Fluid-Structure Interaction Using Localized Lagrange Multipliers

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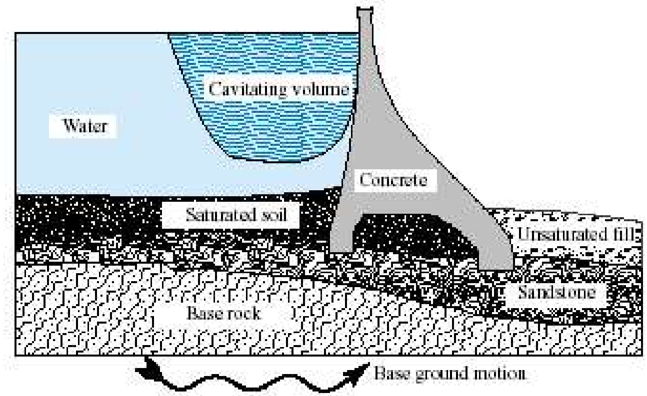
Basic Outline

- Motivation for FSI Research
- Current Methods for Fluid-Structure Interaction (FSI)
- Contributions of Research (Original Work)
- Purposed Benefits
- 1-D Model (Analytical & Computational) for introduction of LLM method
- Dam Model
- Research Plan (Future Work)

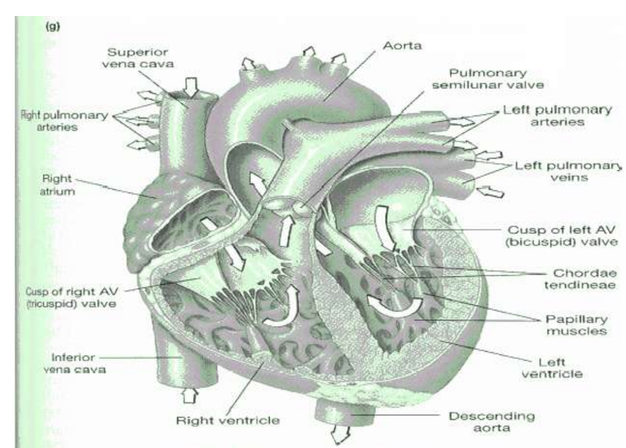
Motivation for Fluid-Structure Interaction

Acoustics



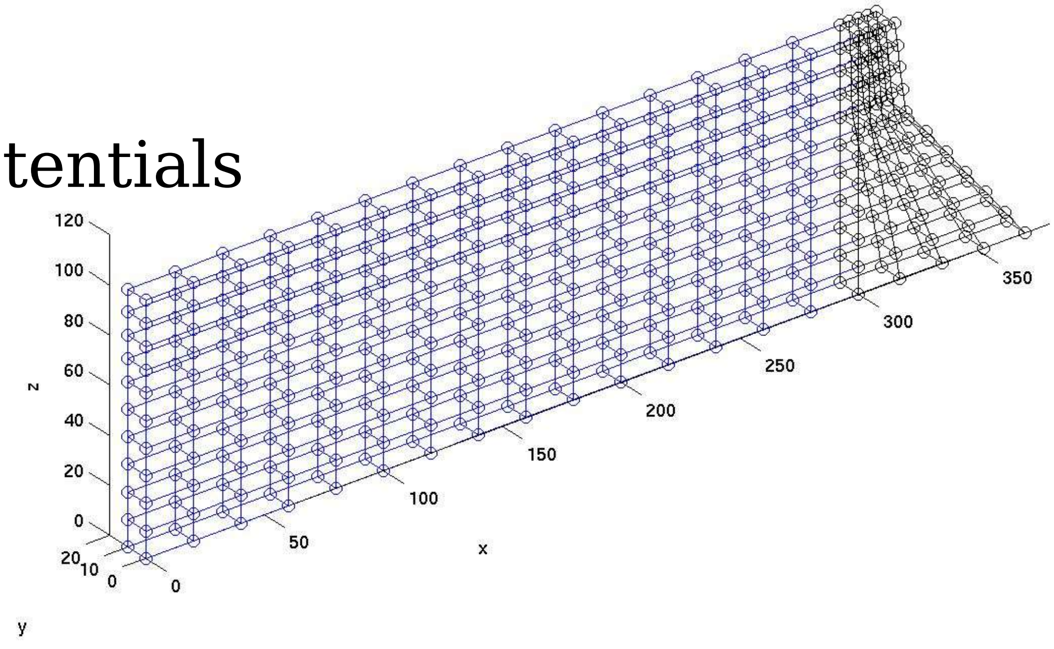
Inviscid Fluid Flow

Viscid Fluid Flow



Current Approaches for Solving Acoustic FSI

- Discretization of Continuum Mathematical Model
  - FEM, BEM, FDM, FVM, Spectral Method
  - Formulation (Fluid) in terms:
    - Displacements
    - Pressures
    - Displacement Potentials
    - Combinations



Current Approaches for Solving Acoustic FSI

How do we Solve the system of Equations?

- Field Elimination:
  - By techniques such as integral transforms or model reduction
- Monolithic Treatment
  - Whole problem is treated as a monolithic entity
  - All components advanced simultaneously in time
- Partitioned Treatment
  - Models are treated as isolated entities
  - Interaction effects are generally viewed as forcing effects
  - Entities are separately stepped in time

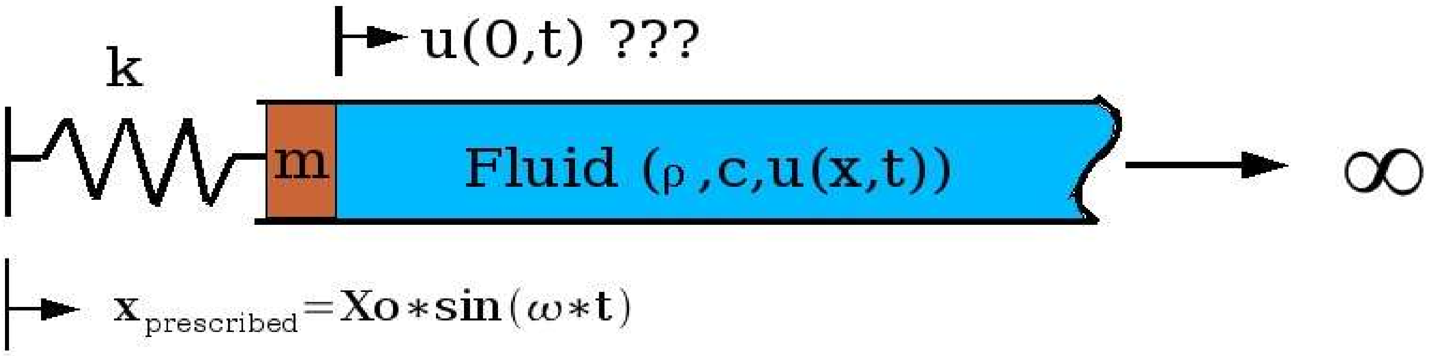
Contributions of Research/ Goals:

- Develop the Localized Lagrange Multiplier (LLM) method for an actual real life FSI problem (Dam)
  - Different implementation techniques
  - Include fluid formed in the displacement potential
- Simple Treatment for Non-matching meshes in FSI
- Parallel computation

Purposed Benefits

- Simplify the treatment of non-matching meshes
- Maintain software modularity
  - Different discretization and solvers for different physics
- Efficient high fidelity simulation
  - Resources can be put on critical parts of the problem
- User friendly
- Robust, Fast, and Accurate

Analytical 1-D Problem



- Assumptions linear acoustic fluid
  - Inviscid, irrotational, and no body forces
  - Small displacements $a = \frac{\partial^2 u}{\partial t^2}$ vs. $\frac{\partial^2 u}{\partial t^2} + \dot{u}\frac{\partial \dot{u}}{\partial x}$
    - Affects Euler so we can write Balance Equation (BE) $\rho \ddot{u} = -\nabla p$
  - Compressible
    - Constitutive Equation (CE): $p = -K \nabla u = -\rho c^2 \nabla u$

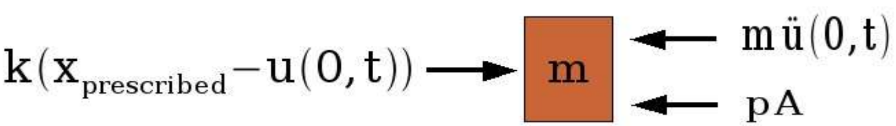
Analytical 1-D Problem

- Insert CE into BE →Wave Equation

$$\frac{\partial^2 u(x,t)}{\partial t^2} = c^2 \frac{\partial^2 u(x,t)}{\partial x^2}$$

- Boundary Conditions

FBD



$$m\ddot{u}(0,t) + \rho c^2 A u'(0,t) + k u(0,t) = k X_o \sin(wt)$$

- at x = L, PWA

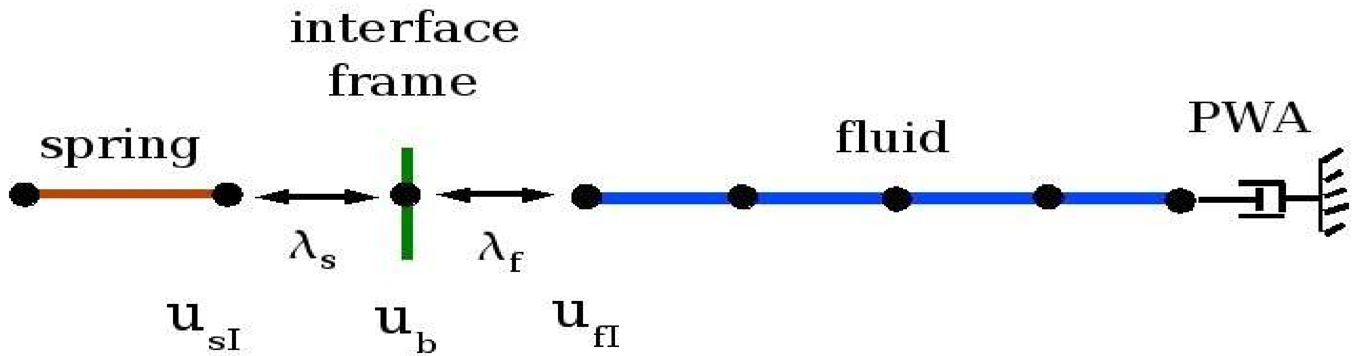
$$p(L) = -\rho c \dot{u}(L,t) \rightarrow -\rho c^2 u'(L,t) = -\rho c \dot{u}(L,t) \rightarrow \dot{u}(L,t) = c u'(L,t)$$

- Initial Conditions

$$u(x,0) = 0$$

$$\dot{u}(x,0) = 0$$

Introduce: Localized Lagrange Multiplier (LLM) method



- Frames are connected to adjacent partitions by force/flux fields
  - Mathematically: Lagrange multipliers are “gluing” the state variables of the partitioned models to that of the frame.
  - Lagrange multipliers at the frame are related by interface constraints and obey Newton’s Third Law.

Variational Principles and Lagrange Multipliers

- Use Lagrange's Method to derive the equilibrium equations of a system of constrained rigid bodies in Newtonian Mechanics Formulation.

1- Treat the problem as if all bodies are entirely free and formulate the virtual work by summing up the contributions of each free body.

2- Identify constraint equations and multiply each by an indeterminate coefficient. Then take the variation and add to the virtual work of the free bodies to yield the total virtual work of the system.

3- The sum of all terms which are multiplied by the same variation are equated to zero. These equations will provide all the conditions necessary for equilibrium.

Introduction: LLM

Total Potential Energy Functional for the system:

$$\Pi = \frac{1}{2}\int_{\Omega_s} \sigma^{\mathbf{T}} \epsilon \, d\Omega_s + \frac{1}{2}\rho_f c_f^2 \int_{\Omega_s} (\nabla \mathbf{u_f})^2 d\Omega_f$$
$$+\int_{\Omega_s} \mathbf{u_s^T}(\rho_s \ddot{\mathbf{u}}_{\mathbf{s}}) d\Omega_s + \int_{\Omega_f} \mathbf{u_f^T}(\rho_f \ddot{\mathbf{u}}_{\mathbf{f}}) d\Omega_f - \int_{\Gamma_s} \mathbf{u_s^T} \mathbf{T_s} d\Gamma_s - \int_{\Gamma_f} \mathbf{u_f^T} \mathbf{T_f} d\Gamma_f$$

<u>1- Subsystem Energy Expressions (Variational Formulation)</u>

$$\partial \Pi_s = \partial \mathbf{u_s^T}(\mathbf{M}\ddot{\mathbf{u}}_{\mathbf{s}} + \mathbf{K}\mathbf{u_s} - \mathbf{f_s})$$

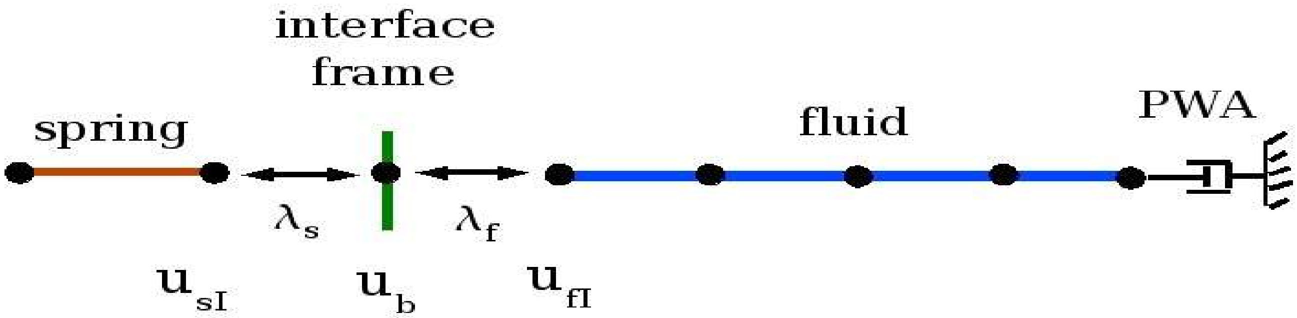
$$\partial \Pi_f = \partial \mathbf{u_f^T}(\rho \mathbf{Q} \ddot{\mathbf{u}}_{\mathbf{f}} + \rho c^2 \mathbf{H} \mathbf{u_f} - \mathbf{f_s})$$

where

$$\mathbf{Q} = \int_{\Omega_f} \mathbf{N} \mathbf{N}^{\mathbf{T}} d\Omega_f$$

$$\mathbf{H} = \int_{\Omega_f} \nabla \mathbf{N} \nabla \mathbf{N}^{\mathbf{T}} d\Omega_f$$

