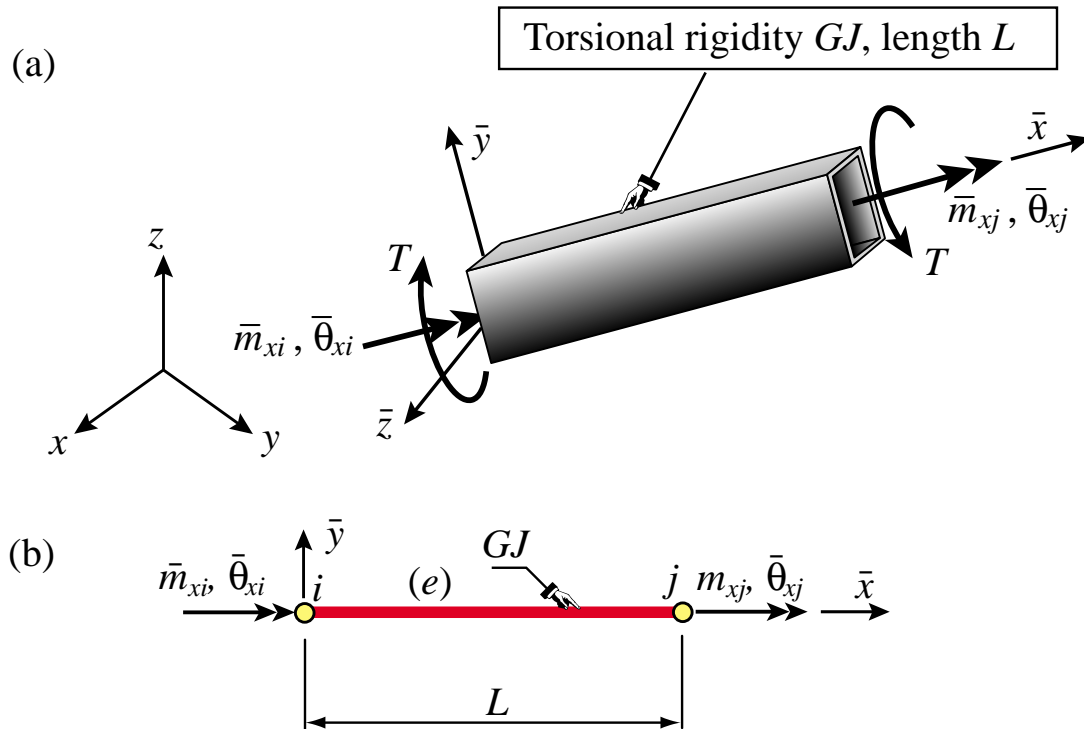
$$V = GA_s \gamma = S \gamma$$

$$\bar{\mathbf{f}} = \begin{bmatrix} \bar{f}_{yi} \\ \bar{f}_{yj} \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \end{bmatrix} V = \mathbf{A}^T V$$

$$\bar{\mathbf{f}} = \begin{bmatrix} \bar{f}_{yi} \\ \bar{f}_{yj} \end{bmatrix} = \mathbf{A}^T S \mathbf{B}\bar{\mathbf{u}} = \frac{GA_s}{L} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} \bar{u}_{yi} \\ \bar{u}_{yj} \end{bmatrix} = \bar{\mathbf{K}}\bar{\mathbf{u}}$$

$$\bar{\mathbf{K}} = \frac{GA_s}{L} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

