Dynamics of Gyroscopes

Matthew Eby
February 10

Questions: Matthew.Eby@Colorado.edu

Gyroscopes

- Basic Definition: A gyroscope measures angular orientation either directly, or through integrating a measured rotational rate or acceleration.
- Uses:
  - Navigation
  - Control Systems (i.e. anti-skid in cars)
  - Combine with accelerometer to form IMU
- Desirable Qualities:
  - High Sensitivity-Minimum detectable rate.
  - Stability-Over time, or environmental change.
  - Broad Range-Accuracy at both low and high rates.

Types of Gyroscopes

- Classic Spinning Disk
  - Based on gyroscopic effect of a of rotating object with angular momentum.

$$\tau = \frac{dh}{dt}$$

- Vibratory
  - Based on Coriolis force on an object in a rotating reference frame.

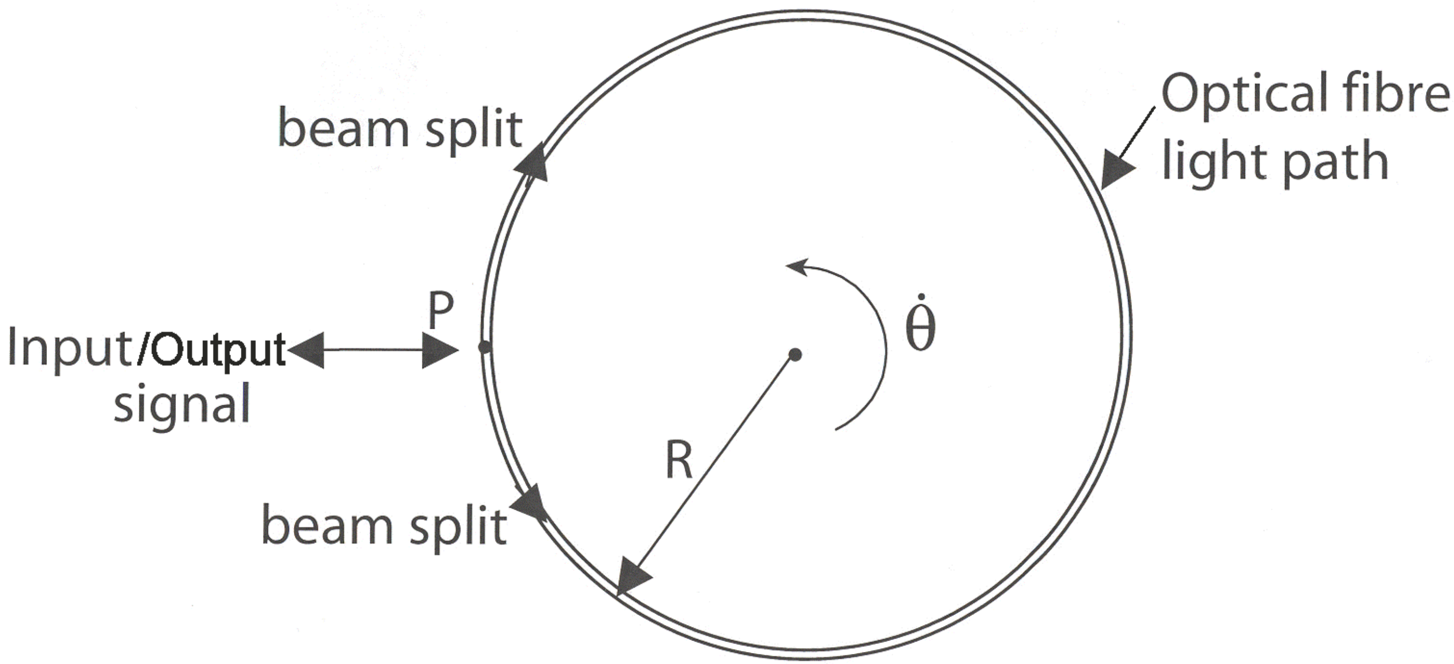
$$F_{coriolis} = 2m\left(\vec{v} \times \vec{U}\right)$$

- Laser (Ring, IFOG)
  - Based on "timing" the travel of light through a course whose length varies with the rotation of the course.

$$Speed\ of\ Light = c = \text{Constant}$$

Interferometric Fiber Optic Gyroscope (IFOG)

- Coherent (laser) light travels in opposite directions around a fiber optic coil.
- Rotation of the coil creates a path difference between the signals.
- Measuring the phase shift between the signals provides a rotation rate measurement.



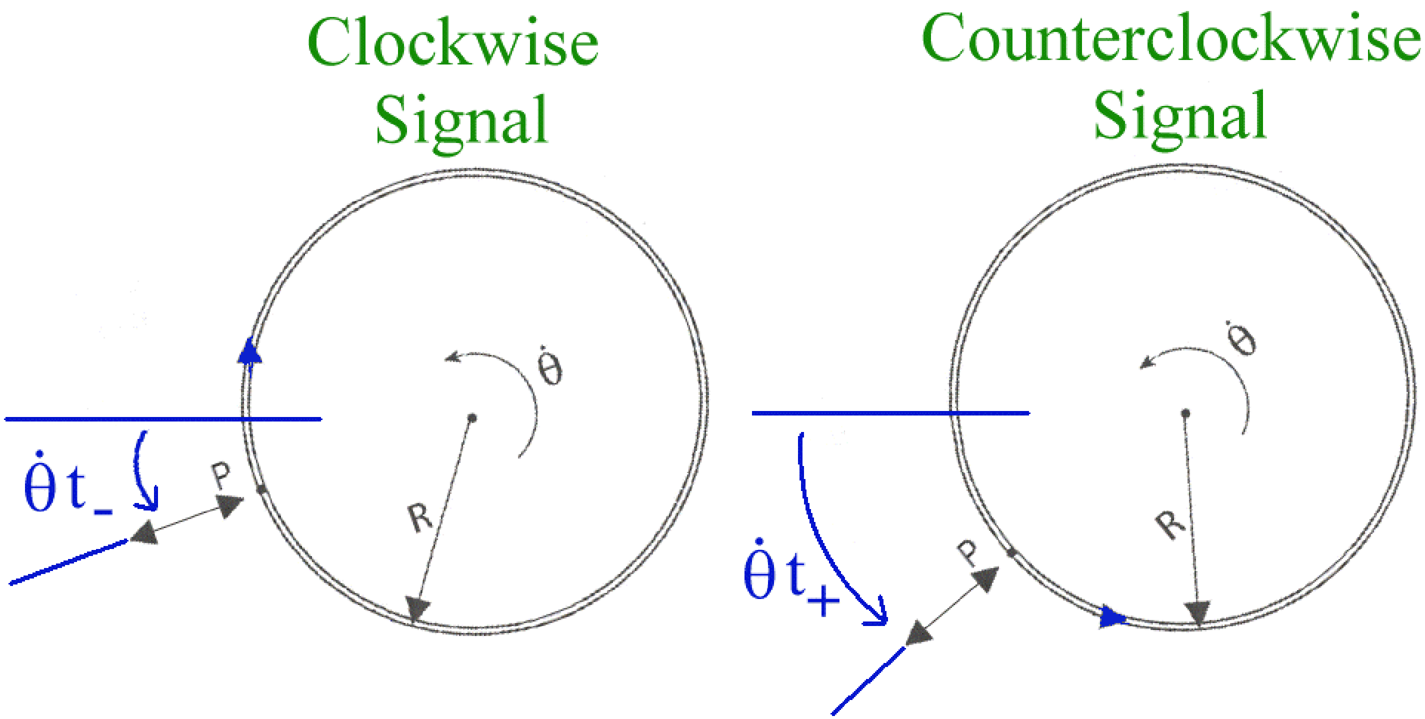
Path Difference

- Distance Traveled by Clockwise Signal:

$$ct_- = 2\pi R - R\dot{\theta}\, t_- \Rightarrow ct_- = \frac{2\pi cR}{c + R\dot{\theta}}$$