Introduction: LLM



2- Identify Interface Constraints

$$\Pi_b = \lambda_s^T(\mathbf{B}_s^T \mathbf{u}_s - \mathbf{L}_s \mathbf{u}_b) + \lambda_f^T(\mathbf{B}_f^T \mathbf{u}_f - \mathbf{L}_f \mathbf{u}_b)$$

$$\partial \Pi_b = \partial \lambda_s^T(\mathbf{B}_s^T \mathbf{u}_s - \mathbf{L}_s \mathbf{u}_b) + \lambda_s^T(\mathbf{B}_s^T \partial \mathbf{u}_s - \mathbf{L}_s \partial \mathbf{u}_b) + \partial \lambda_f^T(\mathbf{B}_f^T \mathbf{u}_f - \mathbf{L}_f \mathbf{u}_b) + \lambda_f^T(\mathbf{B}_f^T \partial \mathbf{u}_f - \mathbf{L}_f \partial \mathbf{u}_b)$$

3- Sum up the Variations and use the stationary condition:

$$\partial \Pi_{total} = \partial \Pi_s + \partial \Pi_f + \partial \Pi_b = 0$$

Eq (1) Eq (2)

$$\partial \Pi = \partial \mathbf{u}_s^T(\mathbf{M}\ddot{\mathbf{u}}_s + \mathbf{K}\mathbf{u}_s + \mathbf{B}_s \lambda_s - \mathbf{f}_s) + \partial \mathbf{u}_f^T(\rho \mathbf{Q}\ddot{\mathbf{u}}_f + \rho c^2 \mathbf{H}\mathbf{u}_f + \mathbf{B}_f \lambda_f - \mathbf{f}_f)$$
$$+ \partial \lambda_s^T(\mathbf{B}_s^T \mathbf{u}_s - \mathbf{L}_s \mathbf{u}_b) + \partial \lambda_f^T(\mathbf{B}_f^T \mathbf{u}_f - \mathbf{L}_f \mathbf{u}_b) + \partial \mathbf{u}_b^T(\mathbf{L}_s^T \lambda_s - \mathbf{L}_f^T \lambda_f) = 0$$

Eq (3) Eq (4) Eq (5)

Introduction: LLM

- Marching through time

  -Central Difference to Eq (1)

$$\left[\frac{1}{\Delta t^2}\mathbf{M}\right]\mathbf{u}_{\mathbf{s}}^{\mathbf{n+1}}=\mathbf{f}_{\mathbf{s}}^{\mathbf{n}}-\mathbf{B}_{\mathbf{s}}\lambda_{\mathbf{s}}^{\mathbf{n}}-\mathbf{K}\,\mathbf{u}^{\mathbf{n}}+\frac{1}{\Delta t^2}(2\,\mathbf{u}^{\mathbf{n}}-\mathbf{u}^{\mathbf{n-1}})$$ Eq (6)

  -Same thing for Eq(2)→ Eq (7)

  -Insert Eq (6) into Eq(3) $\mathbf{B}_{\mathbf{s}}^{\mathbf{T}}\mathbf{u}_{\mathbf{s}}-\mathbf{L}_{\mathbf{s}}\mathbf{u}_{\mathbf{b}}=0$

$$\mathbf{B}_{\mathbf{s}}^{\mathbf{T}}\left[\frac{1}{\Delta t^2}\mathbf{M}\right]^{-1}\mathbf{B}_{\mathbf{s}}\lambda_{\mathbf{s}}^{\mathbf{n}}-\mathbf{L}_{\mathbf{s}}\mathbf{u}_{\mathbf{b}}^{\mathbf{n+1}}=\mathbf{g}_{\mathbf{s}}^{\mathbf{n}}$$

$$\mathbf{g}_{\mathbf{s}}^{\mathbf{n}}=\mathbf{B}_{\mathbf{s}}^{\mathbf{T}}\left[\frac{1}{\Delta t^2}\mathbf{M}\right]^{-1}\left\{\mathbf{f}_{\mathbf{s}}^{\mathbf{n}}-\mathbf{K}\,\mathbf{u}_{\mathbf{s}}^{\mathbf{n}}+\frac{1}{\Delta t^2}\mathbf{M}(2\,\mathbf{u}_{\mathbf{s}}^{\mathbf{n}}-\mathbf{u}_{\mathbf{s}}^{\mathbf{n-1}})\right\}$$

  -Same thing for Eq(7) into Eq(4)

Introduction: LLM

- Interface Code: Solve for Lagrange Multipliers

$$\begin{bmatrix} \mathbf{B}_s^T\left[\frac{1}{\Delta t^2}\mathbf{M}\right]^{-1}\mathbf{B}_s & \mathbf{0} & \mathbf{L}_s \\ \mathbf{0} & \mathbf{B}_f^T\left[\frac{1}{\Delta t^2}\rho\mathbf{Q}\right]^{-1}\mathbf{B}_s & \mathbf{L}_f \\ \mathbf{L}_s^T & \mathbf{L}_f^T & \mathbf{0} \end{bmatrix} \begin{Bmatrix} \lambda_s^n \\ \lambda_f^n \\ \mathbf{u}_b^{n+1} \end{Bmatrix} = \begin{Bmatrix} \mathbf{g}_s^n \\ \mathbf{g}_f^n \\ \mathbf{0} \end{Bmatrix}$$

$$\mathbf{g}_s^n = \mathbf{B}_s^T\left[\frac{1}{\Delta t^2}\mathbf{M}\right]^{-1}\left\{\mathbf{f}_s^n - \mathbf{K}\mathbf{u}_s^n + \frac{1}{\Delta t^2}\mathbf{M}(2\mathbf{u}_s^n - \mathbf{u}_s^{n-1})\right\}$$

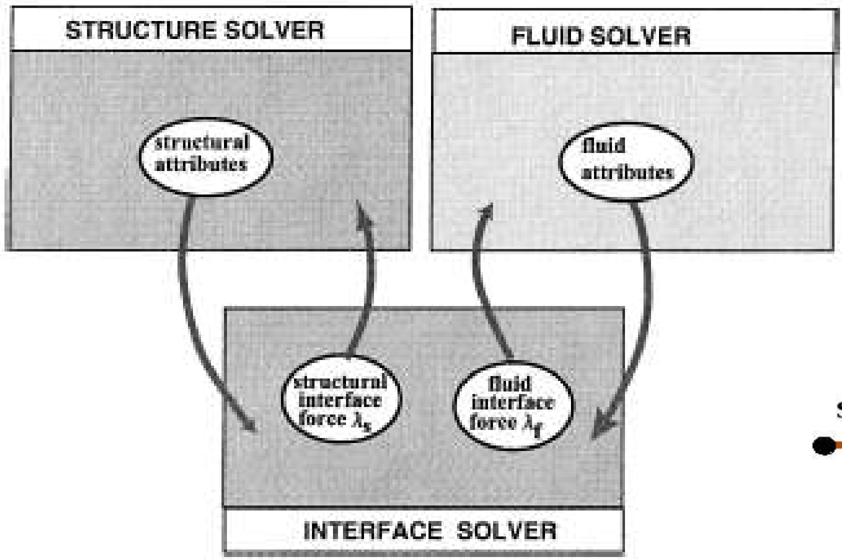
$$\mathbf{g}_f^n = \mathbf{B}_f^T\left[\frac{1}{\Delta t^2}\rho\mathbf{Q}\right]^{-1}\left\{\mathbf{f}_f^n - \rho c^2\mathbf{H}\mathbf{u}_f^n + \frac{1}{\Delta t^2}\rho\mathbf{Q}(2\mathbf{u}_f^n - \mathbf{u}_f^{n-1})\right\}$$

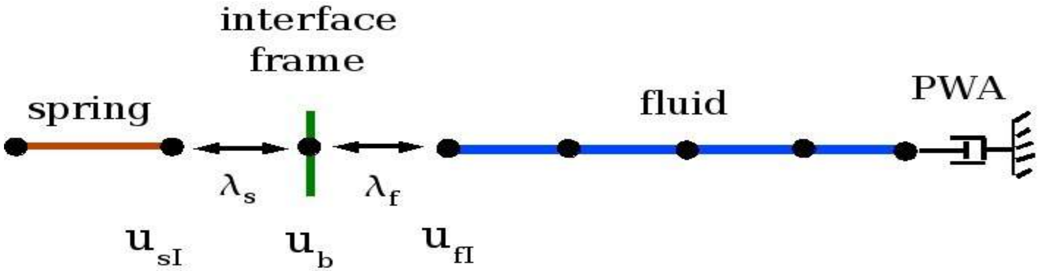
- With solved Lagrange Multipliers: Update displacements

$$\left[\frac{1}{\Delta t^2}\mathbf{M}\right]\mathbf{u}_s^{n+1} = \mathbf{f}_s^n - \mathbf{B}_s\lambda_s^n - \mathbf{K}\mathbf{u}^n + \frac{1}{\Delta t^2}(2\mathbf{u}^n - \mathbf{u}^{n-1})$$

- Repeat

Introduction: LLM





- Algorithm for 1-D Problem:

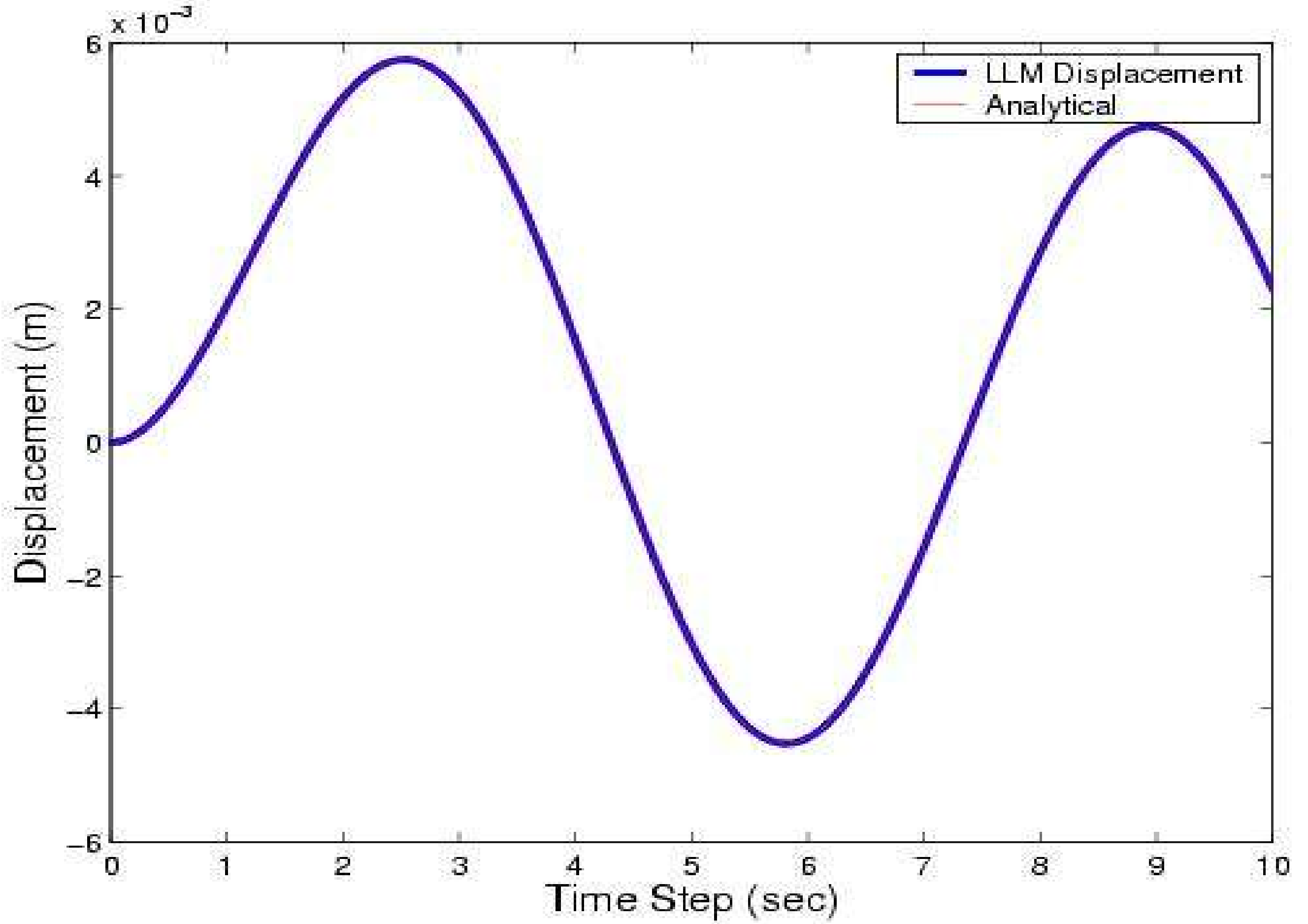
1- Structure Forcing Function & Fluid Force at NRB

$$\mathbf{f}_{\mathbf{f}}^{\mathbf{n}}(\mathrm{NRB}) = -\rho\, c\, \dot{\mathbf{u}}_{\mathbf{f}}^{\mathbf{n}}(\mathrm{NRB})$$

2- Solve for Lagrange Multipliers through Interface Code
3- Update Displacements
4- Repeat

- Stability: CFL condition for lumped mass $\Delta t_{cfl} \le \frac{L}{c}$

1-D Results with LLM in terms of Displacements



In 1-D there is no concern about irrotational effects.