The Shaft Element



For stiffness derivation details see Notes

Matrix Equations for Non-Simplex MoM Element

From node displacements to internal deformations *at each section*

$$\mathbf{v} = \mathbf{B} \bar{\mathbf{u}}$$

Kinematic

From deformations to internal forces *at each section*

$$\mathbf{p} = \mathbf{R} \mathbf{v}$$

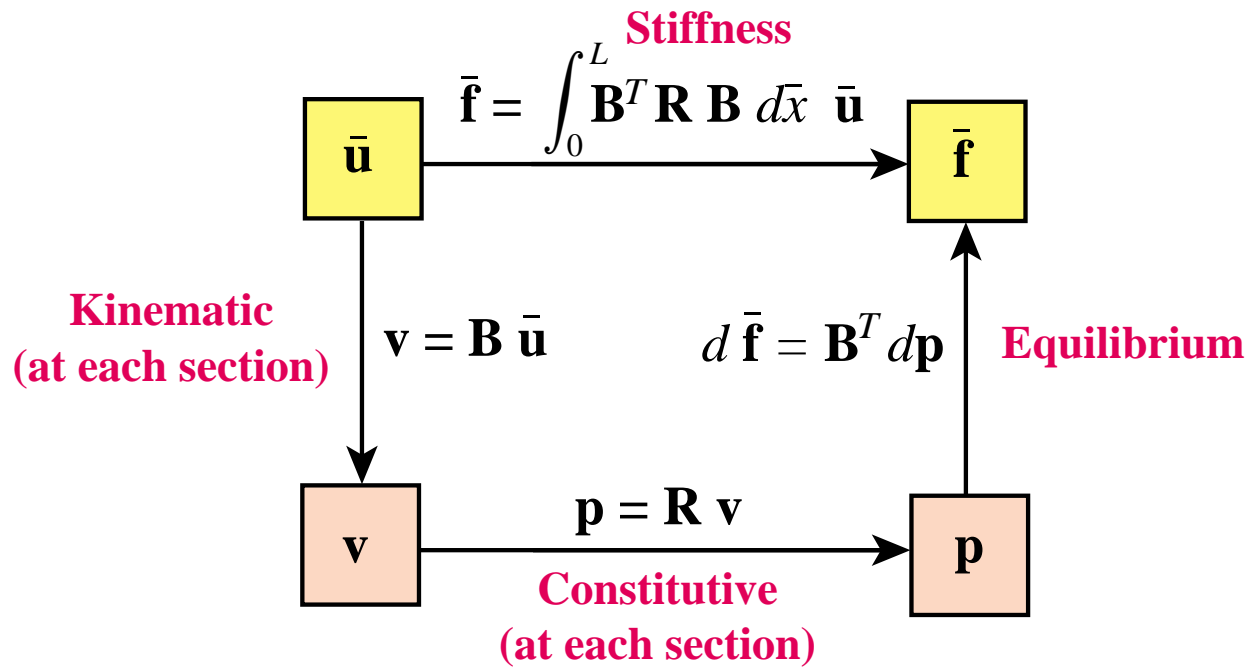
Constitutive

From internal forces to node forces

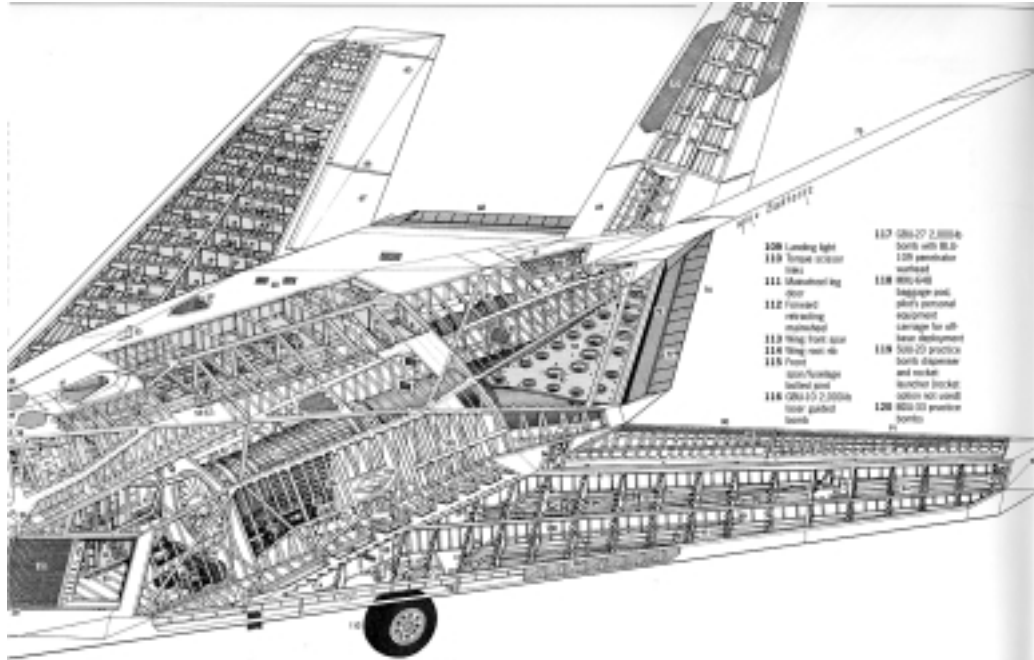
$$d \bar{\mathbf{f}} = \mathbf{A}^T d \mathbf{p}$$