- Distance Traveled by Counterclockwise Signal:

$$ct_+ = 2\pi R + R\dot{\theta}\, t_+ \Rightarrow ct_+ = \frac{2\pi cR}{c - R\dot{\theta}}$$



Path Difference

- Take difference of distance traveled by each signal.

$$dL = c(t_+ - t_-) = 2\pi cR\left[\frac{1}{(c - R\dot{\theta})} - \frac{1}{(c + R\dot{\theta})}\right]$$

$$dL = 2\pi cR\left[\frac{(c + R\dot{\theta})}{(c^2 - R^2\dot{\theta}^2)} - \frac{(c - R\dot{\theta})}{(c^2 - R^2\dot{\theta}^2)}\right] = 2\pi cR\left[\frac{2R\dot{\theta}}{(c^2 - R^2\dot{\theta}^2)}\right]$$

$$dL = \frac{4\pi cR^2\dot{\theta}}{(c^2 - R^2\dot{\theta}^2)}$$

$$c >> R^2\dot{\theta}^2$$

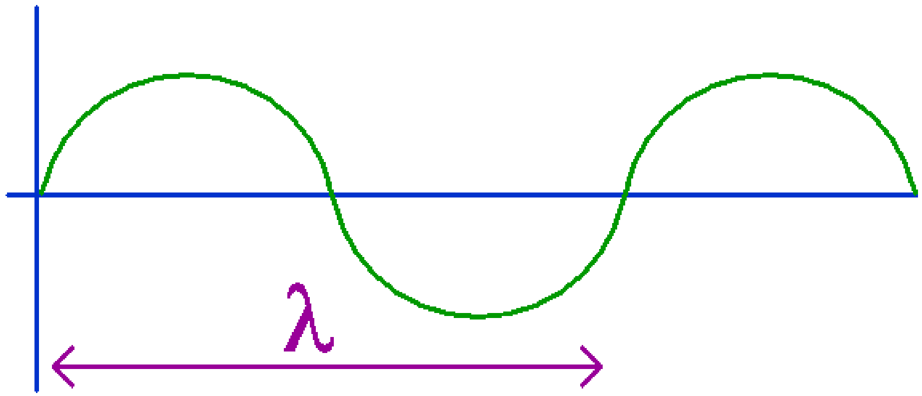
- Path Difference:

$$dL = \frac{4\pi R^2\dot{\theta}}{c}$$

Light Interference

- The coherent light exiting after traveling different distances have a phase difference proportional to rotation rate.

$$dL = \frac{4\pi R^2 \dot{\theta}}{c}$$



$$dL = \lambda \frac{d\phi}{2\pi}$$

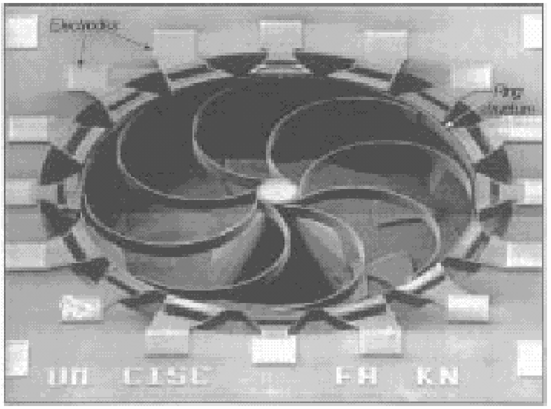
$$\Rightarrow \dot{\theta} = \frac{\lambda c}{8\pi^2 R^2} d\phi$$

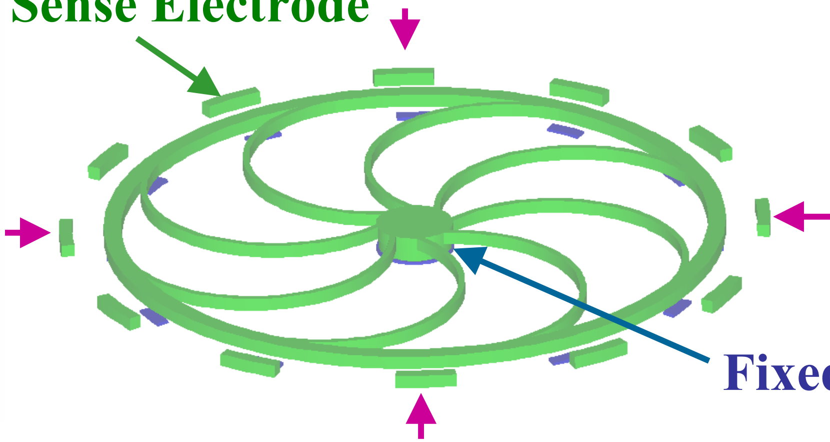
IFOG Summary

- Advantages
  - No mechanical parts
  - Resistant to shock and vibration
  - Long-lived
  - Accurate
  - Commercially Available: First Used in Boeing 777

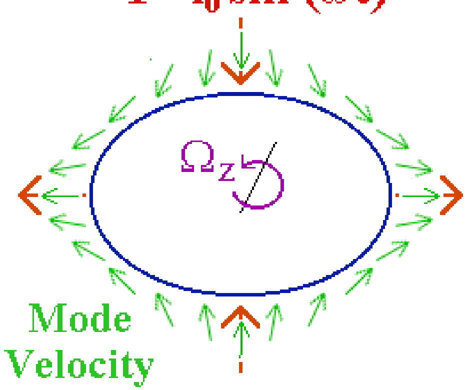
- Disadvantages
  - Speed of light is “fast” and thus requires many loops of fiber optic fiber to create a detectable phase angle.

Coriolis Force Gyroscope

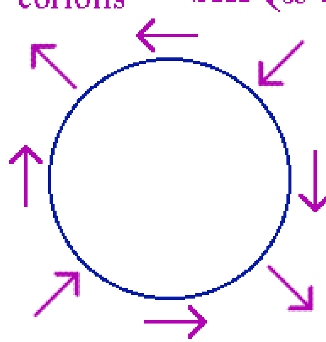




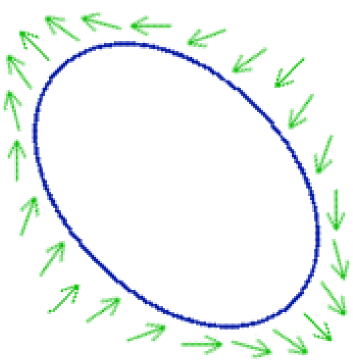
$$F_{coriolis} = 2m\left(\vec{v} \times \vec{U}\right)$$