Spurious Frequencies & Irrotational Fluid

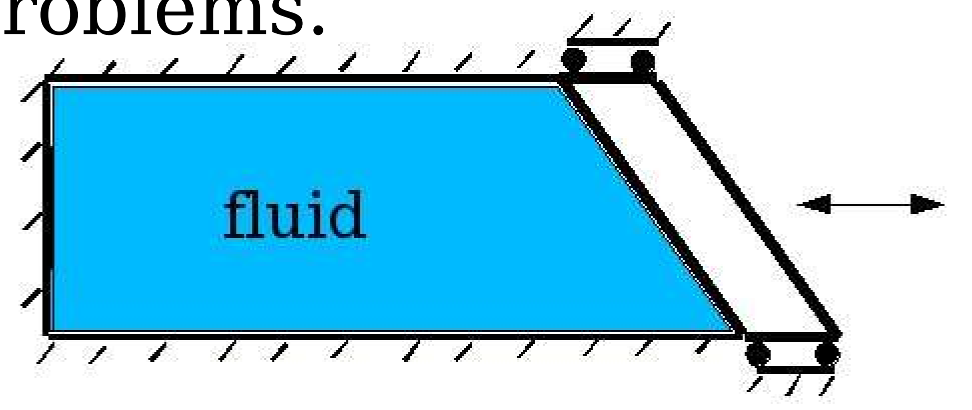
- It has been widely reported that pure displacement-based formulation produces spurious non-zero frequency modes
- Irrotational -> fluid particles do not spin

$$\nabla \times \mathbf{u} = \mathrm{curl}(u) = 0$$

- How to enforce:
  - Apply an irrotational constraint that is enforced by a penalty method.
  - Enforce irrotationality by choice of primary variable ($\nabla\psi$) in the formulation of FE matrices

Spurious Frequencies & Irrotational Fluid

- Wang & Bathe: Displacement based formulations with penalty formulation does NOT reduce these spurious modes in all problems.



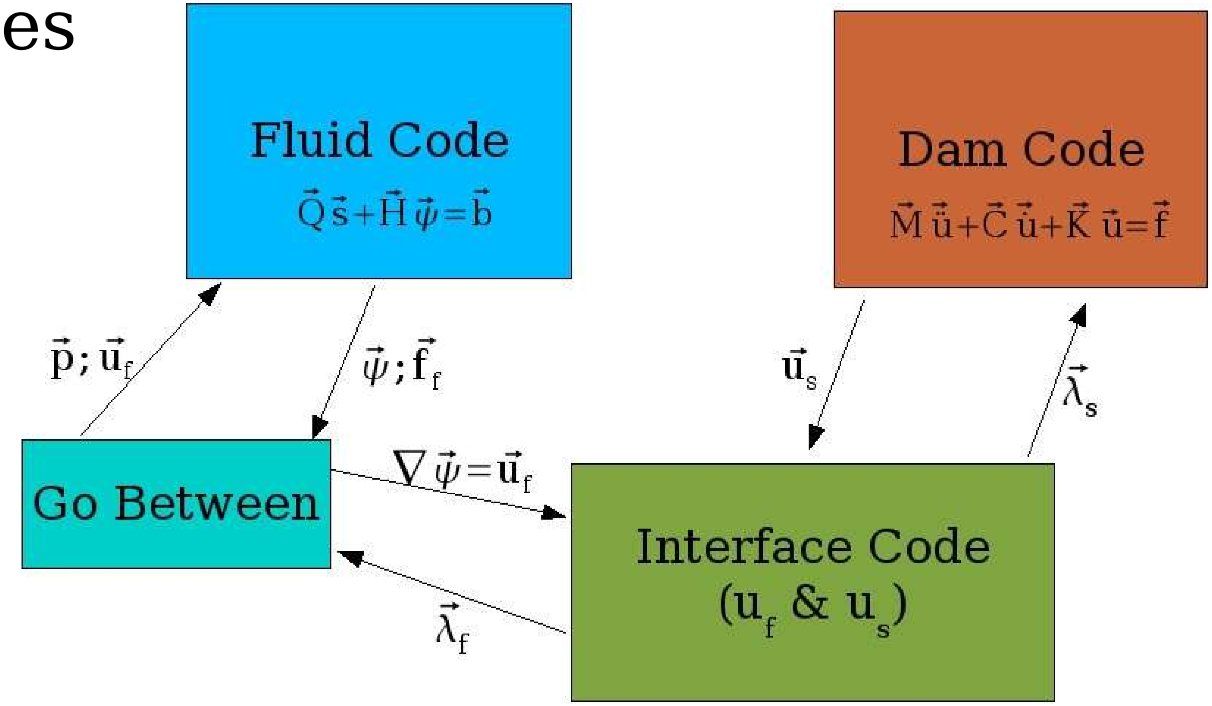
  - Origins of the spurious frequencies are in the use of pure displacement-based formulation (including penalty) and in the mishandling of the FSI interface (no transport of fluid across)

How to Incorporate Irrotationality in LLM

- Modify Variational Formulation by including constraint or change the displacement of the fluid variable to preferred variable.

$$\partial \Pi = \partial \mathbf{u}_\mathbf{s}^\mathbf{T}(\mathbf{M}\ddot{\mathbf{u}}_\mathbf{s} + \mathbf{K}\mathbf{u}_\mathbf{s} + \mathbf{B}_\mathbf{s}\lambda_\mathbf{s} - \mathbf{f}_\mathbf{s}) + \partial \mathbf{u}_\mathbf{f}^\mathbf{T}(\rho \mathbf{Q}\ddot{\mathbf{u}}_\mathbf{f} + \rho c^2 \mathbf{H}\mathbf{u}_\mathbf{f} + \mathbf{B}_\mathbf{f}\lambda_\mathbf{f} + \mathbf{C}_\mathbf{f}\mu - \mathbf{f}_\mathbf{f}) \\ + \partial \lambda_\mathbf{s}^\mathbf{T}(\mathbf{B}_\mathbf{s}^\mathbf{T}\mathbf{u}_\mathbf{s} - \mathbf{L}_\mathbf{s}\mathbf{u}_\mathbf{b}) + \partial \lambda_\mathbf{f}^\mathbf{T}(\mathbf{B}_\mathbf{f}^\mathbf{T}\mathbf{u}_\mathbf{f} - \mathbf{L}_\mathbf{f}\mathbf{u}_\mathbf{b}) + \partial \mathbf{u}_\mathbf{b}^\mathbf{T}(\mathbf{L}_\mathbf{s}^\mathbf{T}\lambda_\mathbf{s} - \mathbf{L}_\mathbf{f}^\mathbf{T}\lambda_\mathbf{f}) = 0$$

- Maintain modularity but modify inputs/outputs from module codes



Fluid Code with Displacement Potentials: CAFE

- Momentum Equation with assumption that the fluid's body force is the gradient of a time independent potential

$$\nabla B_{\mathrm{g}} - \nabla \mathrm{p} = \rho \ddot{\mathbf{u}}_{\mathbf{f}}$$ Eq (10)

- Displacement Potential (Irrotational Condition)

$$\nabla \psi = -\rho \mathbf{u}_{\mathbf{f}} \rightarrow \nabla \ddot{\psi} = -\rho \ddot{\mathbf{u}}_{\mathbf{f}}$$ Eq (11)

- Insert Eq (11) into Eq (10) and <u>spatial integrate</u>

$$-\ddot{\psi} = B_{\mathrm{g}} - \mathrm{p} + \mathrm{C}$$

- At static equilibrium ($\ddot{\psi}=0$) , fluid pressure equals hydrostatic (p = p^{h}). Only body force is gravity ($B_{\mathrm{g}}=\mathrm{p}^{\mathrm{h}}$). Thus, C =0.

$$\ddot{\psi} = \mathrm{p} - \mathrm{p}^{\mathrm{h}}$$ Eq (12), Equation of motion

Fluid Code with Displacement Potentials: CAFE

- Constitutive equation (CE) for linear acoustic fluid

$$p - p^h = -K \nabla \mathbf{u_f}; \qquad K = \rho c^2$$

- Define "relative condensation" ($s = -\rho \nabla \mathbf{u_f}$) and insert into the CE

$$p - p^h = c^2 s$$

- Compare to Eq (12) $\ddot{\psi} = p - p^h$

$$\ddot{\psi} = c^2 s$$

- Apply wave Equation $\ddot{\psi} = c^2 \nabla^2 \psi$

$$s - \nabla^2 \psi = 0 \qquad \text{Governing Equation}$$

Fluid Code with Displacement Potentials: CAFE

- Governing Equation is discretized with a standard Galerkin approach:

$$\int_{\Omega^e} \mathbf{N}(s - \nabla^2 \psi)\, d\Omega$$

- Application of the divergence theorem

$$\int_{\Omega^e} \mathbf{N}\, s\, d\Omega + \int_{\Omega^e} \nabla \mathbf{N} \nabla \psi\, d\Omega = \int_{\Gamma^e} \mathbf{N} \nabla \psi \cdot \mathbf{n}\, d\Gamma$$

- Dependent field variables (s,ψ) are interpolated within each element ($s = N^t s^e$) except on RHS

$$\mathbf{Q^e s^e} + \mathbf{H^e} \psi^{\mathbf{e}} = \mathbf{b^e}$$

$$\mathbf{Q^e} = \int_{\Omega^e} \mathbf{N}\mathbf{N}^{\mathbf{T}} d\Omega; \quad \mathbf{H^e} = \int_{\Omega^e} (\nabla \mathbf{N})(\nabla \mathbf{N})^{\mathrm{T}} d\Omega; \quad \mathbf{b^e} \int_{\Gamma^e} \mathbf{N} \nabla \psi \cdot \mathbf{n}\, d\Gamma$$

Fluid Code with Displacement Potentials: CAFE

- Basic Algorithm:
  - Introduce artificial damping into Equation of Motion Eq (12) to reduce frothing.

$$\ddot{\psi}^{\mathbf{n}} = \mathbf{p}^{\mathbf{n}} - \mathbf{p}^{\mathbf{h}} + \beta \Delta t c^2 \dot{\mathbf{s}}^{\mathbf{n}} ; \text{where}$$
$$\dot{\mathbf{s}} = \frac{1}{\Delta t} (\mathbf{s}^{\mathbf{n}} - \mathbf{s}^{\mathbf{n-1}})$$

  - Update fluid displacement potential

$$\dot{\psi}^{\mathbf{n+1}} = \dot{\psi}^{\mathbf{n}} + \Delta t \ddot{\psi}^{\mathbf{n}}$$
$$\psi^{\mathbf{n+1}} = \psi^{\mathbf{n}} + \Delta t \dot{\psi}^{\mathbf{n+1}}$$

  - Solve for s^{n+1}

$$\mathbf{Q} \mathbf{s}^{\mathbf{n+1}} = \mathbf{b}^{\mathbf{n+1}} - \mathbf{H} \psi^{\mathbf{n+1}}$$

  - Update pressure include bilinear affect (cavitation)

$$p^{n+1} = \max \begin{Bmatrix} p^h + c^2 s^{n+1} \\ 0 \end{Bmatrix}$$

CAFE Stability

- Upper bound for the critical time increment

$$\Delta t_{cr} = \frac{2}{c\sqrt{\lambda_{max}(1+2\beta)}}$$

β = artificial damping constant

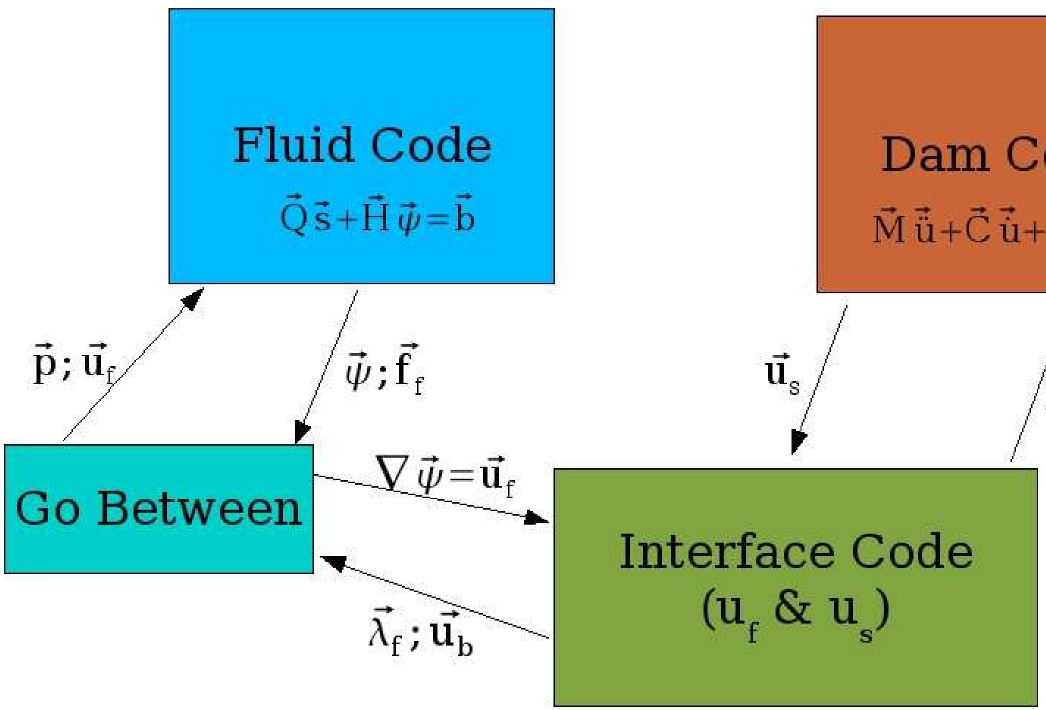
λ_{max} is the maximum eigenvalue of the global generalized eigenproblem.

