ASEN5519 Topics in Multiphysics Modeling

Classification of Shape (or Basis) Functions

1.1 The Fourier transform:

$$Y(\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} y(x)e^{-j\omega x}dx \Longleftrightarrow \quad Y(k) = \frac{1}{N}\sum_{n=0}^{N-1} y(n)e^{-j(2\pi kn/N)}, \quad k = 0, 1, \ldots, N-1$$

$$y(x) = \int_{-\infty}^{\infty} Y(\omega)e^{j\omega x}d\omega \quad \Longleftrightarrow \quad y(n) = \sum_{k=0}^{N-1} Y(n)e^{j(2\pi kn/N)}, \quad n = 0, 1, \ldots, N-1$$

Observe that the basis function, $\{e^{j(2\pi kn/N)}, \quad n = 0, 1, \ldots, N-1\}$, spans the entire domain. In other words, it is a global interpolation function.

1.2 The Fourier series:

$$y(x) = a_0 + \sum_{n=1}^{\infty}(a_n \cos\frac{2n\pi x}{L} + b_n \sin\frac{2n\pi x}{L}), \quad 0 \le x \le L$$

2. The finite element method:

$$y(x) = \sum_{k=1}^{N} N_k(x)\, y_k, \qquad x_i \leq x \leq x_j$$

The FEM basis function, $\{N_k(x)\}$, spans the interval $x_i \leq x \leq x_j$. In other words, it is a compactly supported, local interpolation function.

Of all the forms of the shape functions proposed and tried in the early developmental stage of the finite element method, polynomials

$$N_k(x) = \langle\, 1 \quad x \quad x^2 \quad x^3 \quad \ldots \quad x^p \,\rangle \tag{1}$$

which satisfies the sufficient conditions for a continuum to be in self-equilibrium or rigid-body motion states.

3. The wavelet transform:

$$
\begin{aligned}
y(x) &= b_0\phi(x) + \sum_{J,k} b_{2^J+k} \cdot \psi(2^J x - k) \\
&\Downarrow \\
y(x) &= b_0\phi(x) + b_1\psi(x) \\
&\quad + [b_2 \;\; b_3] \begin{Bmatrix} \psi(2x) \\ \psi(2x-1) \end{Bmatrix} \\
&\quad + [b_4 \;\; b_5 \;\; b_6 \;\; b_7] \begin{Bmatrix} \psi(4x) \\ \psi(4x-1) \\ \psi(4x-2) \\ \psi(4x-3) \end{Bmatrix} + \\
&\quad + ... + \sum_{k=0}^{J} b_{(2^j+k)}\psi(2^J x - k)
\end{aligned}
\tag{2}
$$

where ψ are the dilated and translated wavelet basis functions, ϕ is the scaling function (also a mother wavelet basis function). The result of the inner product of y with the basis functions is the transform of that signal, b, termed the wavelet coefficients.

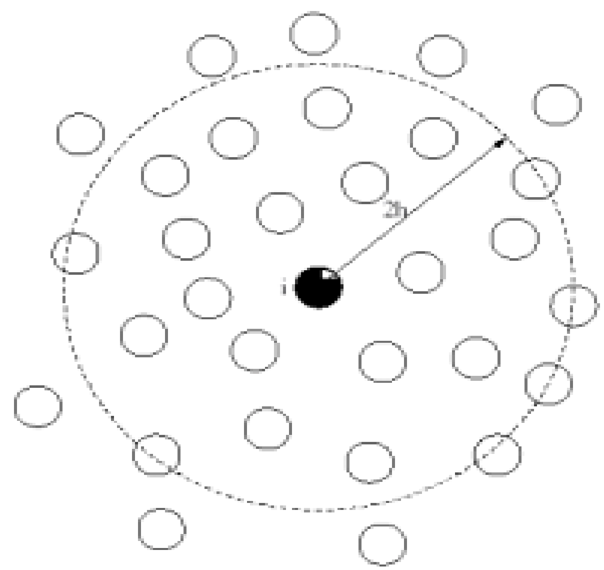
Note that the first two function $\{\phi(x), \psi(x)\}$ span the entire domain, and the subsequent groups of the waelet basis functions span a half, quarter, so on. Hence, the wavelet transform is in essence a multi-level or multiscale method.