a.) Excite Primary Mode



b.) Coriolis Force Develops



c.) Excites Secondary Mode

Coriolis Force Gyroscope

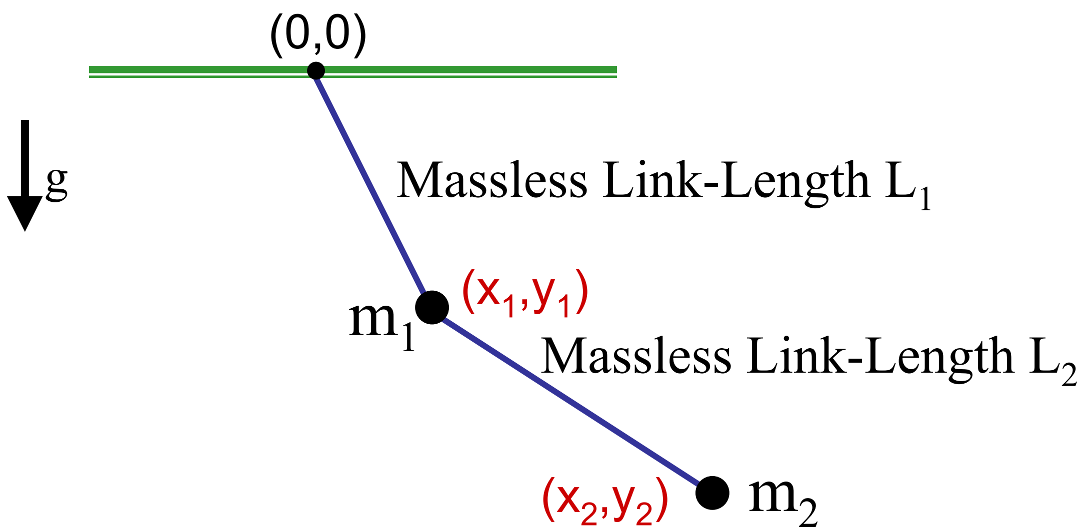
- To characterize the dynamics of a vibratory gyroscope we need to:
  - Create a simple model to account for the principle features.
  - Find generalized coordinates.
  - Calculate system energy as a function of the generalized coordinates.
  - Develop the equations of motion by the Lagrangian method.
  - Evaluate the E.O.M. to determine the system response to an input rotation rate.
- Simple Model: One Axis Gyroscope
  - Vibrating mass with two orthogonal vibration modes (one for forcing and the other for sensing).
  - Rotation axis perpendicular to vibration plane.

Generalized Coordinates Review

- A set of coordinates (q_1, q_2,…) which:
  - Fully describes the position and orientation of the system at any time.
  - Each coordinate is independent from every other coordinate. In other words each coordinate can be set independently without violating the physical system.

$$q_1 \neq f(q_2, q_3, \ldots)$$

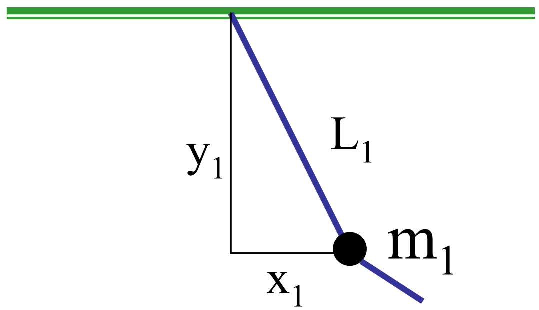
- Example: Find Coordinates for a Compound Pendulum (2 D.O.F.)



- Correct?

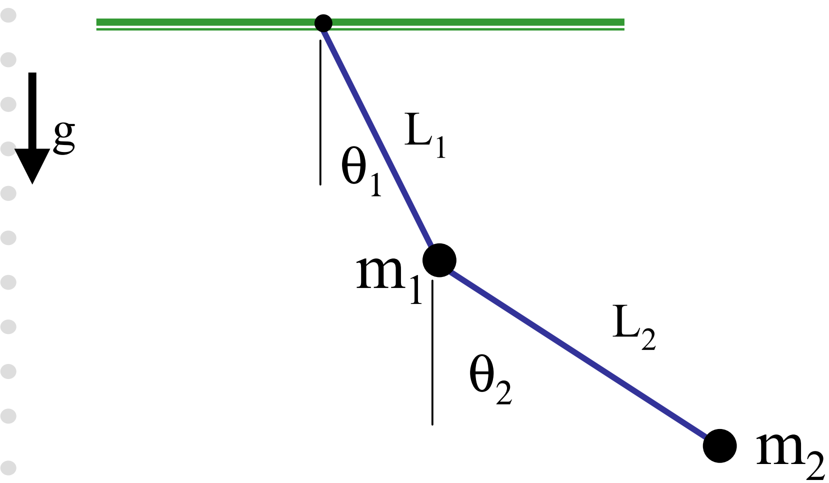
Generalized Coordinates Review

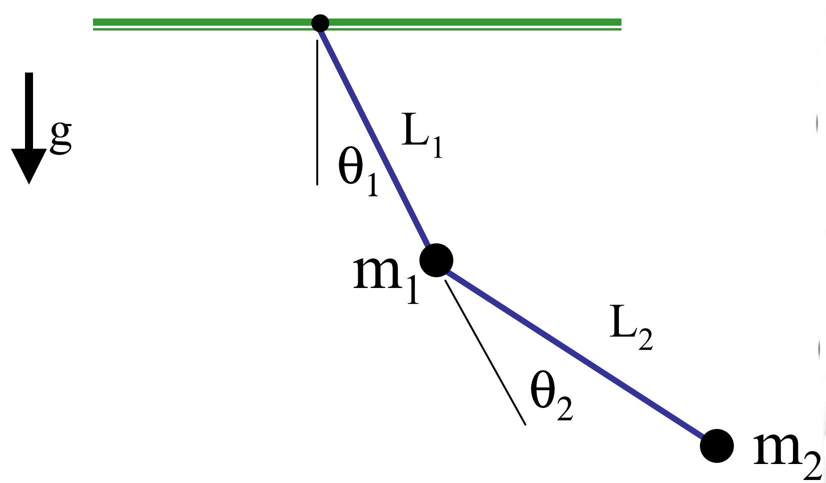
- Fulfills condition 1. but not 2. (Note dependence of coordinates):



$$y_1 = \sqrt{L_1^2 - x_1^2}$$

- Instead Consider Angles (generalized coordinates are often non-unique)