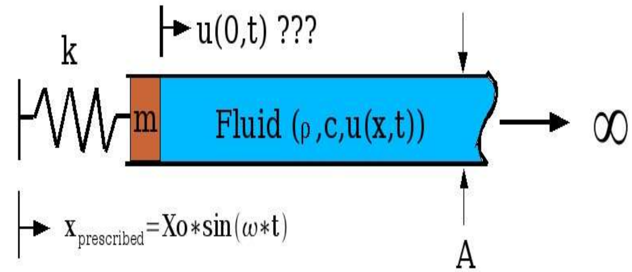
$$(\mathbf{H} - \lambda\mathbf{Q})\mathbf{z} = 0$$

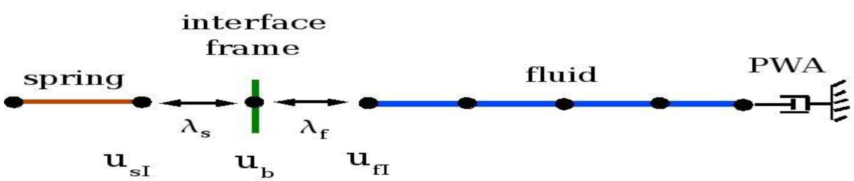
For CAFE, λ_{max} is bounded above by $4/\Delta x_{min}^2$

$$\Delta t_{cr} = \frac{\Delta t_{cfl}}{\sqrt{(1+2\beta)}}$$

LLM with CAFE







<u>- Inputs to the fluid code from output of interface code</u>

$$\mathbf{p}_i^n=\frac{\lambda_f^n}{A}$$

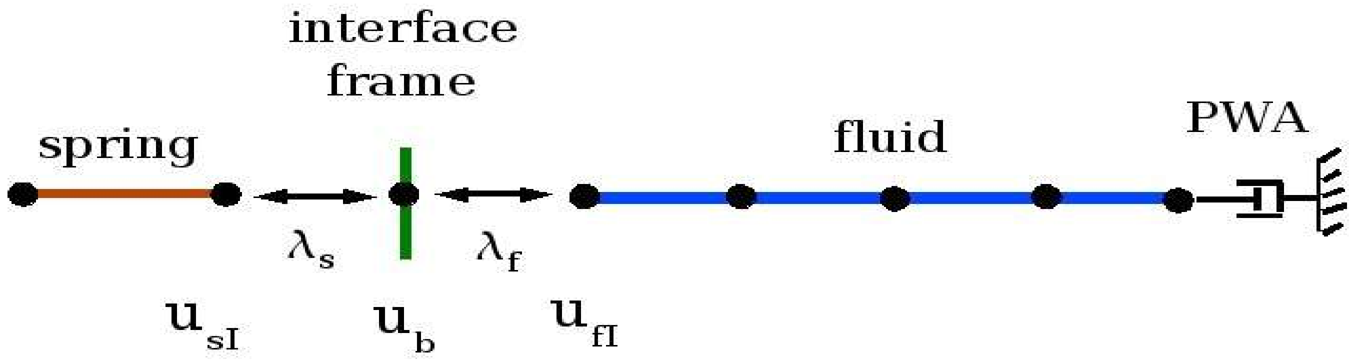
$$\dot{\mathbf{u}}_{fi}^n=\frac{\mathbf{p}_i^n}{\rho C} \qquad \text{(PWA)}$$

$$\mathbf{u}_{fi}^{n+1}=\mathbf{u}_{fi}^n+\Delta t\,\dot{\mathbf{u}}_{fi}^n \quad \rightarrow \mathbf{b}^{n+1}$$

Or

$$\mathbf{u}_b^{n+1} \rightarrow \mathbf{b}^{n+1}$$

LLM with CAFE



- Inputs to the interface code from output of fluid code

$$\mathbf{u_f} = \frac{-\nabla \psi}{\rho}$$

- 2 pt. Forward difference

$$\psi'(x) \approx \frac{1}{h}[\psi(x+h) - \psi(x)]$$

- Use multiple nodes by Differentiation via Polynomial Interpolation

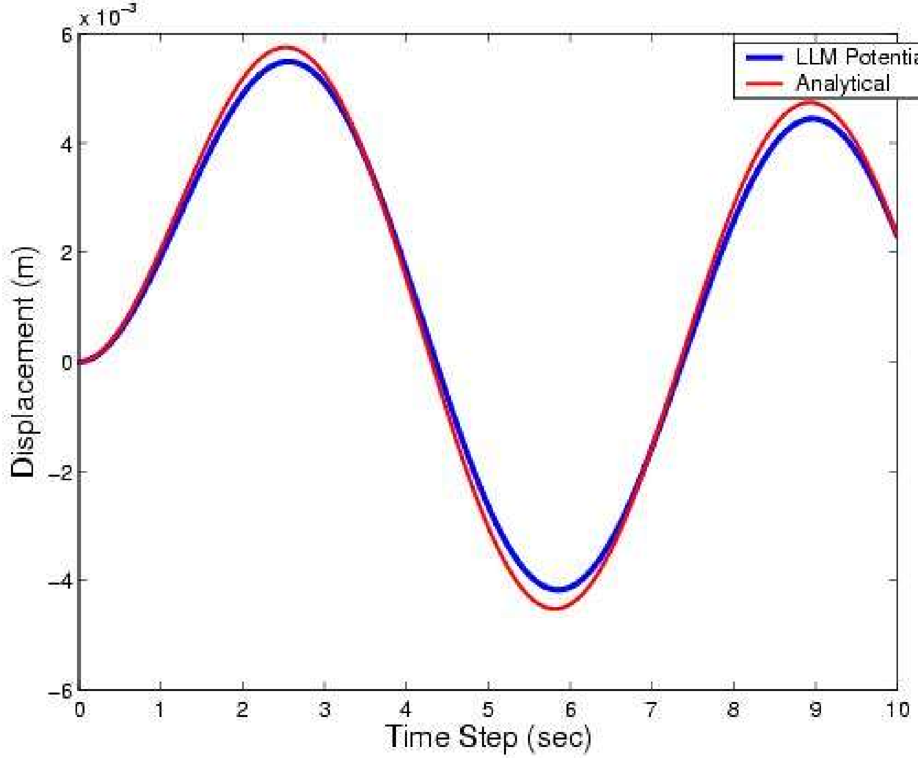
$$\psi'(x) = \psi(x_0) l_0'(x) + \psi(x_1) l_1'(x) + \psi(x_2) l_2'(x) + \cdots$$

$$l_i(x) = \prod_{j=0,\, j\neq i}^{n} \frac{x - x_j}{x_i - x_j}$$ Cardinal functions

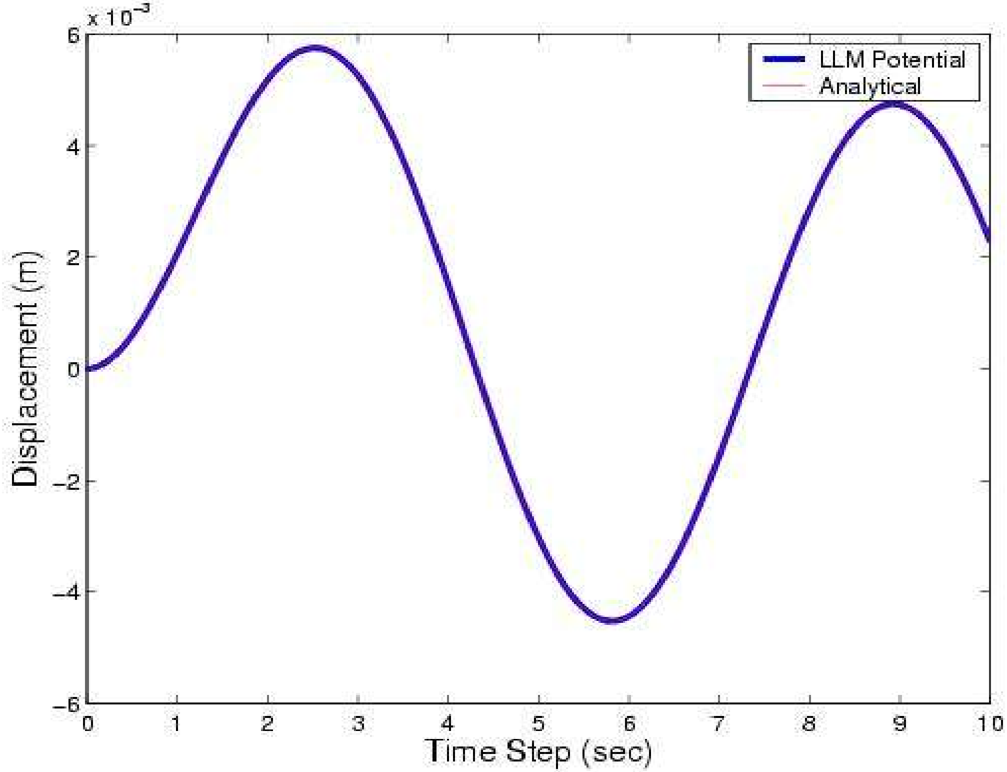
Results of LLM with CAFE

$$\mathbf{p_i^n} = \frac{\lambda_\mathbf{f}^\mathbf{n}}{A}$$

Pressure Input with 2 pt.
Forward difference, dt = 1e-5



Pressure Input with 2 pt.
Forward difference, dt = 1e-6



$$\ddot{\psi}^\mathbf{n} = \mathbf{p^n} - \mathbf{p^h} + \beta \Delta t c^2 \dot{\mathbf{s}}^\mathbf{n}$$

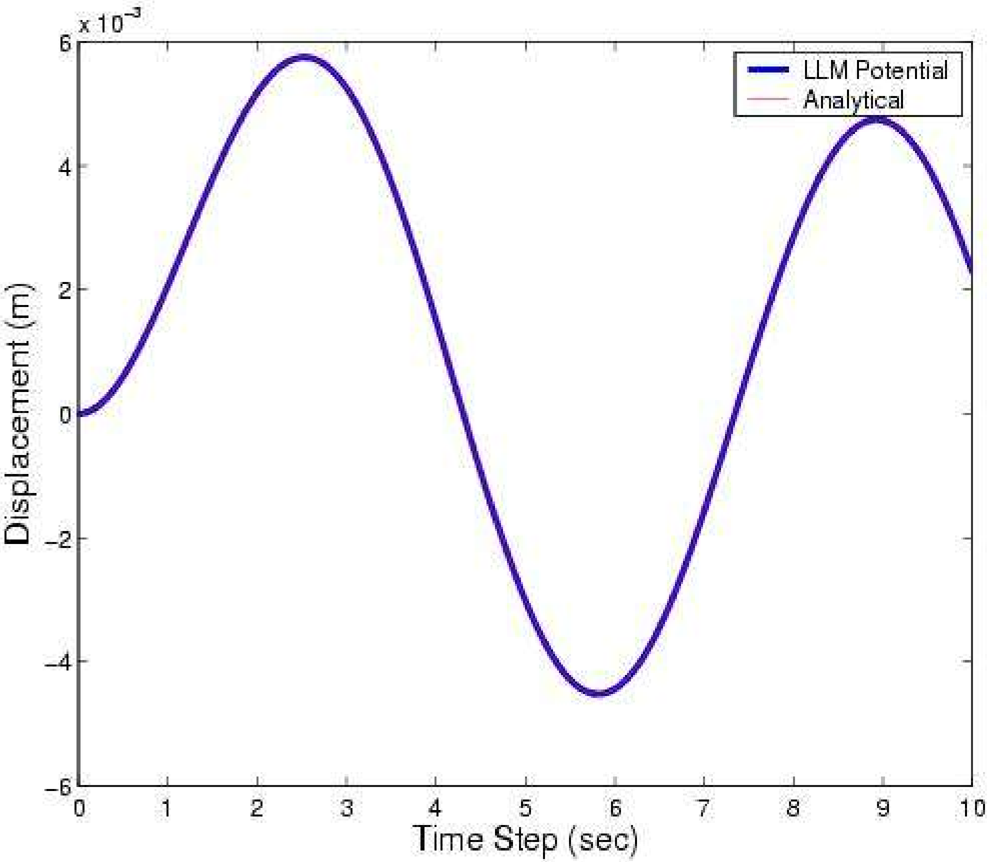
Results of LLM with CAFE

$$\mathbf{p_i^n} = \frac{\lambda_{\mathbf{f}}^{\mathbf{n}}}{A}$$

$$\mathbf{\dot{u}_{fi}^n} = \frac{\mathbf{p_i^n}}{\rho C} \qquad \text{(PWA)}$$

$$\mathbf{u_{fi}^{n+1}} = \mathbf{u_{fi}^n} + \Delta t\, \mathbf{\dot{u}_{fi}^n} \quad \rightarrow \mathbf{b^{n+1}}$$

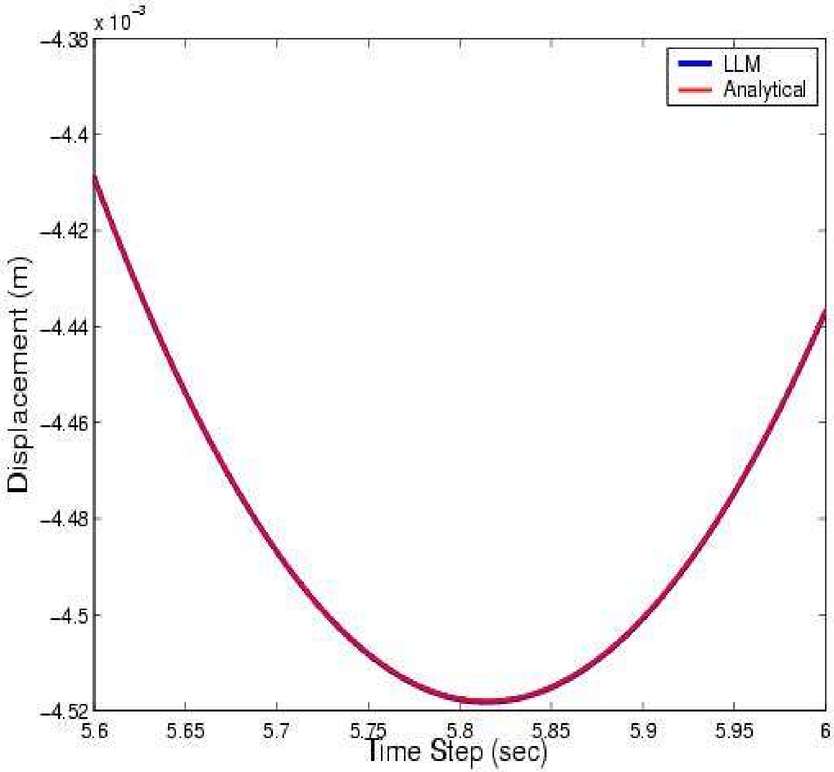
Displacement Input with 2 pt. Forward difference, dt = 1e-6