4. The SPH basis function (node-based interpolation):

$$y(x) = \sum_{j=1}^{N} y(x_j)\ W(x - x_j, h), \int W(x - x', h)\ dx' = 1, \ \lim_{h \to 0} W(x - x', h) = \delta(x - x')$$

where h is called the smoothing length. $W(\mathbf{x}, h)$ is a C^2 spline based interpolation kernel of radius $2h$, and normally approximates the shape of a Gaussian function with compact support.

Other methods such as meshless methods, reproducing kernel methods, particle finite element method, etc. are essentially node-based methods.



5. The partitioned multiscale method:

$$y(x) = y_c(x) + y_f(x) + \tilde{y}_r(x)$$

$y_c(x)$: interpolation of coarse meshes(slowly varying components)

$y_f(x)$: interpolation of fine meshes(rapidly varying components)

$y_r(x)$: unresolved components

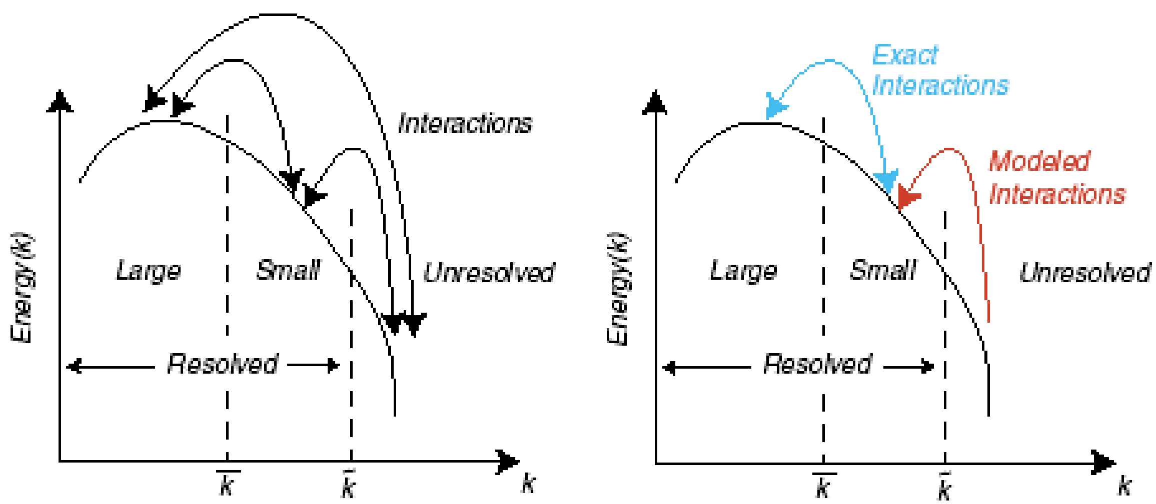


Figure 1. Schematic of interscale transfers in VMS modeling of turbulence. Left: no model, Right: model.

From : A Mathematical Framework for Multiscale Science and Engineering: The Variational Multiscale Method and Interscale Transfer Operators

Pavel Bochev, Mark Christon, S. Scott Collis, Richard Lehoucq, John Shadid, Alex Slepoy, and Greg Wagner, SAND2004-2871, Printed June 29, 2004

Basis Functions and Methods

We have examined five basis functions that are widely used in approximating physical variables in mechanics. It should be noted that *a new method* must be based on a new basis function, otherwise a researcher cannot claim that he or she has developed a new method. One rare exception, an important one though, may be in the computer implementation. For example, both the finite difference and finite element methods utilize the same basis functions in a broader sense. However, whereas the finite difference method applies the basis function to approximate the field variable at a node at a time, the finite element method approximates the field variables over a closed spatial domain, viz., element. This subtle yet important difference in the implmentation procedures resulted in a dramatic difference!

The so-called XFEM (eXtended finite element method) is a latest attempt to combine the nodal and elemental features in the implementation details.

To conclude, when someone entreats youwith a new method, ask or determine if the method truly employs a new basis functions. It is the same intime integration method.