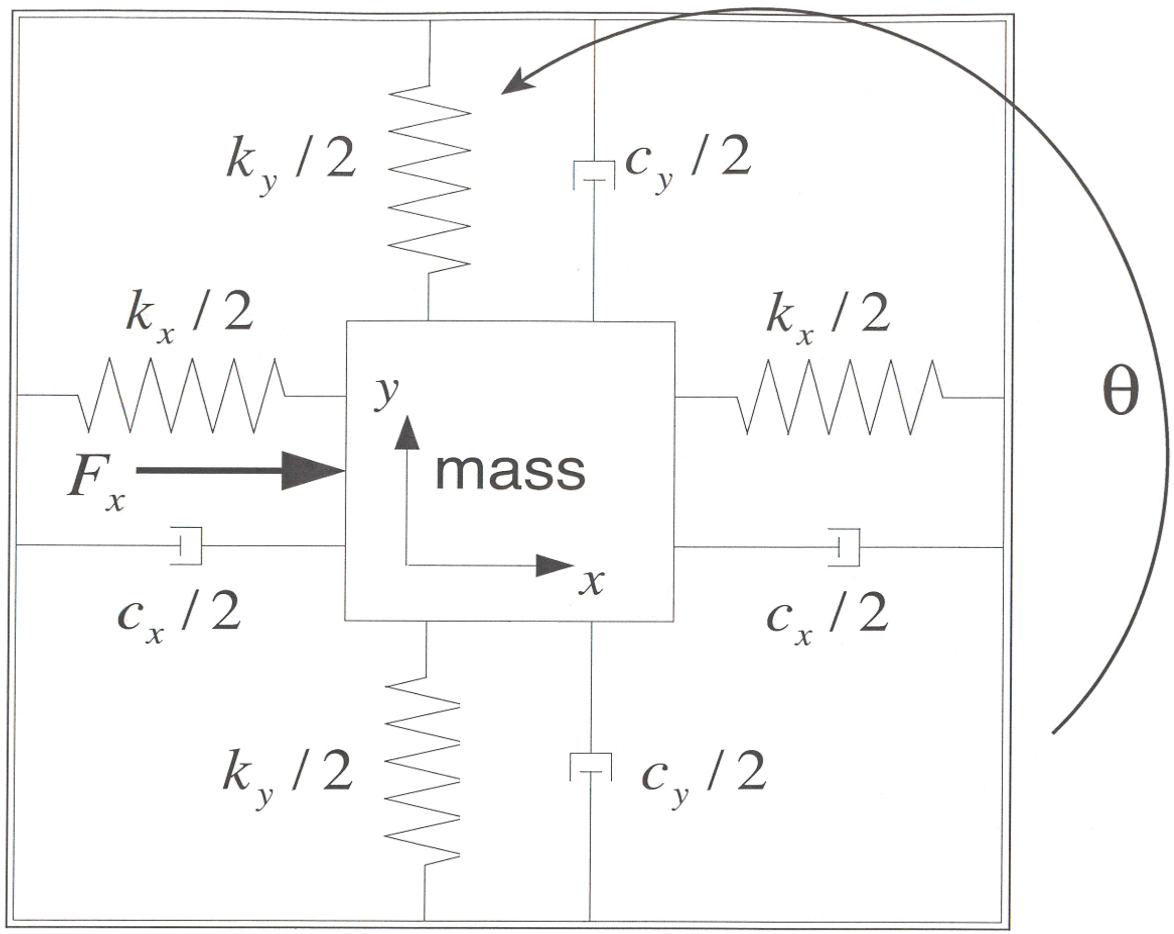




Generalized Coordinates For One Axis Coriolis Gyroscope

- Generalized coordinates θ, x, y with x,y relative to frame.
- Rotation about z-axis, with steady oscillation along x-axis.
- Response develops along y-axis.

Frame →

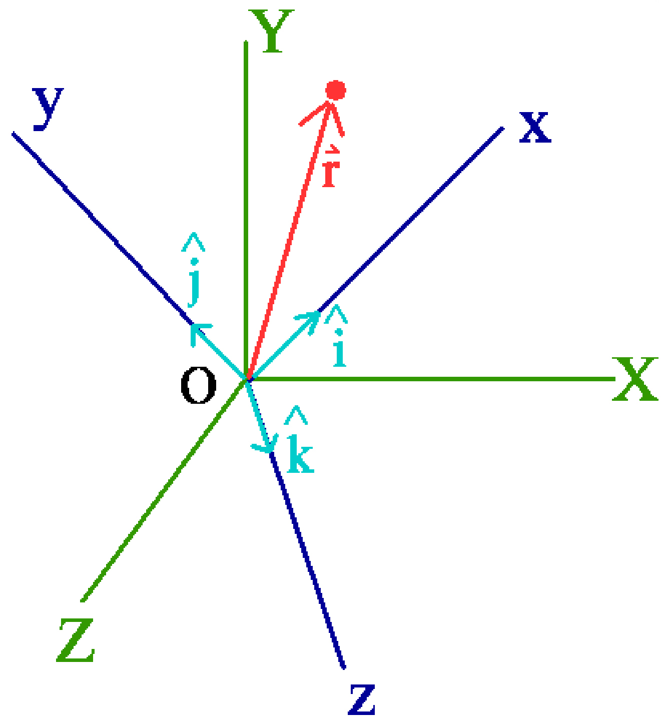


Rotating Reference Frame

- Inertial frame XYZ and rotating frame xyz.
- Position/velocity in rotating reference frame:

$$\vec{r}_{xyz} = r_x\hat{i} + r_y\hat{j} + r_z\hat{k}$$

$$\dot{\vec{r}}_{xyz} = \dot{r}_x\hat{i} + \dot{r}_y\hat{j} + \dot{r}_z\hat{k}$$



- The velocity in the fixed frame can be found by treating the unit vectors as variable:

$$\dot{\vec{r}}_{XYZ} = \dot{r}_x\hat{i} + \dot{r}_y\hat{j} + \dot{r}_z\hat{k} + r_x\dot{\hat{i}} + r_y\dot{\hat{j}} + r_z\dot{\hat{k}}$$

Rotating Reference Frame

$$\dot{\vec{r}}_{XYZ} = \dot{\vec{r}}_{xyz} + r_x\dot{\hat{i}} + r_y\dot{\hat{j}} + r_z\dot{\hat{k}}$$

- Consider r = constant

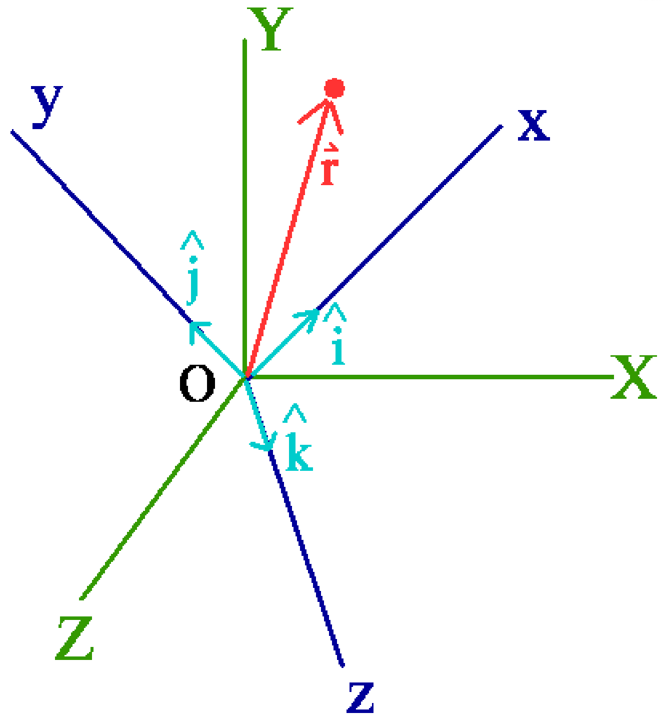
$$\dot{\vec{r}}_{XYZ} = r_x\dot{\hat{i}} + r_y\dot{\hat{j}} + r_z\dot{\hat{k}}$$

- Velocity of Pure Angular Motion:

$$\dot{\vec{r}}_{XYZ} = \Omega \times r_{xyz}$$

$$\Rightarrow \dot{\vec{r}}_{XYZ} = \dot{\vec{r}}_{xyz} + \Omega \times r_{xyz}$$

$$v = v_{rotating} + \Omega \times r_{rotating}$$



System Energy

K.E. and P.E. as a Function of Generalized Coordinates:

$$r_{rotating} = x\hat{i} + y\hat{j} \qquad v_{rotating} = \dot{x}\hat{i} + \dot{y}\hat{j}$$

$$v = v_{rotating} + \Omega \times r_{rotating}$$

$$v = (\dot{x}\hat{i} + \dot{y}\hat{j}) + \dot{\theta}\hat{k} \times (x\hat{i} + y\hat{j})$$

$$v = (\dot{x} - \dot{\theta} y)\hat{i} + (\dot{y} + \dot{\theta} x)\hat{j}$$

$$T = \tfrac{1}{2} m[(\dot{x} - \dot{\theta} y)^2 + (\dot{y} + \dot{\theta} x)^2]$$

$$V = \tfrac{1}{2} k_x x^2 + \tfrac{1}{2} k_y y^2$$

Lagrangian

- Three generalized coordinates x, y, θ.
  - The angular displacement is imposed on the device, while the x direction is forced with sinusoidal oscillation amplitude of x_0.
  - We are interested in the response occurring in the y coordinate, so lets examine the equation of motion for q_i=y.

$$\mathrm{L} = T - V$$

$$L = \tfrac{1}{2} m[(\dot{x} - \dot{\theta}\, y)^2 + (\dot{y} + \dot{\theta}\, x)^2] - \left[\tfrac{1}{2} k_x x^2 + \tfrac{1}{2} k_y y^2\right]$$

$$\frac{d}{dt}\left(\frac{\partial \mathrm{L}}{\partial \dot{q}_i}\right) - \frac{\partial \mathrm{L}}{\partial q_i} = Q_i$$

$$\frac{d}{dt}\left(\frac{\partial \mathrm{L}}{\partial \dot{y}}\right) - \frac{\partial \mathrm{L}}{\partial y} = Q_y$$