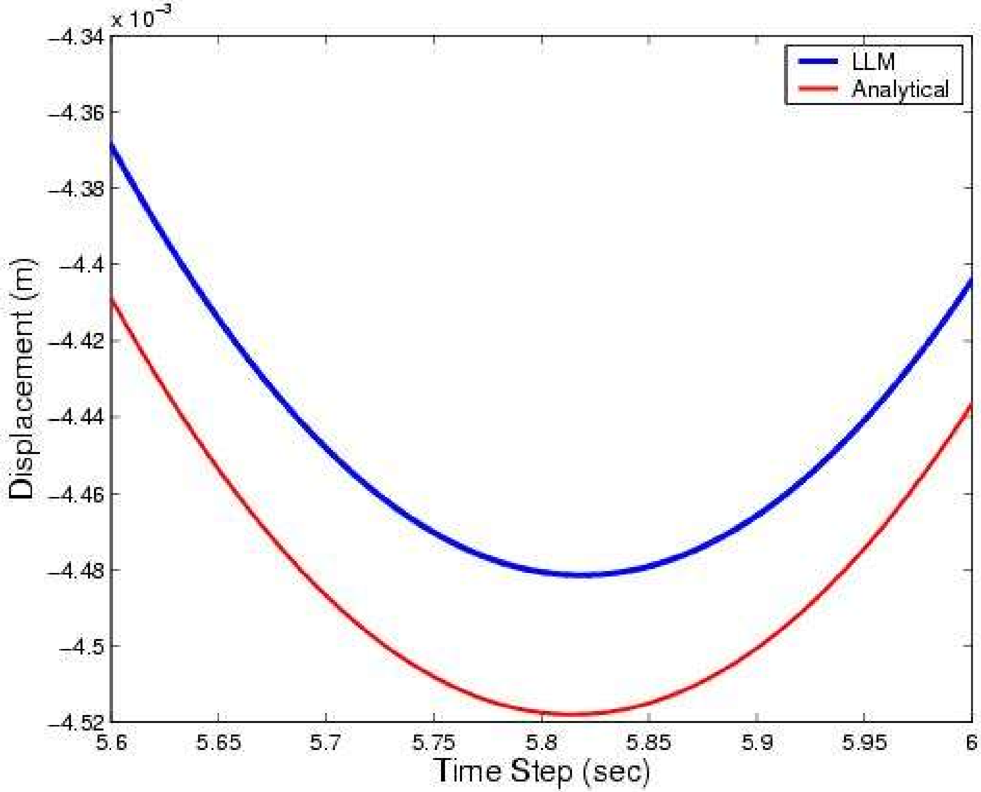
Input from u_{fi} or p

- Comparison of different inputs at dt = 1e-6

Displacement Input with 2 pt. Forward difference, dt = 1e-6



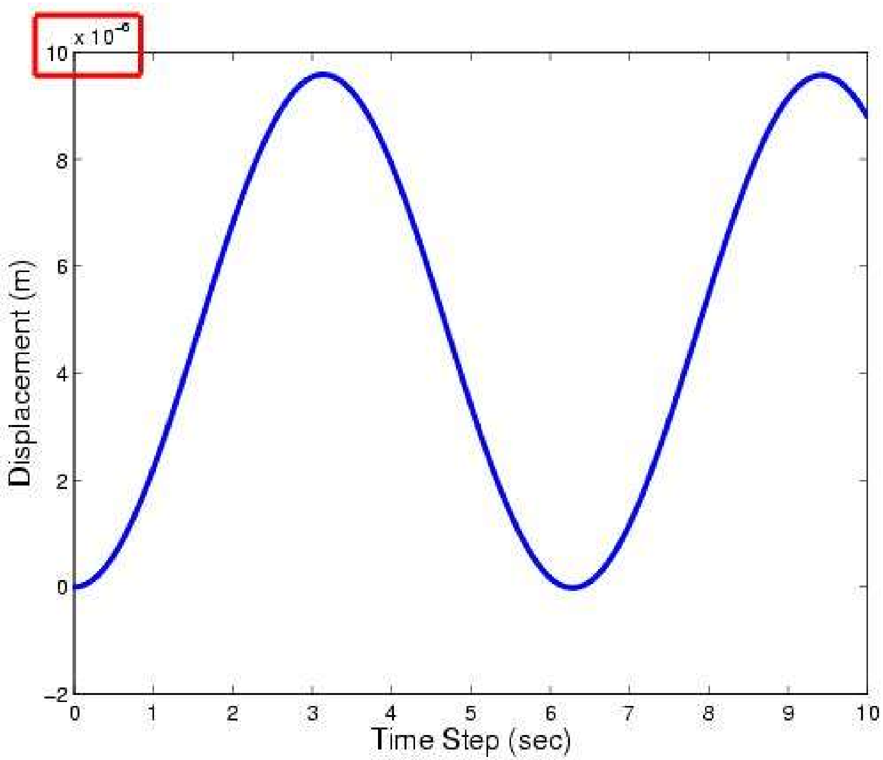
Pressure Input with 2 pt. Forward difference, dt = 1e-6



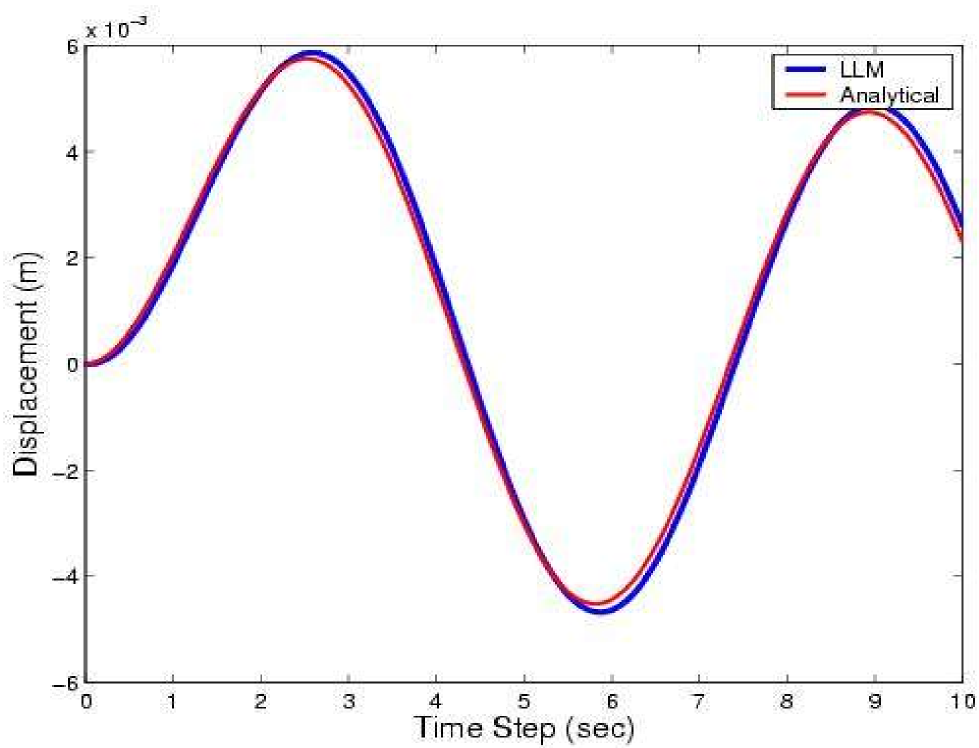
Results of LLM with CAFE

$\mathbf{u}_b^{n+1} \rightarrow \mathbf{b}^{n+1}$

u$_b$ Input with 2 pt. Forward difference, dt = 1e-6



u$_b$ Input with gradient from 3 nodes, dt = 1e-6

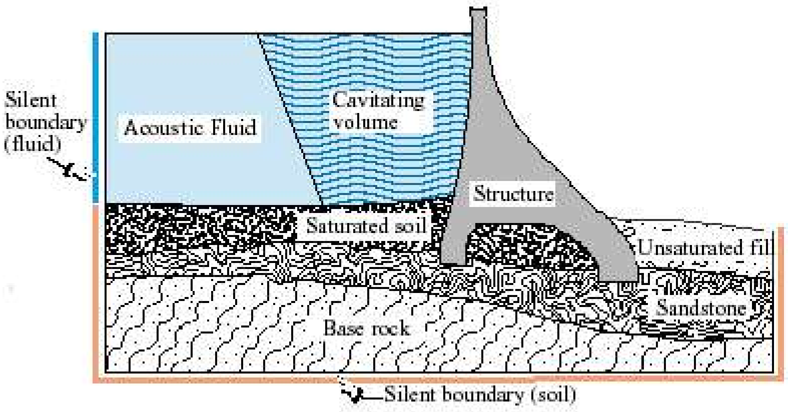


Summary of 1-D Problem

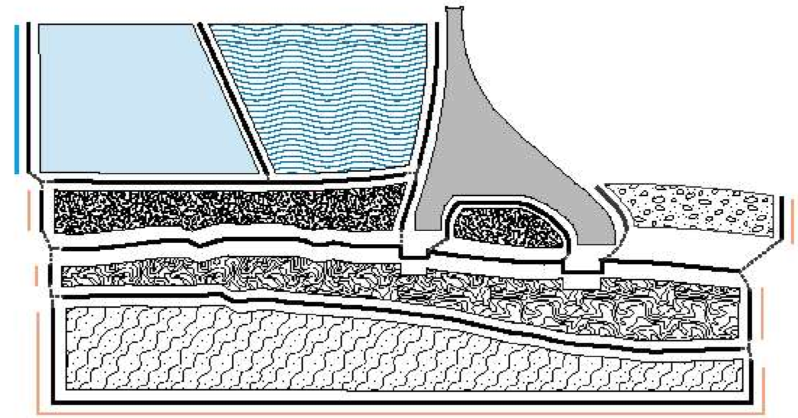
- Modularity Concept of FSI with LLM is viable.
- Converting the lagrange multiplier to either a pressure or a displacement is more accurate then using the interface frame displacement.
  - Plus, this will enable non-matching meshes.
  - Converting to a displacement is better then the pressure, but has additional issue with stability.
- Taken Gradients in multiple Dimensions may cause problems.
- Need to look at other methods for enforcing the irrotationality condition.

Dam: Application Problem

Multi-physics system



Modular Systems

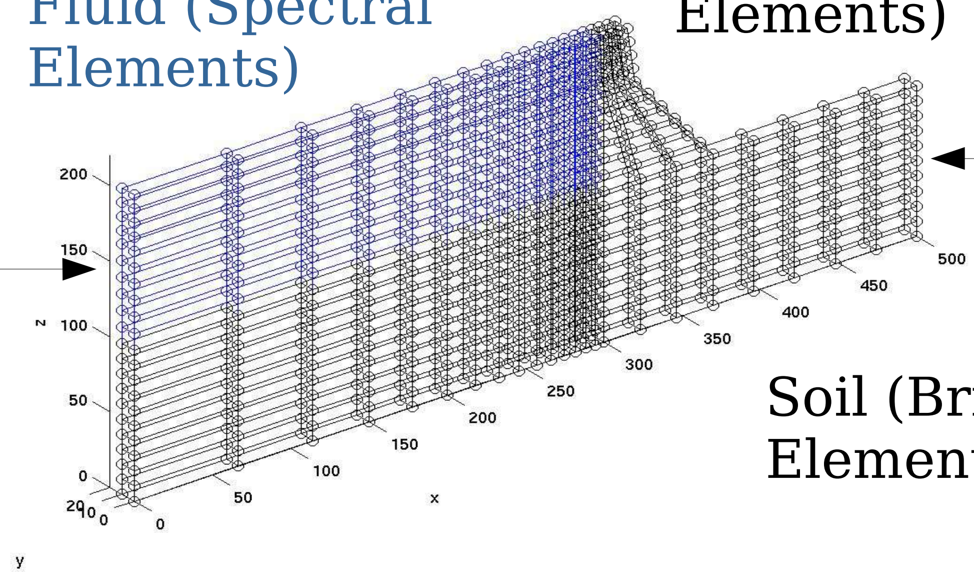


Fluid (Spectral Elements)

Dam (Brick Elements)

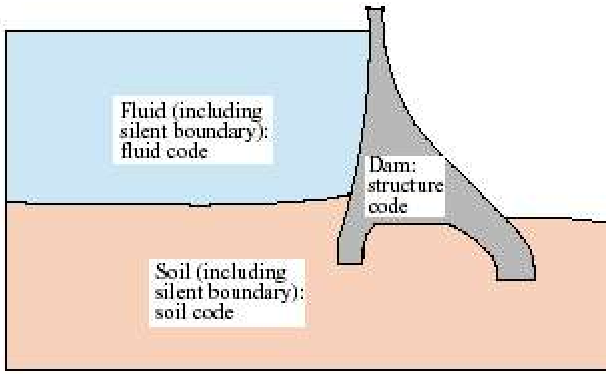
VDB for NRB

PWA for NRB



Soil (Brick Elements)

Dam: CAFE/CASE Code Concept



displacements → pressures →

Structure (seismic displacement)

Fluid Volume

Non-Reflecting Boundary (NRB)

← pressures ← displacements

Structure Equations:

$$M\ddot{x}_s + C\dot{x}_s + Kx_s = f_w - MI_x a_q$$

- **x = relative displacements** $\mathbf{u}_{\mathbf{s}(\mathbf{total})} = \mathbf{x}_\mathbf{s} + \mathrm{u}_{\mathrm{qx}}$
- **Acceleration (a) is from Berkeley's PEER database**
- **Damping modeled with Rayleigh damping**
- **Solved with a Central Difference Method (Explicit). Easy to implement. Also the physics are represented with small time steps**
- **Silent Boundary on Soil is modeled with a Viscous Damping Boundary Method**
- **Soil and Dam are modeled monolithically with different properties (i.e. Young's Modulus, etc.)**

Stability of Uncoupled Structural Time Integration

- Central Difference is conditionally stable (General for explicit methods).
- By going through a Fourier/Spectral Stability: $\Delta t \leq \frac{2}{\omega_{max}}$
  - For propagating waves we bound ω_{max}:

    $$(\omega_{max})_e = \frac{2c}{L};\quad \text{Lumped Mass Matrix}$$

    $$c = \sqrt{\frac{E}{\rho}};\quad \text{wave speed, L = characteristic length}$$

  - CFL condition:

    $$\Delta t \leq \frac{L_{min}}{c};\quad \text{lumped mass matrix}$$

    $$\Delta t \leq \frac{L_{min}}{c\sqrt{3}};\quad \text{consistent mass matrix}$$

- Δt must be small enough that information does not propagate across more than one element per time step.