Equilibrium

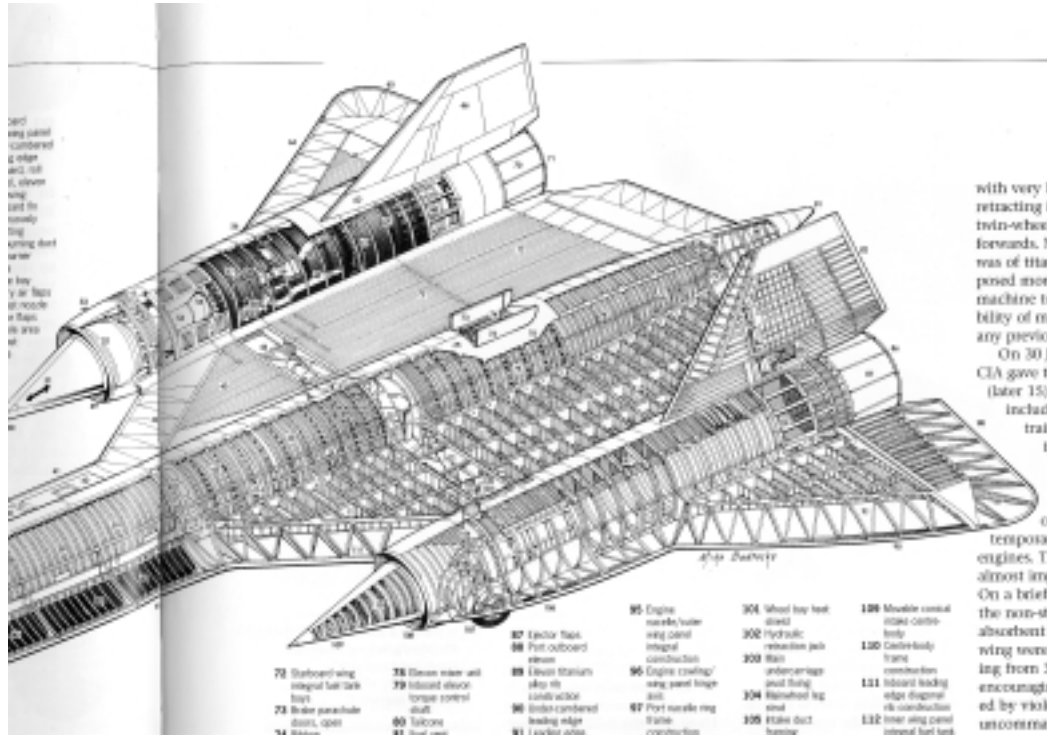
Tonti Diagram of Matrix Equations for Non-Simplex MoM Element (with $A=B$)



Low-Aspect Delta Wing, F-117 (1975)



Low-Aspect Delta Wing, Blackhawk (1972)



Delta Wing Aircraft