Formulate Equation of Motion

$$L = \tfrac{1}{2} m[(\dot{x} - \dot{\theta} y)^2 + (\dot{y} + \dot{\theta} x)^2] - \left[\tfrac{1}{2} k_x x^2 + \tfrac{1}{2} k_y y^2\right]$$

- y coordinate:

$$\frac{\partial L}{\partial \dot{y}} = m(\dot{y} + \dot{\theta} x)$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{y}}\right) = m(\ddot{y} + \ddot{\theta} x + \dot{\theta} \dot{x})$$

$$-\frac{\partial L}{dy} = -m(\dot{x} - \dot{\theta} y)(-\dot{\theta}) + k_y y$$

$$m(\ddot{y} + \ddot{\theta} x + \dot{\theta} \dot{x}) + m\dot{\theta}(\dot{x} - \dot{\theta} y) + k_y y = Q_y$$

$$m\ddot{y} + 2m\dot{\theta} \dot{x} - m\dot{\theta}^2 y + m\ddot{\theta} x + k_y y = Q_y$$

Formulate Equation of Motion

- Add non-conservative damping forces.

$$\delta W_{non-consv.} = F_{damper}\delta y = -c_y \dot{y}\delta y$$

$$Q_y = -c_y \dot{y}$$

$$m\ddot{y} + 2m\dot{\theta}\dot{x} - m\dot{\theta}^2 y + m\ddot{\theta}x + k_y y = -c_y \dot{y}$$

$$\boxed{m\ddot{y} + c_y\dot{y} + 2m\dot{\theta}\dot{x} - m\dot{\theta}^2 y + m\ddot{\theta}x + k_y y = 0}$$

Coriolis Force

Additional Inertial Forces Due to Rotating Ref. Frame

- Assume for our case the Coriolis force dominates the other introduced inertial forces. (rotation rate small and steady)

Find Response Amplitude

- Generally interested in the response amplitude not the full solution.
- Consider the Coriolis force as an external forcing function:

$$m\ddot{y} + c_y \dot{y} + 2m\dot{\theta}\,\dot{x} + k_y y = 0$$

$$\Rightarrow m\ddot{y} + c_y \dot{y} + k_y y = -2m\dot{\theta}\,\dot{x}$$

- Invoke complex analysis of forced (particular) response.
- Control scheme provides a sinusoidal oscillation in x with amplitude of x_0 and frequency ω_d (Drive Frequency).

$$x = x_o \sin(\omega_d t) \Rightarrow \dot{x} = x_o \omega_d \cos(\omega_d t)$$

$$e^{\pm i\theta} = \cos(\theta) \pm i \sin(\theta)$$

$$\dot{x} = \mathrm{Re}[x_o \omega_d e^{i\omega_d t}]$$

Find Response Amplitude

- Complex representation of forcing function:

$$F_{coriolis} = \text{Re}\left[-2m\dot{\theta}\, x_o \omega_d e^{i\omega_d t}\right]$$

- Particular solution has the form:

$$y_p = \text{Re}\left[Y e^{i\omega_d t}\right]$$

- Insert these equations back into the equation of motion:

$$\dot{y}_p = \text{Re}\left[iY\omega_d e^{i\omega_d t}\right] \qquad \ddot{y}_p = \text{Re}\left[-Y\omega_d^2 e^{i\omega_d t}\right]$$

- Leave out Re symbol for now (just remember we want the real part)

$$m\left(-Y\omega_d^2 e^{i\omega_d t}\right) + c_y\left(iY\omega_d e^{i\omega_d t}\right) + k_y\left(Ye^{i\omega_d t}\right) = -2m\dot{\theta}\, x_o \omega_d e^{i\omega_d t}$$

$$Y\left(-m\omega_d^2 + ic_y\omega_d + k_y\right) = -2m\dot{\theta}\, x_o \omega_d$$

Find Response Amplitude

- Continued:

$$Y\left(-m\omega_d^2 + ic_y\omega_d + k_y\right) = -2m\dot{\theta}\, x_o\omega_d$$

$$c_y = 2m\xi\omega_{n-y}$$

$$Y\left(-\omega_d^2 + i2\xi\omega_{n-y}\omega_d + \omega_{n-y}^2\right) = -2\dot{\theta}\, x_o\omega_d$$

$$Y = \frac{-2\dot{\theta}\, x_o\omega_d}{\left[\left(-\omega_d^2 + \omega_{n-y}^2\right) + i2\xi\omega_{n-y}\omega_d\right]}$$

- Multiply top and bottom by:

$$\left[\left(-\omega_d^2 + \omega_{n-y}^2\right) - i2\xi\omega_{n-y}\omega_d\right]$$

$$Y = \frac{-2\dot{\theta}\, x_o\omega_d\left[\left(-\omega_d^2 + \omega_{n-y}\right) - i2\xi\omega_{n-y}\omega_d\right]}{\left[\left(-\omega_d^2 + \omega_{n-y}^2\right)^2 + \left(2\xi\omega_{n-y}\omega_d\right)^2\right]}$$

Find Response Amplitude

- We now have complex amplitude Y:

$$Y = a + ib$$

$$y_p = \mathrm{Re}\left[Y e^{i\omega_d t}\right]$$

- Complex Identity:

$$a + ib = Ae^{i\phi} \text{ , } A = \sqrt{a^2 + b^2}$$

$$y_p = A\,\mathrm{Re}\left[e^{i\omega_d t + \phi}\right]$$

- Thus, the real response has an amplitude of A.

$$A^2 = \left(-2\dot{\theta}\, x_o \omega_d\right)^2 \left[\frac{\left(-\omega_d^2 + \omega_{n-y}^2\right)^2}{\left[\left(-\omega_d^2 + \omega_{n-y}^2\right)^2 + \left(2\xi\omega_{n-y}\omega_d\right)^2\right]^2} + \frac{\left(2\xi\omega_{n-y}\omega_d\right)^2}{\left[\left(-\omega_d^2 + \omega_{n-y}^2\right)^2 + \left(2\xi\omega_{n-y}\omega_d\right)^2\right]^2} \right]$$

$$A^2 = \left(-2\dot{\theta}\, x_o \omega_d\right)^2 \left[\frac{\left(-\omega_d^2 + \omega_{n-y}^2\right)^2 + \left(2\xi\omega_{n-y}\omega_d\right)^2}{\left[\left(-\omega_d^2 + \omega_{n-y}^2\right)^2 + \left(2\xi\omega_{n-y}\omega_d\right)^2\right]^2} \right]$$

Gyroscopic Response (Accuracy)

- Simplification yields the response amplitude for small, steady rotation input $\dot{\theta}$.

$$A = \frac{-2\dot{\theta}\, x_o \omega_d}{\sqrt{\left(\omega_{n-y}^2 - \omega_d^2\right)^2 + \left(2\xi\omega_{n-y}\omega_d\right)^2}}$$

- Equation extremely useful in designing a gyroscope with maximum response amplitude (increased accuracy).
- Foremost notice the amplitude decreases if the drive frequency differs from the natural frequency in the y-direction.
- Since one would select the drive frequency roughly equal to the natural frequency in the x-direction (maximize x_o), an optimal gyroscope has matched modes.