Fluid Equations:

- Assumptions
  - Fluid is inviscid & irrotational
  - Displacements are small, and density is constant
  - Fluid is compressible with a bulk modulus
  - Bilinear acoustic fluid (bilinear to account for cavitation)
    - Fluid cannot transmit negative pressures
  - System is initial in static equilibrium
  - Steady body force field (gravity)
- Goal is to develop continuum fluid models that are discretized with the spectral-element method.

$$\mathbf{Q}^{\mathbf{e}}\mathbf{s}^{\mathbf{e}}+\mathbf{H}^{\mathbf{e}}\psi^{\mathbf{e}}=\mathbf{b}^{\mathbf{e}}$$

$$\mathbf{Q}^{\mathbf{e}}=\int_{\Omega^{\mathrm{e}}}\mathbf{N}\mathbf{N}^{\mathbf{T}}\mathrm{d}\Omega;\quad \mathbf{H}^{\mathbf{e}}=\int_{\Omega^{\mathrm{e}}}(\nabla\mathbf{N})(\nabla\mathbf{N})^{\mathrm{T}}\mathrm{d}\Omega;\quad \mathbf{b}^{\mathbf{e}}\int_{\Gamma^{\mathrm{e}}}\mathbf{N}\nabla\psi\cdot\mathbf{n}\,\mathrm{d}\Gamma$$

Spectral Elements (Fluid)

- First-order basis functions are used for geometry

$$x = N^T X; \quad N^T = standard\ trilinear\ shape\ functions$$

- Higher-order basis functions are used for field-variable (s,ψ) representations

$$\psi(\xi,\eta,\zeta,t) = \phi^T(\xi,\eta,\zeta)\psi^e(t) = \sum_{i,j,k=0}^{n} \phi_i(\xi)\phi_j(\eta)\phi_k(\zeta)\psi^e_{ijk}(t)$$

$$\mathbf{Q^e} = \int_{\Omega^e} \phi\phi^{\mathbf{T}} \mathrm{d}\Omega; \quad \mathbf{H^e} = \int_{\Omega^e} (\nabla\phi)(\nabla\phi)^{\mathrm{T}} \mathrm{d}\Omega; \quad \mathbf{b^e} \int_{\Gamma^e} \phi\nabla\psi\cdot\mathbf{n}\,\mathrm{d}\Gamma$$

  – Essence of spectral-elements is the choice of ϕ and the quadrature rule
  – ϕ by Lagrangian interpolants

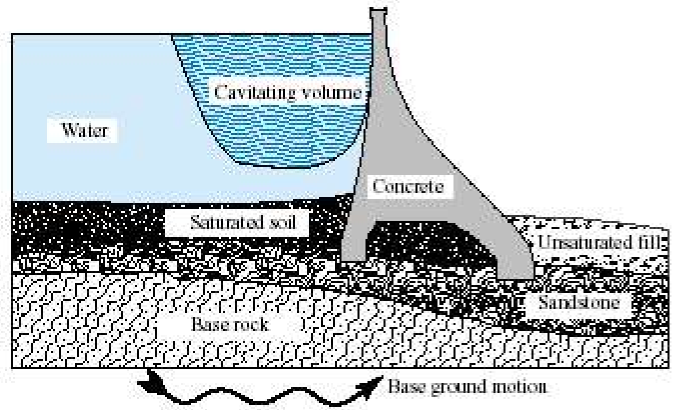
$$\phi_i(\xi) = \frac{-(1-\xi^2)P_n(\xi)}{n(n+1)P_n(\xi_i)(\xi-\xi_i)}$$

  – With Gauss-Lobatto-Legendre (GLL) quadrature
  – Element node locations are coincident with the quadrature points (Makes Q diagonal)

Fluid: Bilinear (Cavitation)

- Cavitation is the spontaneous vaporization of a fluid. It happens when the fluid pressure < vapor pressure.
- Water's vapor pressure << atmospheric pressure
- Simple Mathematical model is that if the total pressure is negative then it is just zero.

$$p = \begin{cases} p^H + c^2 s & \text{if } s > -p^H/c^2 \\ 0, & \text{otherwise} \end{cases}$$



Coupling for CAFE/CASE

- Structure Forcing from fluid

$$\mathbf{f_i^e} = \gamma_i^e \int_{\Gamma_{wet}} \mathbf{N}\,\overline{\mathbf{p}}\,\mathrm{d}\Gamma; \quad \mathrm{i}\,\epsilon\,(\mathrm{X} < \mathrm{Y} < \mathrm{Z})$$

$\overline{\mathbf{p}}$: Average pressure over wet element face

γ_i^e: Cosine of angle between structure normal and ith cartesian dirction

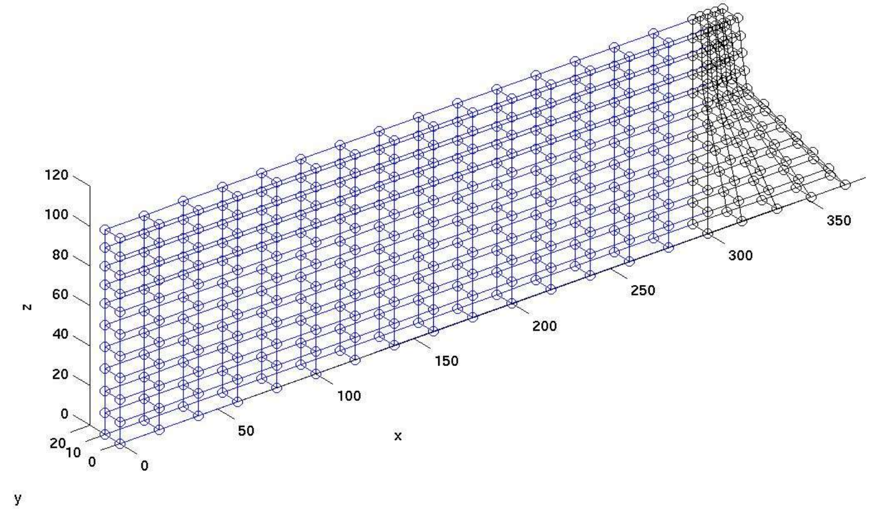
- Fluid Forcing from structure

$$\mathbf{b^e} \int_{\Gamma^e} \phi \nabla \psi \cdot \vec{\mathrm{n}}\,\mathrm{d}\Gamma \qquad \nabla \psi = -\rho\,\overline{\mathbf{u}}_{\mathbf{si}}$$

$\overline{\mathbf{u}}_{\mathbf{si}}$: Average normal structure displacement at the center of the wet element

NRB on the Fluid (PWA)

- Require the NRB to be transparent to incoming incident waves.



- PWA

$$\mathbf{p}_{\mathbf{nrb}} = \rho\, c\, \dot{\mathbf{u}}_{\mathbf{nrb}} \cdot \vec{n};$$

$$\mathbf{u}_{\mathbf{nrb}} \cdot \vec{n} = \frac{1}{\rho\, c} \mathring{\mathbf{p}}_{\mathbf{nrb}}$$

- Predict → - Code gives p → - Correct

$$\mathbf{u}^{\mathbf{n+1}}_{\mathbf{nrb}} \cdot \vec{n} = \mathbf{u}^{\mathbf{n}}_{\mathbf{nrb}} \cdot \vec{n} + \Delta t \frac{\mathbf{p}_{\mathbf{nrb}}}{\rho\, c}$$

$$\nabla \psi \cdot \vec{n} = -\rho\, \mathbf{u}_{\mathbf{nrb}} \cdot \vec{n};$$

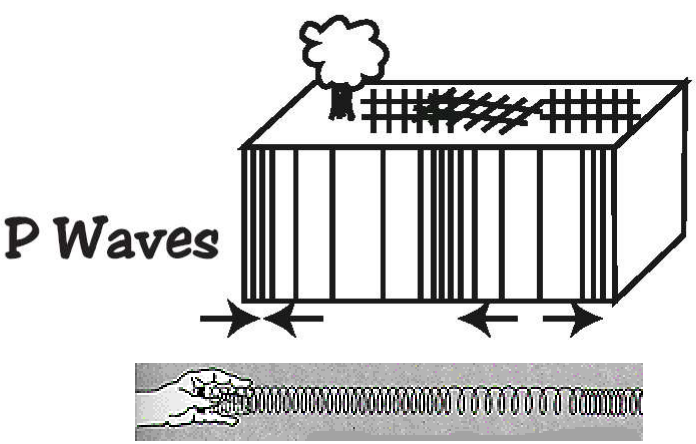
$$\mathbf{b}^{\mathbf{e}} \int_{\Gamma^e} \phi \nabla \psi \cdot \vec{n}\, d\Gamma$$

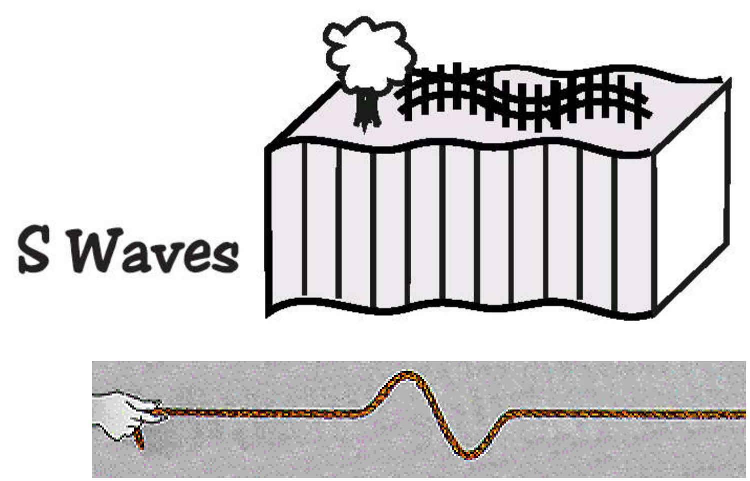
$$\Delta \mathring{p}_{NRB} = \frac{\Delta t}{2} \left(p^{n+1}_{NRB} + p^{n}_{NRB} \right);$$

$$u^{n+1}_{nrb} \cdot \vec{n} = u^{n}_{nrb} \cdot \vec{n} + \frac{\Delta \mathring{p}_{NRB}}{\rho\, c}$$

Viscous Damping Boundary Method (VDB)

- Main concept is to put viscous dampers at the boundary of the finite model.
- Used for Elastic Infinite Media (Soil)
- Earthquake induces P-waves and S-waves



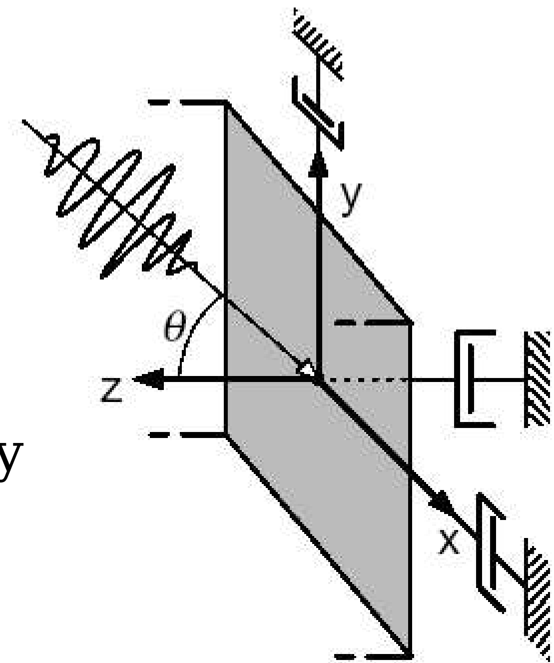


Which one is faster? Which can travel through the center of the earth?

Viscous Damping Boundary

- Assuming a passage of the wave at boundaries, the model gives proper stress components on the boundary.

$\sigma_{zz} = a\rho V_p u_z$: Primary wave
$\tau_{zx} = b\rho V_s u_x$: Secondary wave
$\tau_{yz} = b\rho V_s u_z$: Secondary wave



$$V_p = \sqrt{\frac{(1-\nu)E}{(1-2\nu)(1+\nu)\rho}};$$
$$V_s = \sqrt{\frac{E}{2(1+\nu)\rho}}$$

E: modulus of elasticity
ν: Poisson's ratio

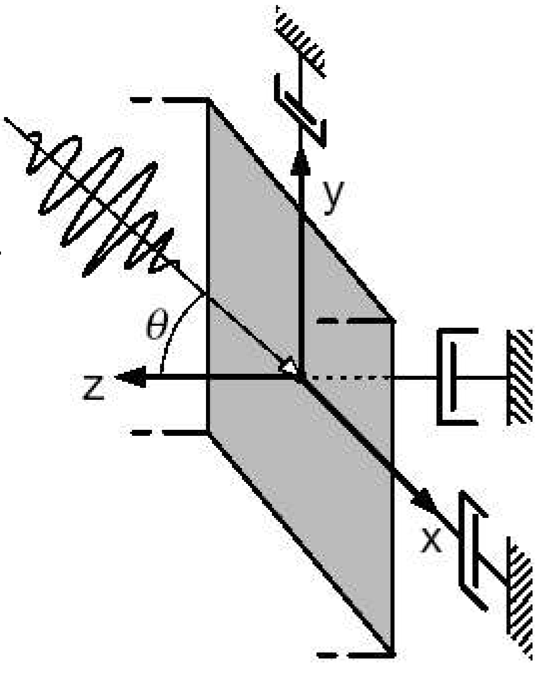
- Where a, b are non-dimensional parameters.
  - Value = 1 most effective to absorb the reflecting energy for small incident angles (θ)

Viscous Damping Boundary

- With the stress components in mind.
- The following lumped damping term is added to boundaries damping terms

$$C_{ii} = \int a \rho V_p d\Gamma : \quad \text{P-wave, z direction}$$

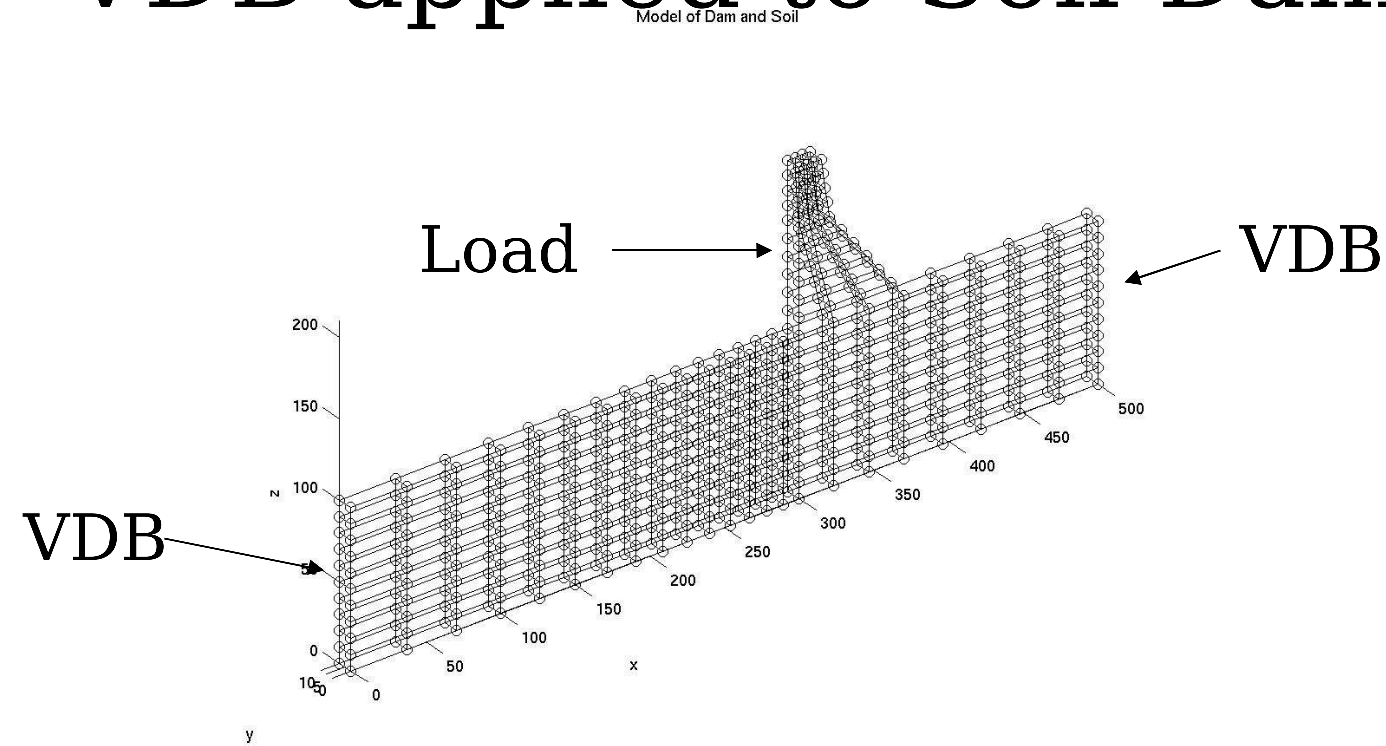
$$C_{ii} = \int b \rho V_s d\Gamma : \quad \text{S-wave, x or y direction}$$



$$\boldsymbol{M}\ddot{\boldsymbol{x}}_s + \boldsymbol{C}\dot{\boldsymbol{x}}_s + \boldsymbol{K}\boldsymbol{x}_s = \boldsymbol{f}_w - \boldsymbol{M}\boldsymbol{I}_x a_q$$

- Notice only applied to diagonal components and only at boundary.
- Take care in making sure the appropriate component.