Gyroscopic Response (Accuracy)

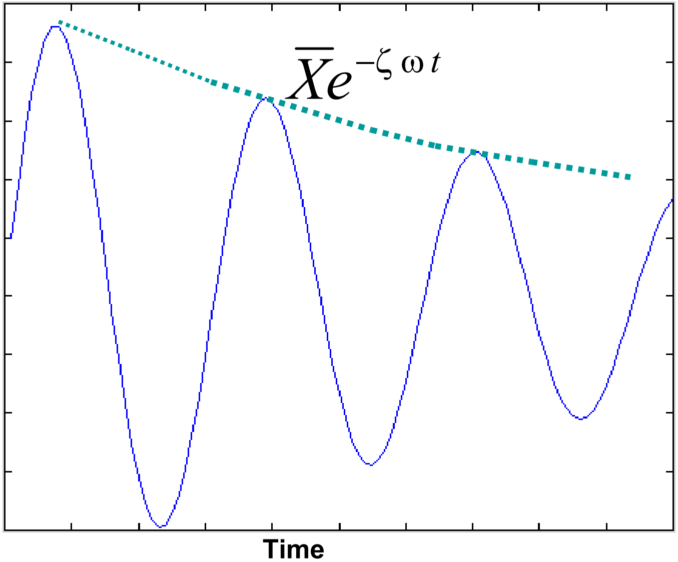
- Now consider a matched mode gyroscope:

$$\omega_d = \omega_{n-x} = \omega_{n-y}$$

$$A = \frac{-\dot{\theta}\, x_o}{\xi \omega_n}$$

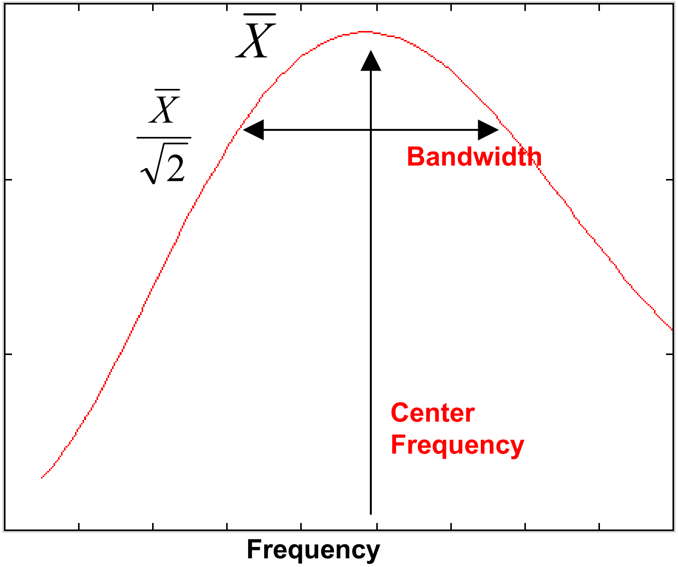
- Introduce the Quality (Q) Factor

Amplitude Decrement Method



$$Q = \frac{1}{2\zeta}$$

Frequency Response Method



$$Q = \frac{\omega_{center}}{bandwidth}$$

Gyroscopic Response (Accuracy)

- Amplitude becomes:

$$A = \frac{-2Qx_o\dot{\theta}}{\omega_n}$$

- Two Key Observations
  - Want to maximize the quality factor (MEMS resonators can have Q-factors > 20,000, which makes MEMS gyroscopes interesting)
  - Want to decrease the fundamental frequency of the device. Usually set at ~20 kHz to avoid interaction with low frequency environmental noise.
  - This last fact leads to some interesting dimensions for MEMS structures.

Group Work (Two Teams)

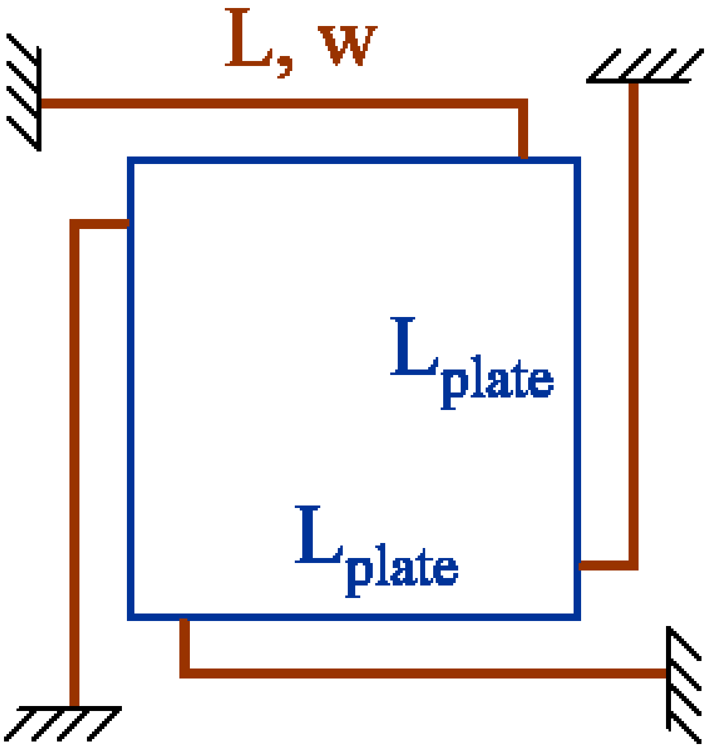
- MUMPs Gyroscope
- Find L,w of beams and L_{plate} such that the gyroscope has a fundamental frequency of ~20 kHz.
- Useful Facts
  - E=160 GPa, ρ=2330 kg/m^3
  - Thickness=2μm
  - Assume cantilever spring

$$k_{eq} = \frac{3EI}{L^3}$$

$$I = \tfrac{1}{12} w t^3$$

$$\omega = \sqrt{\frac{2k_{cantilever}}{m_{plate}}}$$

  - Neglect spring mass
  - Plate to substrate separation is 1.5μm.
  - Minimum feature size also 2μm.



Rigorous Amplitude Derivation

- Equations of Motion for x, y Coordinates:

$$\begin{bmatrix} m & 0 \\ 0 & m \end{bmatrix}\begin{bmatrix} \ddot{x} \\ \ddot{y} \end{bmatrix} + \begin{bmatrix} c_x & -2m\dot{\theta} \\ 2m\dot{\theta} & c_y \end{bmatrix}\begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} + \begin{bmatrix} k_x & 0 \\ 0 & k_y \end{bmatrix} = \begin{bmatrix} f_x \\ 0 \end{bmatrix}$$

- Complex Representation:

$$x_j = X_j e^{i\omega_d t} \qquad f_j = F_j e^{i\omega_d t}$$

$$\begin{bmatrix} -m\omega_d^2 + ic_x\omega_d + k_x & -2m\dot{\theta}\omega_d \\ 2m\dot{\theta}\omega_d & -m\omega_d^2 + ic_y\omega_d + k_y \end{bmatrix}\begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} F_x \\ 0 \end{bmatrix} = \begin{bmatrix} k_x x_0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}\begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} F_{10} \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}^{-1}\begin{bmatrix} F_{10} \\ 0 \end{bmatrix}$$

Rigorous Amplitude Derivation

$$Y = \frac{-Z_{12}F_{10}}{Z_{11}Z_{22} - Z_{21}Z_{12}}$$

$$Y = \frac{-2m\dot{\theta}\,\omega_d\,F_x}{m^2[\left(-\omega_d^2 + \omega_{n-x}^2 + i2\xi_x\omega_{n-x}\omega_d\right)\left(-\omega_d^2 + \omega_{n-y}^2 + i2\xi_y\omega_{n-y}\omega_d\right) + \left(2\dot{\theta}\,\omega_d\right)^2]}$$