VDB applied to Soil-Dam



Extended Mesh

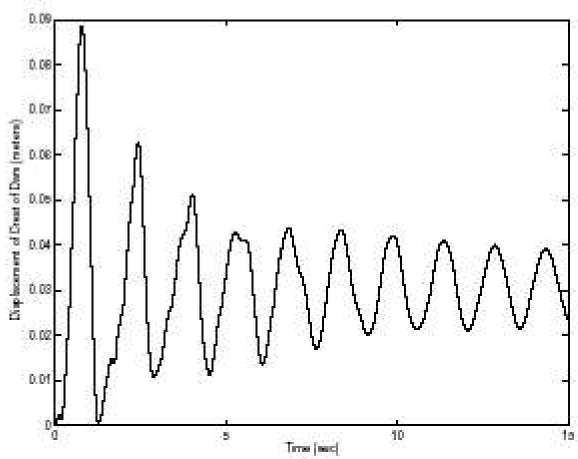


Figure 3: Displacement of Dam Crest with an Extended Mesh

- Nearly identical.
- Only worked with damping in the media

Regular Mesh with VDB

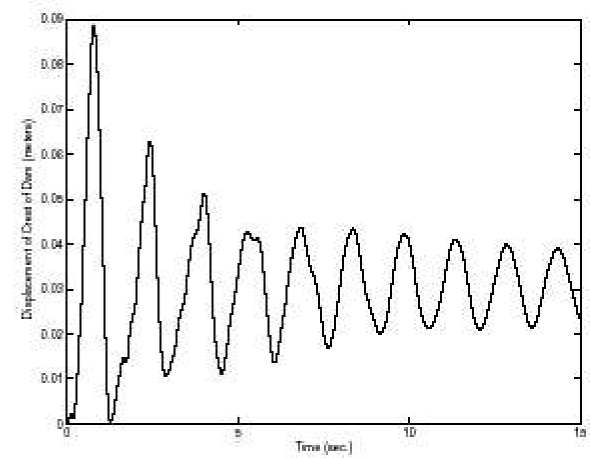
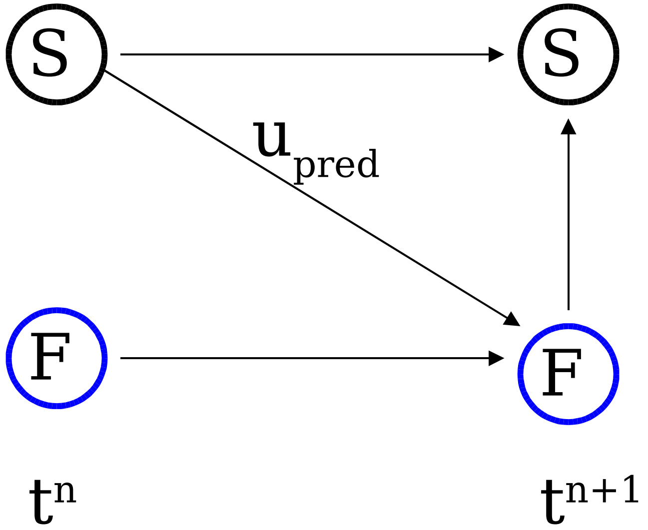


Figure 4: Displacement of Dam Crest with Viscous Boundary Method

Time Stepping:

Staggered Partition

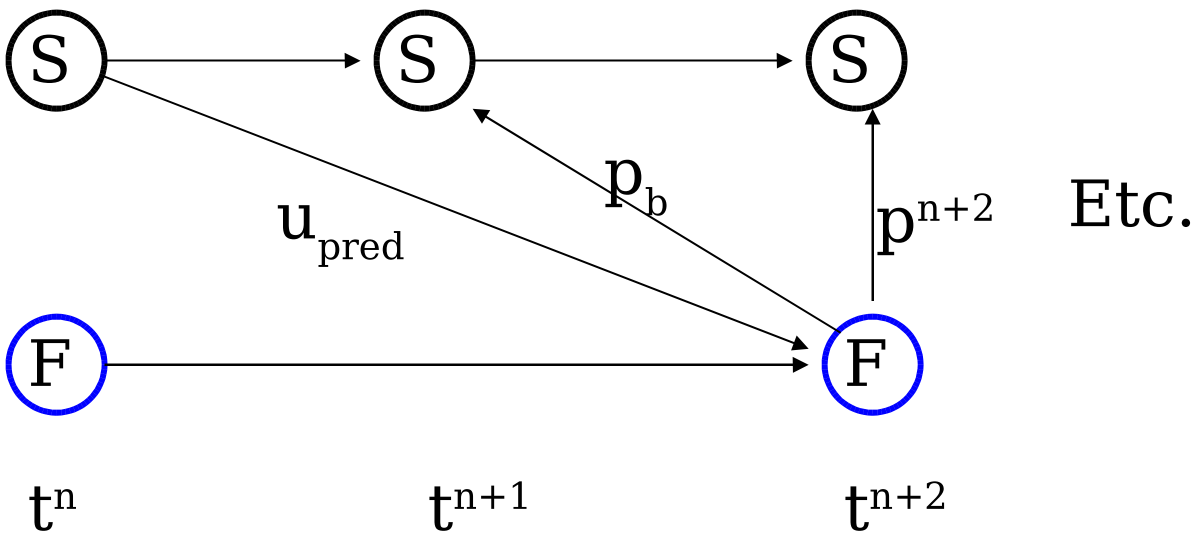


Etc.

$$u_{pred} = u^n + \Delta t \, \dot{u}^n$$

Time Stepping:
Staggered Method add subcycling

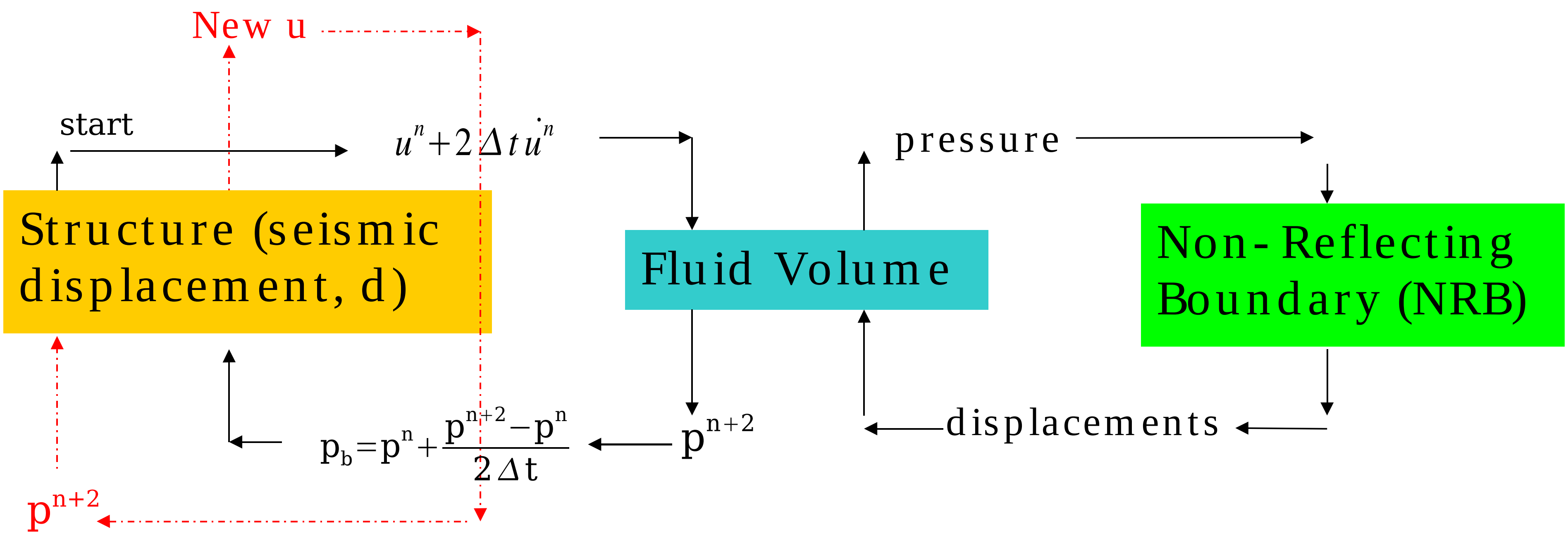
Example: Structure time = Δt; Fluid time = $2\Delta t$;
Subcycling = 2



$$u_{pred} = u^n + 2\Delta t\,\dot{u}_n$$

$$p_b = p^n + \frac{p^{n+2} - p^n}{2\Delta t}$$

Time Stepping: Staggered Method add subcycling



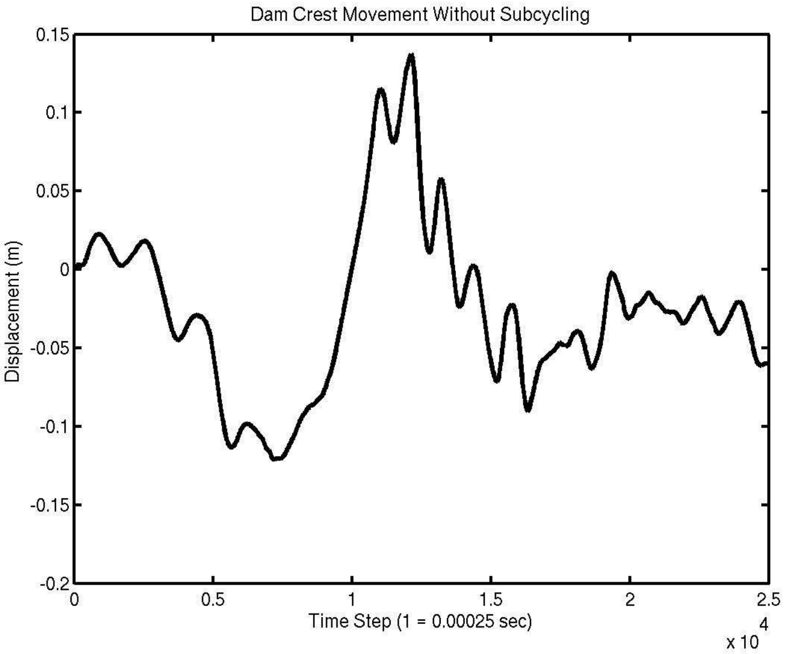
- Used Subcycling to reduce computational time and have the time increments near the upper bound of the stable time region for the fluid and the structure. Using a Euler Scheme for the prediction.

Subcycling

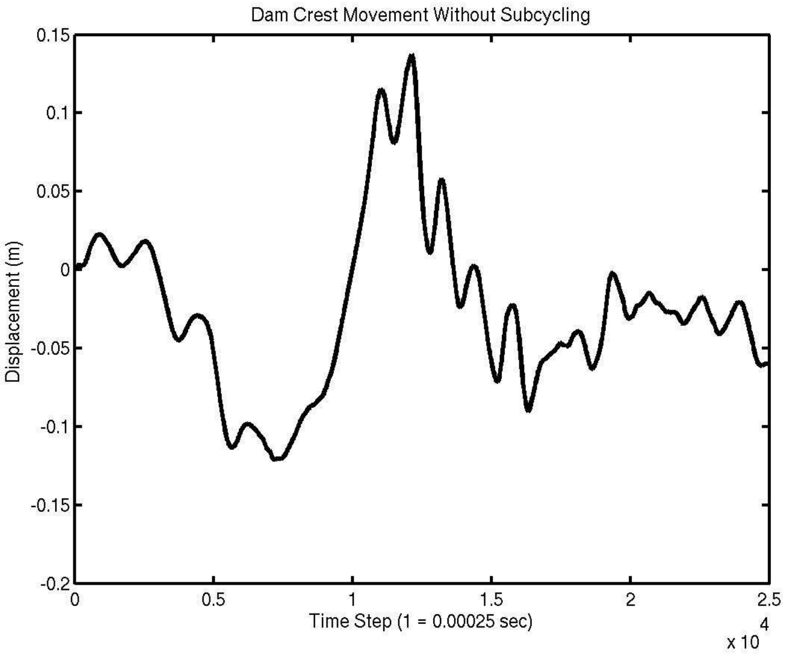
- Trying to integrate the different systems with their critical time for stability
- Critical time (fluid,3.5e-3 sec.)>(structure,2.5e-4 sec.)
- We predict fluid displacements with Euler Scheme.

Comparison of deflections with subcycling and without.