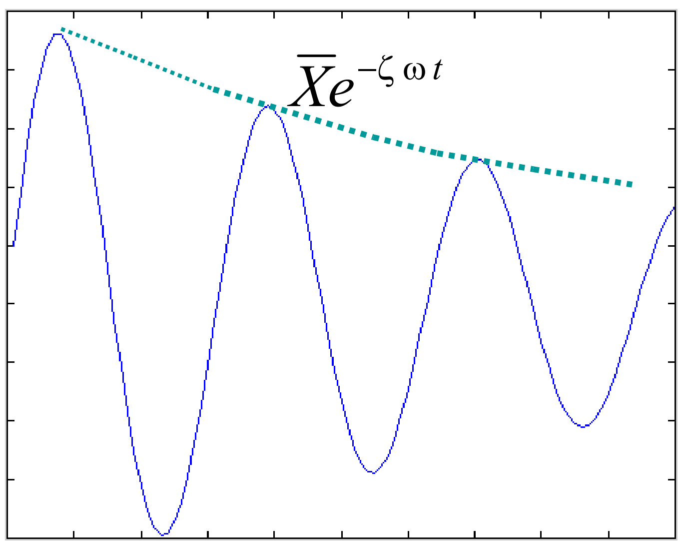
$$Y = \frac{-2\dot{\theta}\,\omega_d\omega_{n-x}^2\,x_0}{[\left(-\omega_d^2 + \omega_{n-x}^2 + i2\xi_x\omega_{n-x}\omega_d\right)\left(-\omega_d^2 + \omega_{n-y}^2 + i2\xi_y\omega_{n-y}\omega_d\right) + \left(2\dot{\theta}\,\omega_d\right)^2]}$$

- Matched Modes Case:

$$Y = \frac{-\dot{\theta}\,x_0}{2\omega\,[\left(-\xi_x\xi_y\right) + \left(\frac{\dot{\theta}}{\omega}\right)^2]}$$

How Steady?

- Derivation assumed a small and steady rotation rate.
- To provide a feel for how steady, find the time required to reduce the homogenous solution to 10% its original value.
- Assumptions:
  - Q = 10,000
  - ω = 20 kHz = 125,000 rad/sec



$$Q = \frac{1}{2\zeta}$$

Coriolis Gyroscope Summary

- Advantages
  - Scales down with increased accuracy
  - MEMS implementation (small and light)
  - MEMS devices can be integrated directly with associated electronics
  - No relative motion (i.e. no bearings, shafts, gears)
  - Low wear
- Disadvantages
  - Lower accuracy due to difficulty inherent in measuring motion of MEMS devices.
  - Devices often “flimsy” with length to thickness (slenderness) ratios often approaching 1000.
  - Susceptible to shock.
  - As with most MEMS devices the reliability is low due to stiction.
  - Still in research phase

Research Project

- Recall, for a Coriolis gyro., the accuracy is proportional to Q-factor.

$$A = \frac{-2Qx_o\dot{\theta}}{\omega_n}$$

- The total quality factor combines the losses attributed to friction, thermoelastic, air, and anchor dissipation mechanisms.

$$\frac{1}{Q} = \sum \frac{1}{Q_i} = \frac{1}{Q_{TED}} + \frac{1}{Q_{Volumetic}} + \frac{1}{Q_{Surface}} + \frac{1}{Q_{Air}} + \frac{1}{Q_{Anchor}}$$

- Despite the often limiting nature of anchor loss, this source has received insufficient study.

Analytical Anchor Loss Relationship

- Analytical model derived by assuming an equivalent single d.o.f. resonator:

$$Q_{anchor} = 2\pi \frac{W_0}{\Delta W}$$

$$W_0 = \tfrac{1}{2} k_r \bar{x}^2$$

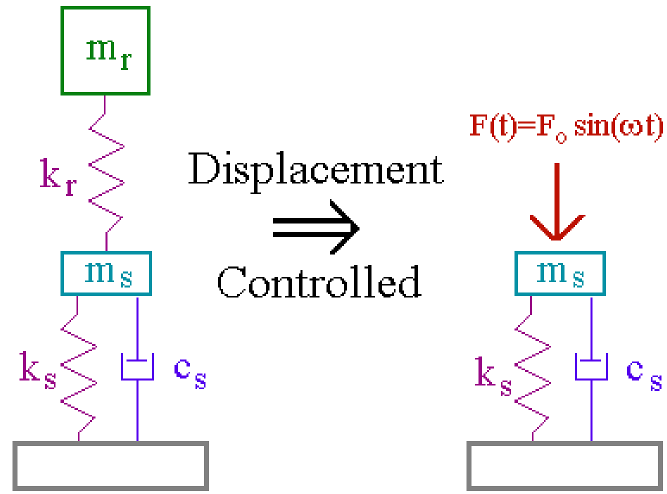
$$P_a(\omega) = \frac{\int_0^T W dt}{T} = \frac{\Delta W}{T} \Rightarrow \Delta W = T P_a(\omega)$$

$$H_s(\omega) = \frac{\left(k_s - m_s\omega^2\right) - i c_s \omega}{\left(k_s - m_s\omega^2\right)^2 + \left(c_s\omega\right)^2}$$

$$P_a(\omega) = \tfrac{1}{2} \mathrm{Re}\left[i\omega F_0^* H_s(\omega) F_0\right]$$

$$P_a(\omega) = \tfrac{1}{2}\omega\, k_r^2 \bar{x}^2 \left(\frac{c_s\omega}{\left(k_s - m_s\omega^2\right)^2 + \left(c_s\omega\right)^2}\right)$$

$$Q_{anchor} = \frac{1}{\omega\, k_r \left(\frac{c_s}{\left(k_s - m_s\omega^2\right)^2 + \left(c_s\omega\right)^2}\right)}$$



$$Q_{anchor} = \frac{1}{k_r \,\mathrm{Im}[-H_s(\omega)]}$$

Parametric Examples-Cantilever

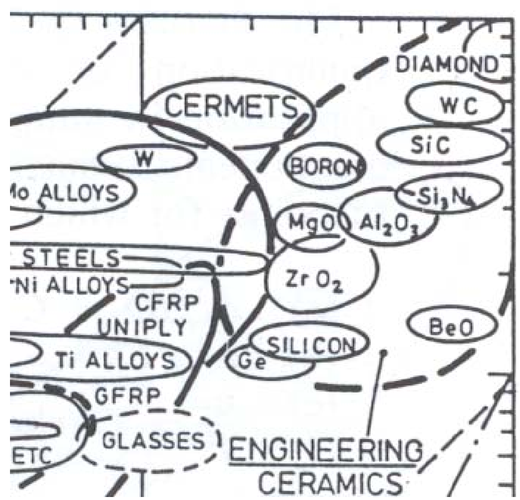
- **A Cantilever example highlights the general trends apparent in many resonator structures.**

$$k_{cantilever} = \frac{E\,w\,th^3}{4L^3}$$

$$Q_{anchor-cantilever} = \frac{1}{\mathrm{Im}[H_s(\omega)]}\frac{4}{Ew}\frac{L^3}{th^3}$$

- **Relationship reveals an anchor loss dependency upon resonator slenderness, width, Young's modulus, and frequency dependent substrate properties.**
- **Alternative substrate materials offer significant rigidity improvement.**

Strength (MPa)



1000

Young's Modulus (GPa)