Dam Crest Movement without subcycling

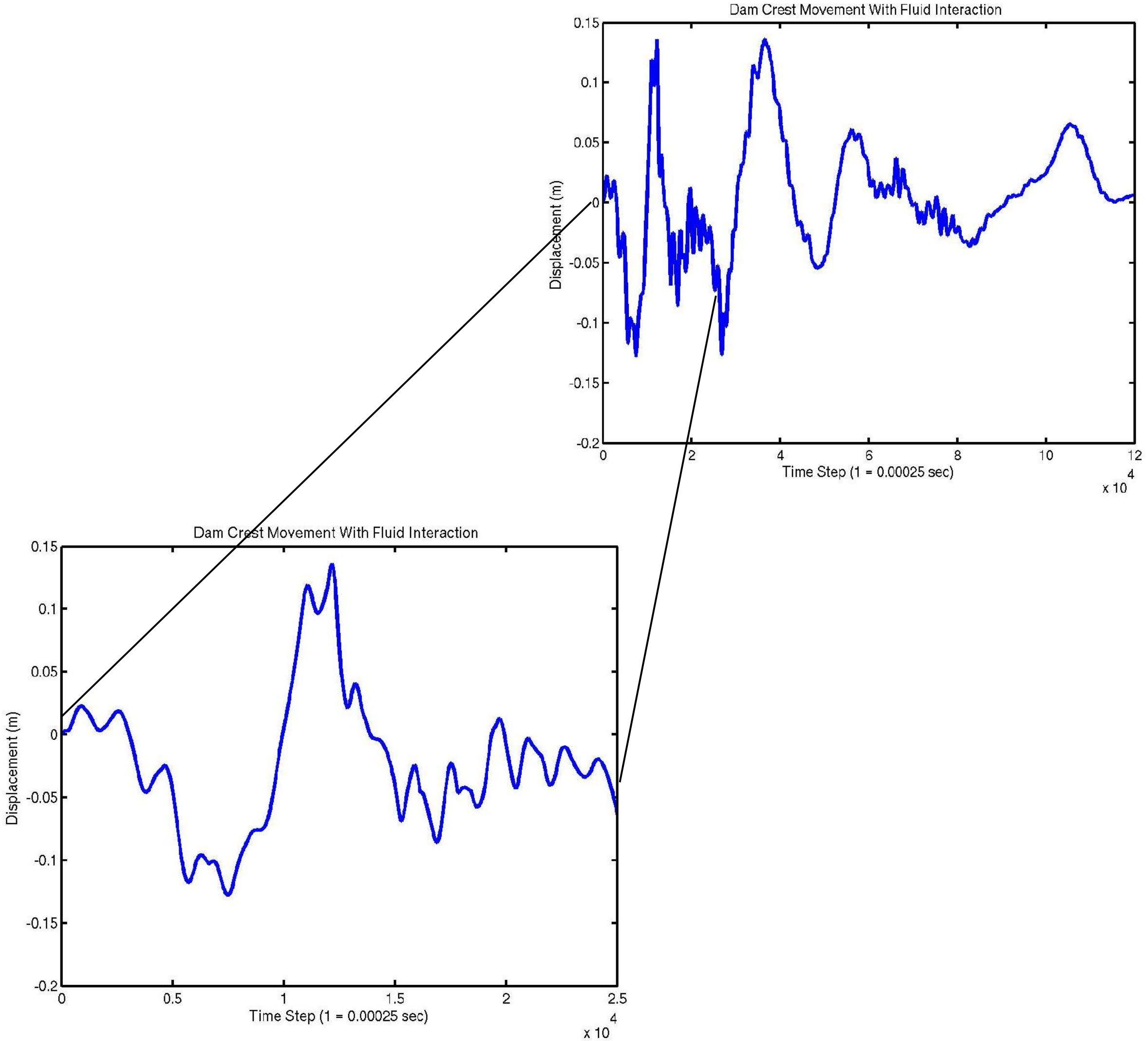


Dam Crest Movement with subcycling

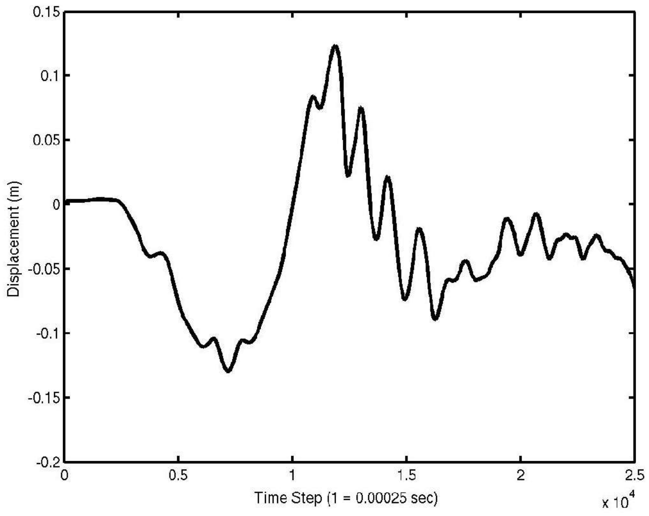


Output for Dam Crest Under Seismic Load no Soil

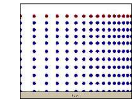
With Fluid Interaction



Without Fluid Interaction

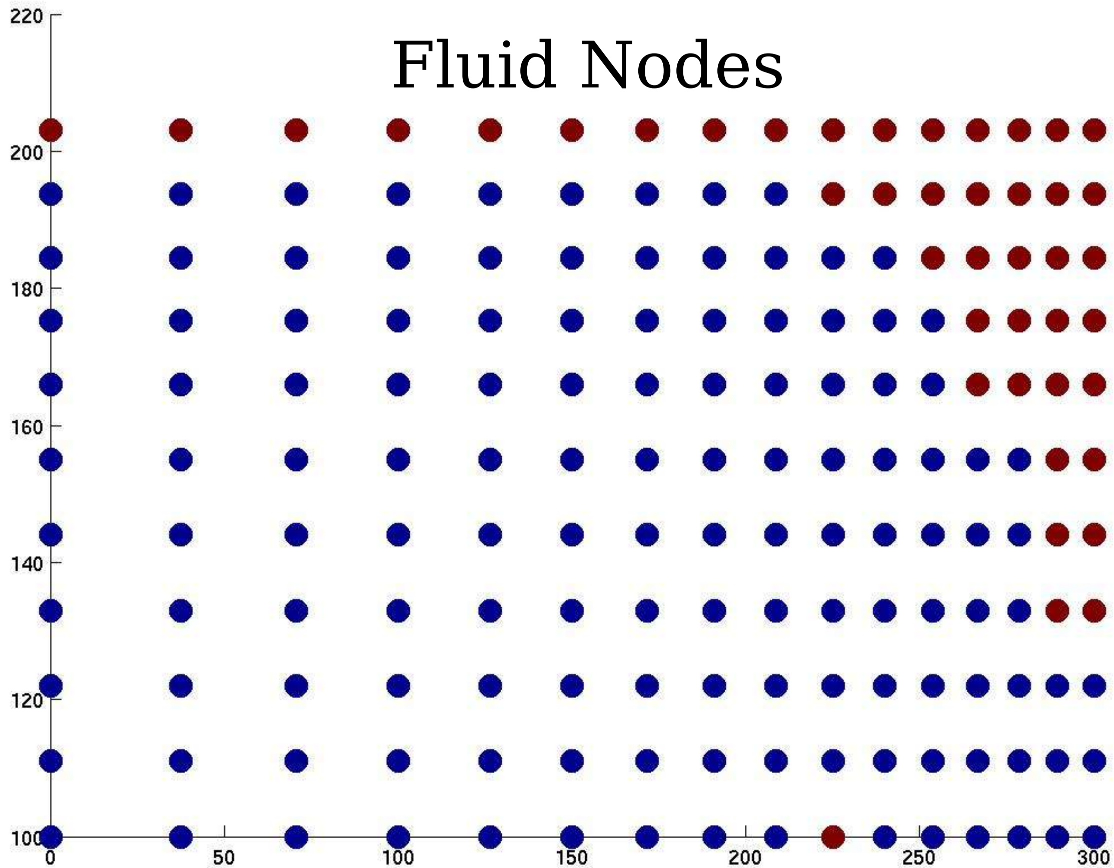


CAVITATION REGION



Fluid Nodes

Silent Boundary



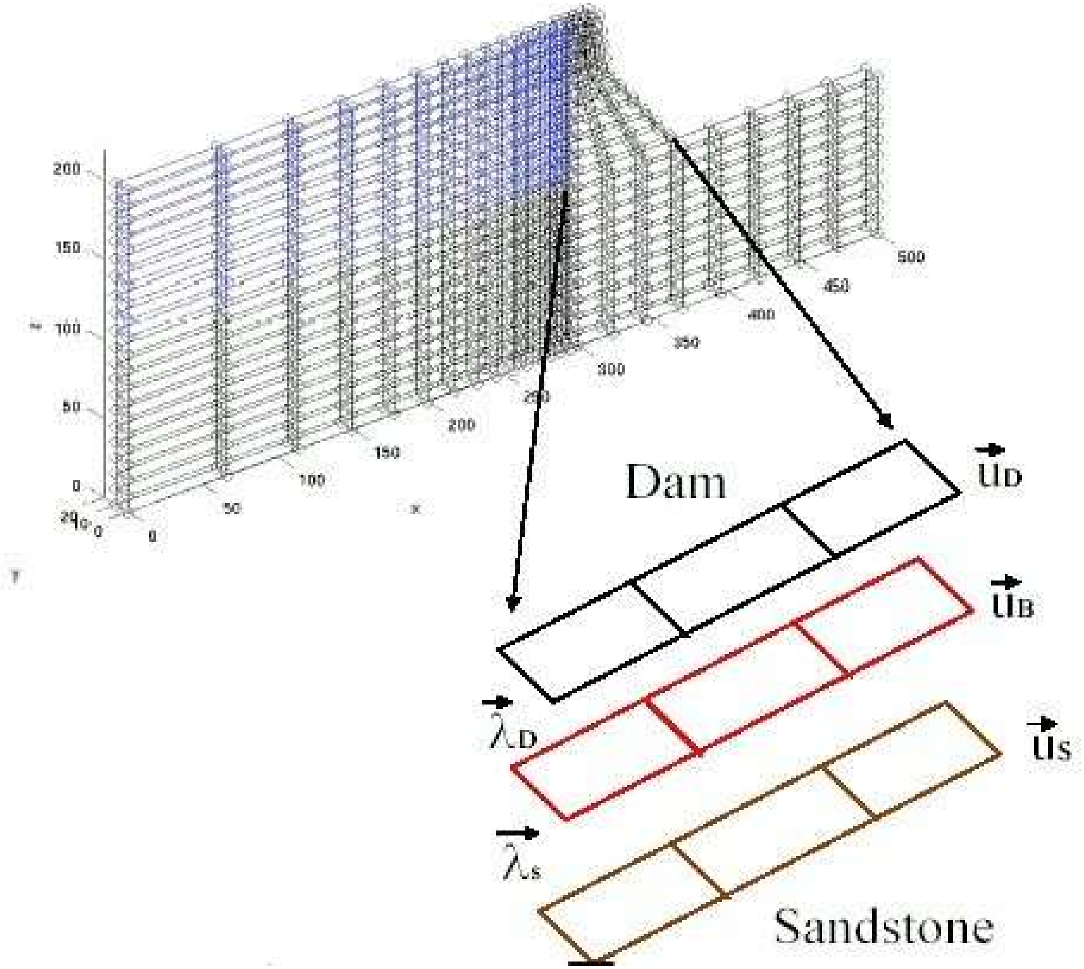
Dam Face

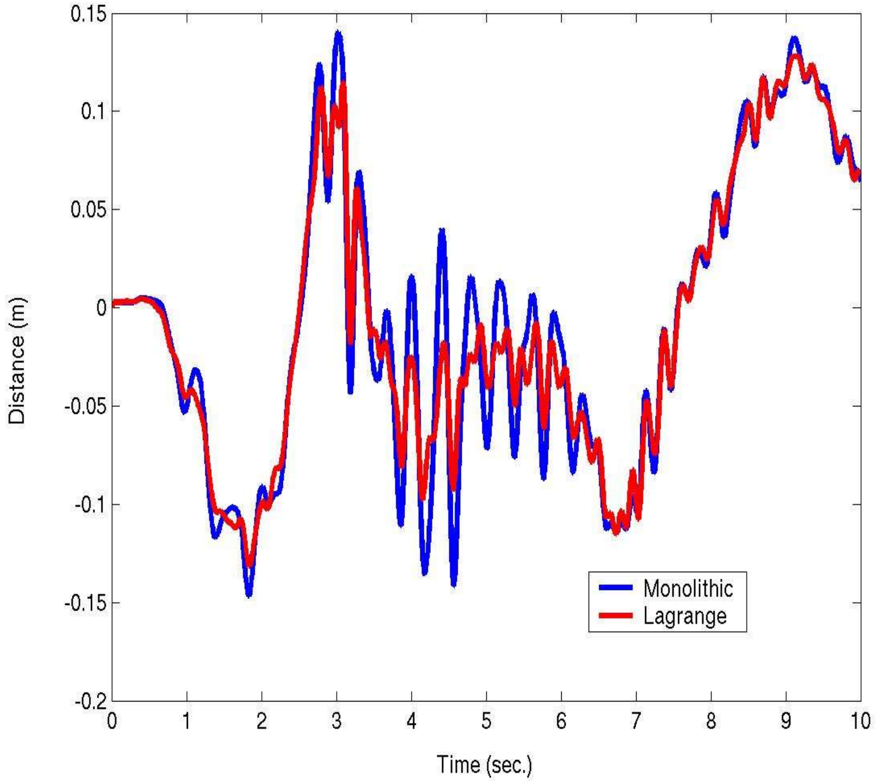
Soil

Proposal for Future Work

- Examine the LLM method in 2-D problem with different fluid formulations that maintain irrotationality as well as the modularity concept
  - Examine the spurious non-zero frequencies
  - Develop the best strategy for LLM in FSI
- Implement the LLM method in 3-D on the Dam (Fluid-Dam-Soil) and compare to present model
  - Test with Parallel Computation
  - Include Subscyling (Stability and Accuracy Analysis with Dr. Sprague)
  - Include <u>non-matching meshes</u>
  - Implement and assess the effects of dynamic model reduction
- Develop rules and clear procedure for LLM method with FSI and different soil media

LLM Method to base of Dam and Sandstone





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Questions?