100

Parametric Examples-Gyroscope

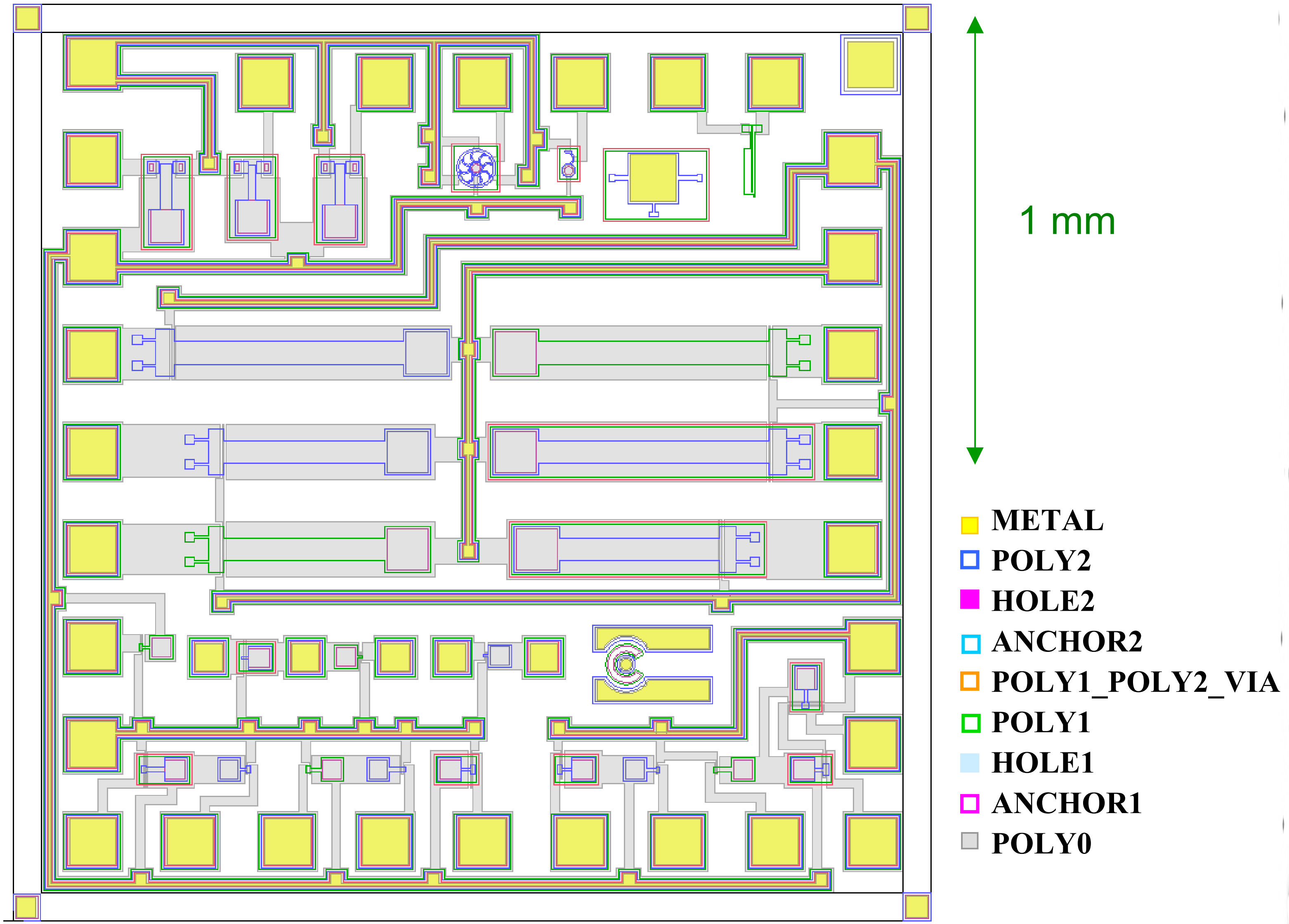
- Combing anchor loss model with modal amplitude equation yields additional insight unique to gyroscopes.

$$A = \frac{-2Qx_o\dot{\theta}}{\omega_n}$$

$$A = \beta\left(\dot{\theta}\ x_o\right) * \left(\frac{1}{E}\right) * \left(\frac{1}{w}\left(\frac{L}{th}\right)^3\right) * \left(\frac{1}{\omega_n}\right) * \left(\frac{1}{\mathrm{Im}[-H_s(\omega)]}\right)$$

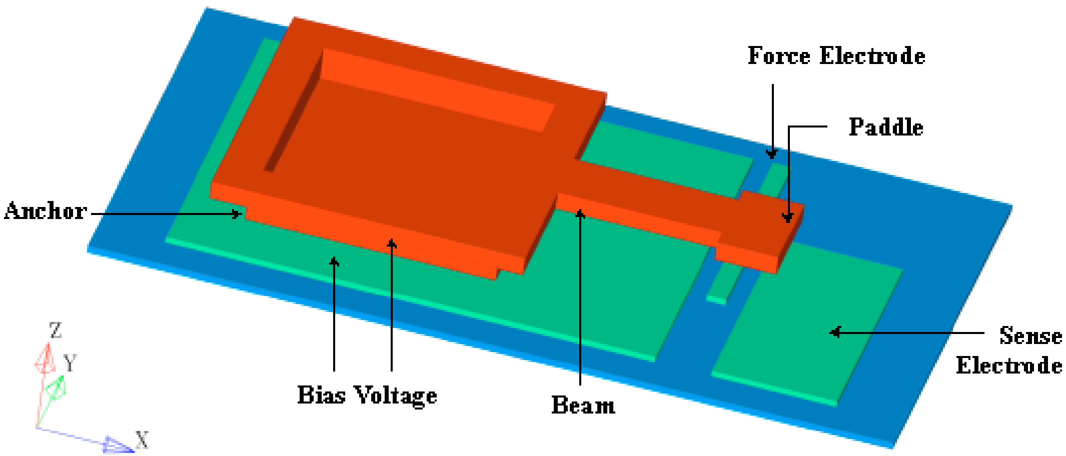
- Five relevent terms to work with in maximizing the sense amplitude; an input term, a ligament material term, a ligament geometry term, a frequency term, and a substrate term.

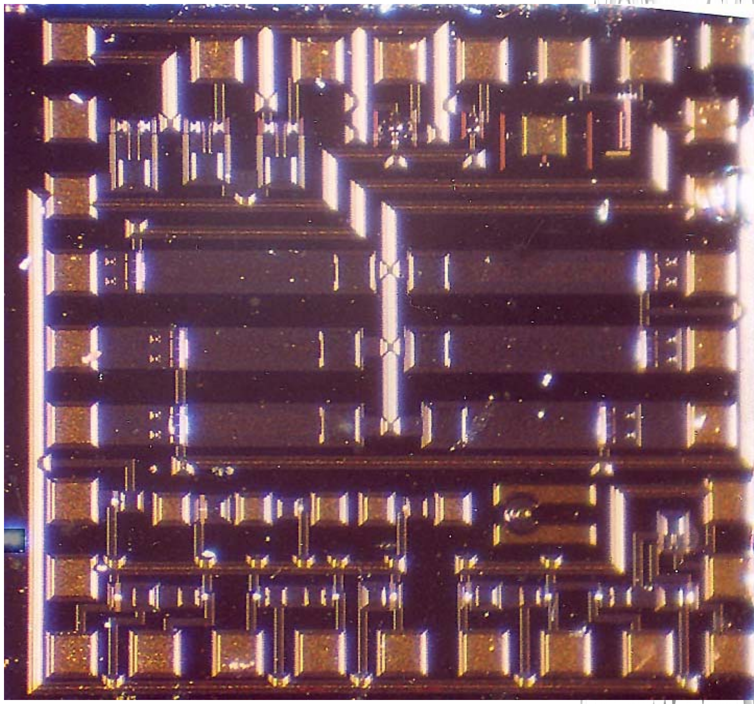
MUMPs Test Chip



MUMPs Test Chip

- Series of cantilever structures to test width, frequency, and slenderness dependence of anchor loss.
- Results applicable not only to gyroscopes but to other applications involving resonators (filters).





Conclusions

- Examining the dynamics of two new classes of gyroscopes provides the design intuition to:
  - Identify the design parameters which control the sensitivity.
  - Maximize the sensitivity to input rotation through parametric design.
  - Identify deficiencies in current understanding and develop new research projects.
  - Point out the relative strengths and weakness of each, and what applications are best suited for each.